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Effect of the Reactance Term on the Charge/discharge Electrical Measurements using Cylindrical Capacitive Probes

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Abstract: In this study, operation of the contact Cylindrical Capacitive Sensor (CCS) for the reactance capacitance measurement of the water liquids by using the charge/discharge measurement module is investigated. The effect of geometric factor on the capacitance measurement of different liquid samples is also studied. The measured capacitances for the distilled, mineral, tap and salt water samples are reported by using two contact CCSs with the active length of 16 and 74 mm. The measured capacitances by small contact CCS for the salt water is 1.76 \( \mu F \) and by big contact CCS is 47.8 \( \mu F \), which shows a big difference for different probes. In another study, the effects of the liquid conductance on the capacitance values are theoretically investigated. The obtained results for different water liquids show that by increasing the conductance of the sample fluids both the stream discharge current and the stored load charge on the cylindrical electrodes are decreased, accordingly.

Key words: Capacitance measurement, electrode, contact type, liquid sample

INTRODUCTION

In recent years capacitive sensors have found many applications (Baxter, 1996). For the capacitance measurements by contact CCS there are many methods such as LCR module, charge/discharge, resonance, oscillation, AC bridge and capacitance to phase conversion method (Ashrafii and Gohnabi, 1999) that are used in different applications (Rodjegard and Loof, 2005; Huang et al., 1988). Measurement and control of the liquid level are important tasks in industrial applications (Reverter et al., 2007). For example, application of a pressure sensor is described by Moe et al. (2000). In another study, the amount of the water content in the soil sample is determined by using the electrical conductivity measurements (Persson and Hariy, 2003). In a paper operation of a cylindrical sensor to investigate the electrical properties of water samples is given (Gohnabi and Azimi, 2008a). In another investigation electrical conductivities of the pure water and different electrolytes are reported (Gohnabi and Sharifian, 2010). Most of our reported experiments are based on the contact probe designs (Sharifian and Gohnabi, 2010; Gohnabi and Azimi, 2008b).

Behzadi and Gohnabi (2009) described the variation of the reactance capacitance parameter of the liquids with temperature in the range of 17-60°C for the water and water mixtures. Following that study the effects of the conductivity on the capacitance measurements of different water samples are reported by Behzadi and Gohnabi (2010). Comparison of the performance of the invasive and non-invasive cylindrical probe is reported recently (Behzadi and Gohnabi, 2011). Effects of the guard ring electrodes of the invasive CCS investigated that revealed important information about the wetness of the electrodes and electrode-liquid interaction process.

Reported experiments indicate that the measured capacitance values are very sensitive to the electrical conductivity of the liquids and this has led to study the reactance capacitance in detail. The goals of this study were threefold. First aim was to describe operation of the small and big contact probes, which are used in similar measurements for the investigation of the geometric factor effect on the capacitance measurement. The second point was to compare the obtained experimental results of the capacitance measurements of different liquid samples. The third aim was to develop a theoretical model to formulate the reactance capacitance effect on the charge/discharge measurement system and compare these results with the experimental measurements.

MATERIALS AND METHODS

A photograph picture of the proposed big/small contact CCSs is shown in Fig. 1. As can be seen in Fig. 1, in this design, a cylindrical geometry is chosen and

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aluminum materials are used as the capacitor tube electrodes. The diameter of the inner electrode for both probes is 14 mm and the inner diameter of the outer main electrode is 22 mm and has a thickness wall diameter of 4 mm. The radial gap between the two tube electrodes is about 4 mm and the overall diameter of the probes is about 30 mm. The overall height of the big probe is 100 mm while the active part has a length of 74 mm and the overall height of the small probe is 16 mm. The measured full liquid volume for the big probe is 17 cc and for the small probe is about 3.5 cc.

The experimental set up for the measurements includes the constructed cylindrical cell probe and a Digital Multimeter (DMM) modules (SANWA, PC 5000), that can be interfaced to a PC. The software (PC Link plus) allows one to log measuring data into PC through RS232 port with the digital multimeter PC-5000 series. It provides a function for the capacitance measurements using the charge/discharge method and capacitances in the range of 0.01 nF to 9.99 mF can be measured with a resolution of about 0.01 nF. The nominal input impedance of the DMM is about 10 MΩ and 30 pF. The specified accuracy of the DMM for 50-500 nF capacitance range is about ±(0.8% rdg+3dgt), for 500 nF-5 μF is about ±(1% rdg+3dgt) and measuring system is capable of measuring precisely the capacitance values in the range of 0.01 nF to 50 mF. For resistance measurements, it is possible to measure values for the range of 50 Ω to 50 MΩ. The specified accuracy of the DMM for 5-500 KΩ resistance range is about ±(0.2% rdg+2dgt) and ±(1.0% rdg+3dgt) for the 5 MΩ range and ±(1.5% rdg+5dgt) for the 50 MΩ range.

