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Numerical Simulation of Coupled Nonlinear Schrödinger Equation by RDTM and Comparison with DTM

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Abstract: In this study, nonlinear coupled Schrödinger equation is solved using two recent semi-analytic methods, Differential Transform Method (DTM) and reduced form of differential transformation method (so called RDTM). The concepts of DTM and RDTM is briefly introduced and their application for nonlinear coupled Schrödinger equations are studied. The results obtained employing DTM and RDTM are compared together and exact solution. As an important result, it is depicted that the RDTM results are more accurate in comparison with those obtained by classic DTM. The numerical results reveal that the RDTM is very effective, convenient and quite accurate to systems of nonlinear equations. It is predicted that the RDTM can be found widely applicable in engineering.

Key words: Coupled schrödinger equations, differential transform method, reduced DTM, closed form solutions

INTRODUCTION

In applied sciences, each physical event may be modeled mathematically. So, it is very important to have information about analytical solutions of the models because these solutions provide information about the character of the modeled event. Therefore, it is very important to find analytical solutions of linear or nonlinear ordinary and Partial Differential Equations (PDEs) in physics, chemistry, biology and engineering areas (Ablowitz and Segur, 1981). Among the possible solutions to nonlinear PDEs, certain special form solutions may depend only on a single combination of variables such as traveling wave variables. In the literature, there is a wide variety of approaches to nonlinear problems for constructing traveling wave solutions, such as the inverse method (Ablowitz and Segur, 1981) Bácklund transformation (Tam and Hu, 2002) Hirota bilinear method (Hirota, 2004), numerical methods (Borhanifar and Abazari, 2009) and the Wronskian determinant technique (Freeman and Nimmo, 1983).

It is more difficult to obtain solutions of nonlinear PDEs than those of linear differential equations. Therefore, it may not always be possible to obtain analytical solutions of these equations. In this case, we use semi-analytical methods giving series solutions. In these kinds of methods, the solutions are sought in the form of series. Semi-analytical methods are based on finding the other terms of the series from given initial conditions for the problem being considered. At this point, we encounter the concept of convergence of the series. So, it is necessary to perform convergence analysis of these methods. As this convergence analysis can be carried out theoretically, one can gain information about the convergence of the series solution by looking at the absolute error between the numerical solution and the analytical solution. In some semi-analytic methods, a very good convergence can be achieved with only a few terms of the series, but more terms can be needed in some problems. That is, if the terms of the series increase, this provides better convergence to the analytical solution.

In this case study, similarity transformation has been used to reduce the governing differential equations into an ordinary nonlinear differential equation. In most cases, these problems do not admit analytical solution, so these equations should be solved by using special techniques. In recent years, some researchers used new methods to solve these kinds of problem (Shabani and Abazari, 2009; Adomian, 1994; He, 1999; Liao, 2003). Integral transform methods such as the Laplace and the Fourier transform methods are widely used in engineering problems. These methods transform differential equations into algebraic equations which are easier to deal with. However, integral transform methods are more complex and difficult when applying to nonlinear problems. The differential transformation method was first applied in the engineering domain by Zhou (1986). The differential transform method is based on Taylor expansion. It constructs an analytical solution in the form of a polynomial. It is different from the traditional high order Taylor series method, which requires symbolic computation of the necessary derivatives of the data functions. The Taylor series method is computationally taken long time for large orders. The differential transform is an iterative procedure for obtaining analytic Taylor series solutions of differential equations. DTM has been successfully applied to solve many nonlinear problems arising in engineering, physics, mechanics, biology etc. Ayaz (2004) applied DTM for solution of system of differential equations. Arikoglu and Ozkol (2006) employed DTM on differential-difference equations. Furthermore, the method may be employed for the solution of partial differential equations. DTM employed on some PDEs and their coupled version (Borhanifar and Abazari, 2011, 2010; Abazari and Borhanifar, 2010; Abazari and Ganji, 2011; Abazari and Abazari, 2012; Reza and Abazari, 2010) and extended DTM to solve the first and second kind of the Riccati matrix differential equations by Abazari (2009).

The propagation of pulses with equal mean frequencies in birefringent nonlinear fiber is governed by the coupled nonlinear Schrödinger equation (CNLSE) (Menyuk, 1988):

$$i\left(\frac{\partial\Phi}{\partial t}+\eta\frac{\partial\Phi}{\partial x}\right)+\frac{1}{2}\frac{\partial^2\Phi}{\partial x^2}+(|\Phi|^2+e|\Psi|^2)\Phi=0,$$
$$i\left(\frac{\partial\Psi}{\partial t}-\eta\frac{\partial\Psi}{\partial x}\right)+\frac{1}{2}\frac{\partial^2\Psi}{\partial x^2}+(|\Psi|^2+e|\Phi|^2)\Psi=0$$

where, i^2 = -1 and ϕ, ψ are the wave amplitudes in two polarizations and η is the normalized strength of the linear birefringence. There are various analytical and numerical results on solitary wave solutions of the general N coupled Schrödinger equations (Chow, 2001; Chow and Lai, 2003; Cipolatti and Zumpichiatti, 2000; Biswas and Khalique, 2010; Cattani, 2005).