In Fig. 2, the equivalent circuit of the contact cylindrical probe and that of the DMM measuring modules is displayed.

The equivalent circuit of the contact cylindrical probe considered as a sample dielectric capacitance $C_x$ in parallel with a sample resistance, $R_s$, which results in the reactance capacitance of the water liquids. The charge operation is based on the charging of an unknown sample capacitance under study $C_x$ to a supply voltage $V_s$ through a resistance $R_{in}$ that is about 6.65 MΩ for this circuit. Discharge process starts when the stored charge load transfers through the contact probe to the parallel resistances network (C). In the parallel resistances network, the stored load on the cylindrical electrodes discharges from a resistance step range of 1 KΩ when C is approximate 99 μF. Then it discharges through a resistance 10 MΩ when the discharge capacitance C is approximately 99 nF.

In order to measure the liquid sample conductance a conductometer (Sension 5) is used in this study. It can measure the conductance, Total Dissolved Solid (TDS) of the liquid samples at room temperature. The Sension 5 conductometer is a contacting style conductivity sensor in which two measuring electrodes are in direct contact.

![Fig. 1: A photograph picture of the big and small contact cylindrical capacitive sensors](image)

![Fig. 2: Equivalent circuit of the contact cylindrical probe and measuring module (DMM)](image)
with the solution. Electrical conductivities in the range of resolution of approximately 0.1 μS cm⁻¹. The specified accuracy of the Sension 5 for electrical conductivity is about ±0.5% of the full reading scale.

RESULTS

In the initial experiment, the capacitance measurement results by the big probe \( C_b \) and small probe \( C_s \) of different water liquids are presented. Figure 3 shows that the measured capacitance for the distilled water by big probe, which is 2.28 μF and for the small probe is 0.45 μF. The measured capacitance by the small probe is 19.7% of the measured capacitance of the big probe. The measured capacitance for the salt water by big probe is 47.8 μF while by the small probe is 1.76 μF that is 3.7% of the measured capacitance by the big probe. As can be seen in Fig. 3, by increasing the length of the cell probe, the measured capacitance for all water liquids is increased. The difference between the measured capacitances of the two probes for the distilled water is 1.83 μF, mineral water 20.36 μF, tap water 37.66 μF and for the salt water is 46.04 μF.

The capacitance measurement for the cylindrical cell probe depends on the permittivity \( \varepsilon \) (F/m) of the liquid, geometric factor of probe (GF) and reactivity capacitance that depends on the conductivity \( \sigma \) (S/m) of the liquid as well as the geometrical factor.

\[
C_n = f_1(\varepsilon, \text{GF}) + f_1(\sigma, \text{GF})
\]

Precise formulation of the electrical capacitance for a cylindrical capacitive sensor was reported by Azimi and Gornabi (2009). Using Coulomb law for the water liquids with same permittivity \( \varepsilon = \varepsilon_0 \) at \( T = 25^\circ\text{C} \) the geometric factor of the probe can be obtained from:

\[
f_1(\text{GF}) = L \left\{ \frac{z - \frac{1}{2} \sqrt{a^2 + z^2 - 2a^2 \cos \phi' + z^2}}{a^2 + z^2 - 2a^2 \cos \phi' + z^2} \right\}^n + \left\{ \frac{z - \frac{1}{2} \sqrt{b^2 + z^2 - 2b^2 \cos \phi' + z^2}}{b^2 + z^2 - 2b^2 \cos \phi' + z^2} \right\}^m
\]

where, \( L \) is the length of probe, \( a \) is the inner electrode radius and \( b \) is the outer electrode radius. The calculated ratio of \( C_b/C_s \) by coulomb law for all the water liquids is approximately 1.644 while for the experimental measurement, the ratio of \( C_b/C_s \) for the distilled water is 5.066, mineral water 15.139, tap water 25.776 and for the salt water is 27.159. Equation 1 shows that the difference between the calculated and experimental results of the ratio \( C_b/C_s \) that is due to the reactance capacitance \( f_1(\sigma) \) of different water liquids. The ratio of \( C_b/C_s \) by considering the reactance capacitance for the distilled water is 3.422, mineral water 13.495, tap water 24.132 and for salt water is 25.515. For a better comparison the ratio of \( C_b/C_s \) and electrical conductivity of different water liquids, are plotted together in Fig. 4. As can be seen in Fig. 4, the ratio of \( C_b/C_s \) is increased by increasing the electrical conductivity of the water liquids. Figure 4 clearly shows that the reactance capacitance has a significant role in the measured capacitances.

In another study to investigate the effect of the parameter \( f_1(\sigma, \text{GF}) \) the role of such a factor on the charge/discharge measurement system is investigated. The equivalent circuit of the contact cylindrical probe and that of the DMM measuring modules is considered for this analysis (Fig. 2). The equivalent circuit of the contact cylindrical probe is considered as a dielectric capacitance

![Fig. 3: Comparison of the measured capacitances by the big and small probes](image)

![Fig. 4: Comparison of the electrical conductivity of different water liquids with the ratio of \( C_b/C_s \)](image)
C\text{r} related to the f\text{r} (\rho, GF) term. This capacitance is in parallel with the resistance R\text{m} which depends on the reactance capacitance term f\text{j} (\sigma, GF) of the water liquids. As described, the charge operation is based on the charging of an unknown capacitance under study C\text{r} to a supply voltage V\text{s} through a resistance R\text{m} (about 6.65 M\Omega). When the contact probe is filled up by a perfect ideal dielectric liquid with \sigma = 0 the electrical charge that is stored on the cylindrical electrodes can be obtained from:

\[
Q\text{m} = C\text{r}V\text{s} \left(1 - \exp \left(-t/\tau\text{j}\right)\right)
\]

and the charge time constant \tau\text{j} is defined by:

\[
\tau\text{j} = R\text{m}C\text{r}
\]

When the contact probe is filled up by a dielectric liquid that has an electrical conductivity the electrical charge, which is stored on the electrodes, is decreased to:

\[
Q\text{m} = \left[R\text{r}/(R\text{r} + R\text{m})\right] C\text{r}V\text{s} \left(1 - \exp \left(-t/\tau\text{j}\right)\right)
\]

Whereas, the charge time constant \tau\text{j} is described via:

\[
\tau\text{j} = \left[R\text{r}/(R\text{r} + R\text{m})\right] \tau\text{j}
\]

The effect of the sample resistance, R\text{m}, can be neglected if the turn-on resistance (R\text{m}) of the charge switch is small compared to the R\text{r}. Using Coulomb’s law for the medium filling with the water liquids the electrical resistance of the small contact cylindrical probe can be obtained from Eq. 7:

\[
R\text{r} = \frac{\rho}{8\pi K f\text{r}(GF)}
\]

where, \rho is the resistivity and K is the dielectric constant of the filling medium, L is the active length of the probe, a is the inner electrode radius and b is the outer electrode radius. The calculated resistance by Coulomb’s law for the distilled water is 107.373 K\Omega, mineral water is 1.642 K\Omega, tap water is 1.151 K\Omega and salt water is 0.117 K\Omega.

Figure 5 shows a comparison of the ratio of \tau\text{j}/\tau\text{j} (the effect of the liquid resistance on the charging time) for the distilled water, mineral water, tap water and the salt water. For a better comparison, the electrical conductance of different water liquids for the small contact probe is also shown in Fig. 5. The ratio of the charging time constants for two cases, \tau\text{j}/\tau\text{j}, for the distilled water is 1.6146 x 10^{-6}, mineral water is 246.9 x 10^{-6}, tap water is 173.1 x 10^{-6} and for the salt water sample is 17.6 x 10^{-6}. Comparative measurements of the calculated results show that in measurements by the contact CCS the ratio of \tau\text{j}/\tau\text{j} is decreased by increasing the electrical conductance of the water liquids. As can be seen in Fig. 5, the electrical conductance of the distilled water is 9.313 \muS, for mineral water is 609.013 \muS, for tap water is 868.809 \muS and for the salt water is 6024.096 \muS. The obtained results verify that the reactance capacitance term f\text{j} (\sigma, GF) causes the obtained decrease in the charge time constant.

In the next study, the reactance capacitance effect on the accumulated electrical charge, Q, which is stored on the capacitance electrodes is investigated. Figure 6 shows the stored charge load on the cylindrical electrodes as function of the charging time when the contact probe cell is filled up by the distilled water. For a better presentation of the result, the stored load on the cylindrical electrodes when the contact probe is filled up by a perfect dielectric sample (ultra pure distilled water with \sigma = 0 and dielectric constant K = 78.54) is also shown in Fig. 6. As can be seen in Fig. 6, the contact probe filled with the ideal dielectric medium without conductivity can be charged in
1.33 ms and the value of the stored load \( Q_{10} \) is 0.2384 nC. When the cylindrical gap is filled up by the regular distilled water it can be fully charged in 10.32 \( \mu \)s interval and the stored load \( Q_{20} \) is reduced to 1.846 pC. The notable difference in the two obtained values of the stored charges clearly shows that the reactance capacitance is causing the reduction of the stored load charge on the electrodes. On the other hand, the conductance effect of the gap medium shortens the charge time and therefore the circuit time constant.

In a similar experiment, the reactance capacitance effect on the discharge process is investigated. Discharge process starts when the stored load transfers through the discharge resistances to a network capacitance \( (C) \). In the parallel resistances network, the stored load on the cylindrical electrodes discharge through a resistance 1 \( K\Omega \) when, \( C \) is approximately 99 \( \mu \)F and discharges through a big resistance of 10 \( M\Omega \) when \( C \) is small in the range of 10-99 \( n\)F. For the case of a perfect dielectric medium with the zero conductivity the discharge stream current can be obtained from:

\[
I_t = \frac{(V_t}{R}) \left( \exp \left(-t/\tau_{id}\right) \right)
\]

where, \( \tau_{id} = RC \) is the discharge time constant and \( R \) is the discharge resistance that is approximately 10 \( M\Omega \). When the contact probe is filled up by the regular distilled water the discharge current stream can be obtained from the given relations:

\[
I_t = \left[ R_c \left( \frac{R_{eq}}{R_c} \right) \right] \left( \frac{V_t}{R} \right) \exp \left(-t/\tau_{id}\right)
\]

\[
\tau_{id} = \left[ R_c \left( \frac{R_{eq}}{R_c} \right) \right] \tau_{id}
\]

where, \( t = RC \) is the discharge time constant for the distilled water filling case and \( R \) is the discharge resistance (10 \( M\Omega \)). Figure 7 shows the comparison of the discharge stream current as a function of discharge time. In this presentation when the contact probe is filled up by the regular distilled water the electrical current is denoted by \( (I_1) \) and when filled up by the ideal dielectric medium with no conductivity is indicated by \( (I_2) \). As can be seen in the Fig. 7, for the perfect dielectric medium filling without any conductivity the discharge stream current is 0.6 \( \mu \)A and the stored load on the cylindrical electrodes is completely discharged after 2 msec. The discharge current stream for the regular distilled water filling with minor conductance is 4.648 \( n\)A and the discharge time is shortened to around 10.32 \( \mu \)s. The main reason for the decrease in the discharge current stream and time constant of the regular distilled is the conductance effect of the filling sample.

In the last study, effects of the liquid conductance on the stored load charge and discharge current stream of the different liquid samples such as the mineral water, tap water and salt water are investigated. For a better comparison, the stored load charge on the electrodes for different water liquids as function of the charge time are shown together in Fig. 8. Comparison of the stored charge load for different water liquids shows a notable decrease in the charge load charge by any increase in the electrical conductivity of the filling sample. The charge load stored for the regular distilled water filling is 1.846 pC, for mineral water 0.027 pC, for tap water 0.019 pC and finally for the salt water is 0.002 pC. On the other hand, the charge time for the distilled water is 10.32 \( \mu \)s, mineral water 0.15 \( \mu \)s, tap water 0.11 \( \mu \)s and for the salt water is 0.011 \( \mu \)s. Figure 8

\[ A = \frac{R_f}{R} \]  

(11)

where, \( R \) is the discharge resistance and the output voltage of the amplifier can be obtained:

\[ V_o = I_1 R_f \]  

(12)

with the substitution of Eq. 9 for current, \( I_s \) into Eq. 12 and using Eq. 11 for \( A \) we obtain:

\[ V_o = A \left[ \frac{R_f}{(R_m + R_m)} \right] V_s \exp (-\frac{1}{\tau_m}) \]  

(13)

The measured capacitance \( C_{in} \) can be obtained from:

\[ C_{in} = \frac{Q_s}{V_s} \]  

(14)

where, the maximum steady state load charge (infinite time), \( Q_s \) and \( V_s \) are, respectively given by:

\[ Q_s = A C_p V_s \]  

(15)

\[ V_s = A \left[ \frac{R_f}{(R_m + R_m)} \right] V_s \]  

(16)

By using Eq. 15 and 16 in 14 one can write:

\[ C_{in} = (1 + \frac{R_m}{R_f}) C_p \]  

C_{in} \text{ in Eq. 17 becomes:}  

\[ C_{in} = (1 + G_s R_m) C_p \]  

(18)

In the alternate way it can be written as:

\[ C_{in} = C_p + \tau_i G_s \]  

(19)

where, \( \tau_i \) is the time constant as described in the given formulation. In Eq. 19, two points are considerable. First point is that the measured capacitance by the contact probe has a direct relationship with the electrical conductance of the dielectric medium. The second point and perhaps the most important one, is that one can determine the reactance capacitance from:

\[ f_i (\sigma, DF) - \tau_i G_s \]  

(20)

Equation 20 shows a good agreement with the obtained results. As indicated in Fig. 5, the charge time constant and stored load on the electrodes decrease by increasing the electrical conductance. As indicated in

clearly shows that the stored load and charge time strongly depend on the reactance capacitance of the cell probe gap medium. Finally, Fig. 9 shows the comparison of the discharge current stream of different water liquids as function of the discharge time. As indicated in Fig. 9, by increasing the electrical conductivity both the stream discharge current and the discharge time show decrease, accordingly. The discharge current for the regular distilled water is 4.648 nA, while for the mineral water 0.072 nA, tap water 0.05 nA and finally for the salt water is 0.005 nA.

**DISCUSSION**

The amplification factor, \( A \), for the current transfer, \( I_s \) through the feedback resistance \( R_f \) of an amplifier (not shown in Fig. 2) can be found from:
Fig. 9, a comparison of the different water liquids shows that by increasing the electrical conductance, the decrease in the discharge current stream is very sharp. The obtained results verify that the reported contact CCS is a good tool to test the electrical characteristic of the single-phase liquids.

Now it is useful to compare the results of this study with the previous ones. A result of measured capacitance with different lengths shows that by increasing the probe length, the electrical capacitance increased. Figure 3 shows that the measured capacitance for the mineral water by big probe (74 mm), which is 1.44 μF and for the small probe (16 mm) is 21.8 μF. The measured capacitance for the case the liquid level is increase and decrease, is reported by Behzadi and Gholami (2011). Results show that the amount of the measured capacitance is higher for the case, which liquid level is higher. For example in this experiment, when the liquid level is about 37 mm the measured capacitance for mineral water is 23.74 μF and for the case the liquid level is about 74 mm is 32.84 μF. In another study, length effects on capacitance measurements of cylindrical cell probe are reported by Sharifian and Gholami (2010). In this study, the electrical capacitance of different water calculated by Gauss law and Coulomb law. As a result, the theoretical values obtained by Coulomb law are closer to the experimental results and by increasing in the probe length the measured capacitance is increased.

As can be seen in Fig. 4, the ratio of $C_2/C_1$ is increased by increasing the electrical conductivity of the water liquids. As indicated by Behzadi and Gholami (2011), measurement of the capacitance by invasive CCS for different solutions and mixtures shows that by increasing the sample electrical conductivity the measured capacitance is also increased. This indicates that for the invasive CCS the reactance capacitance is the dominating term in measurements. In general, there is a reasonable agreement between our experimental data and the one reported by Behzadi and Gholami (2011).

The effects of reactance capacitance term are negligible when one uses a dielectric without electrical conductivity or a non-contact cylindrical probe. Therefore, the measured capacitance for two cases related to the dielectric permittivity of liquid. Equation 19 shows that the measured capacitance has direct relationship with the dielectric permittivity of liquid. For non-contact cylindrical capacitive sensor the measured capacitance for solutions and mixtures increased by increasing the dielectric permittivity of the tested samples (Behzadi and Gholami, 2011).

CONCLUSIONS

This study described the reactance capacitance effect on the capacitance measurement of the different water liquids by the contact cylindrical capacitive sensor. The measured capacitances for different water liquids are compared for the two similar contact probes with different lengths. The measured dielectric capacitance of the contact probe depends on the permittivity of the liquid and the geometric factor of the probe. The reactance capacitance is strongly related to the electrical conductivity of the tested liquids. Comparison of the capacitance ratio of the two probes ($C_2/C_1$) for the experimental and calculated results shows that the difference between measurements is due to the reactance capacitance. Comparison of the ratio of $C_2/C_1$ for the distilled water, mineral water, tap water and salt water shows that by increasing the electrical conductivity of the water liquids the ratio of $C_2/C_1$ is also increased. This result indicates that the reactance capacitance has a significant role in the measured capacitance values by a contact CCS. Theoretical results of the charge/discharge process for different water liquids indicate that the stored charge load on the electrodes and the discharge stream current decrease by increasing the conductance effect. Such a theoretical model for the capacitance of the contact type CCS demonstrates that the reactance capacitance has a major role on the measured capacitance. On the other hand, a contact CCS such as the one reported here can provide a sensitive mean to study the conductance effect on the capacitance measurement.

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REFERENCES