Following the discussion of Wadati *et al*. (1992), the exact solution of Eq. (1) is:

$$\Phi(x,t)=\sqrt{\frac{2\alpha}{1+e}}\operatorname{sech}(\sqrt{2\alpha}(x-vt))\exp\left(i\{(v-\eta)x-[(\frac{v^2-\eta^2}{2})-\alpha]t\}\right),$$
$$\Psi(x,t)=\pm\sqrt{\frac{2\alpha}{1+e}}\operatorname{sech}(\sqrt{2\alpha}(x-vt))\exp\left(i\{(v+\eta)x-[(\frac{v^2-\eta^2}{2})-\alpha]t\}\right)$$

where, α and v are real parameters and i^2 = -1.

Recently, Abazari and Ganji (2011) extended RDTM to study the partial differential equation with proportional delay and Abazari and Abazari (2012) applied the RDTM on generalized Hirota-Satsuma coupled KdV equation and shown that as a special advantage of RDTM rather than DTM, the reduced differential transform recursive equations produce exactly all the Poisson series coefficients of solutions, whereas the differential transform recursive equations produce exactly all the Taylor series coefficients of solutions.

In this study, we employed the DTM and RDTM on coupled nonlinear Schrödinger Eq. 1 and compared the obtained results with the exact solution. As an important result, notwithstanding the simplicity and robustness of the RDTM, it is depicted that the RDTM results are more accurate in comparison with those obtained by classic DTM.

BASIC DEFINITIONS

With reference to the studies of Borhanifar and Abazari (2011) and Reza and Abazari (2010) the basic definitions of two-dimensional differential transformation are introduced as follows:

Two-dimensional DTM: Consider a function of two variables w (x, t) and suppose that it can be represented as a product of two single-variable function, i.e., w (x, t) = f (x) g (t). On the basis of the properties of the one-dimensional differential transform, the function w (x, t) can be represented as:

$$w(x,t)=\sum_{i=0}^{\infty}F(i)x^i\sum_{j=0}^{\infty}G(j)t^j=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}W(i,j)x^it^j \quad (1)$$

where, w (i, j) = F (i) G (j) is called the spectrum of w (x, t).

The basic definitions and operations for two-dimensional differential transform are introduced as follows:

Definition 1: If w (x, t) is analytic and continuously differentiable with respect to time t in the domain of interest, then:

$$W(k,h)=\frac{1}{k!h!}\left[\frac{\partial^{k+h}}{\partial x^k\partial t^h}w(x,t)\right]_{x=x_0,\,t=t_0}, \quad (2)$$

where, the spectrum function w (k, h) is the transformed function, which is also called T-function in brief.

In this study, (lower case)w (x, t) represents the original function while (upper case) w (k, h) stands for the transformed function (T-function).

The differential inverse transform of w (k, h) is defined as:

$$w(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}W(k,h)(x-x_0)^k(t-t_0)^h \quad (3)$$

Combining Eq. 2 and 3, it can be obtained that:

$$w(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\frac{1}{k!h!}\Big[\frac{\partial^{k+h}}{\partial x^k\partial t^h}w(x,t)\Big]_{\substack{x=x_0\\t=t_0}}(x-x_0)^k(t-t_0)^h$$

When (x_0, t_0) are taken as (0, 0), then Eq. 3 can be expressed as:

$$w(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}W(k,h)x^k t^h \tag{4}$$

In real applications, the function w (x, t) is represented by a finite series of Eq. 4 can be written as:

$$w(x,t)=\sum_{k=0}^{n}\sum_{h=0}^{m}W(k,h)x^k t^h+R_{nm}(x,t) \tag{5}$$

and Eq. 4 implies that

$$R_{nm}(x,t)=\sum_{k=n+1}^{\infty}\sum_{h=m+1}^{\infty}W(k,h)x^k t^h$$

is negligibly small. Usually, the values of n and m are decided by convergency of the series coefficients.

From the above definitions, it can be found that the concept of the two-dimensional differential transform is derived from the two-dimensional Taylor series expansion. With Eq. 2 and 3, the fundamental mathematical operations performed using the two-dimensional differential transform be readily obtained and these are listed in Theorem 1 (Borhanifar and Abazari, 2011, 2010; Abazari and Borhanifar, 2010; Abazari and Ganji, 2011; Abazari and Abazari, 2012; Reza and Abazari, 2010).

Theorem 1: Assume that W (k, h), U (k, h) and V (k, h) are the differential transforms of the functions w (x, t), u (x, t) and v (x, t) respectively, then:

- If w (x, t) = u (x, t)±v (x, t) then W (k, h) = U (k, h)±V (k, h)
- If w (x, t) = cu (x, t) then W (k, h) = cU (k, h) where c∈R
- If

  $$w(x,t)=\frac{\partial^{r+s}}{\partial x^r\partial t^s}u(x,t),$$

  then

  $$W(k,h)=\frac{(k+r)!(h+s)!}{k!h!}U(k+r,h+s)$$

- If w (x, t) = $x^m t^n$, then

  $$W(k,h)=\delta(k-m,h-n)=\begin{cases}1 & k=m,h=n\\0 & \text{otherwise}\end{cases}$$

- If w (x, t) = u (x, t) v (x, t) then

  $$W(k,h)=\sum_{r=0}^{k}\sum_{s=0}^{h}U(r,h-s)V(k-r,s)$$

Proof: From Abazari and Borhanifar (2010), Abazari and Ganji (2011) and Abazari and Abazari (2012) and their references.

Two-dimensional RDTM: Consider a function of two variables w (x, t) and suppose that it can be represented as a product of two single-variable function, i.e.,w (x, t) = f (x) g (t). Based on the properties of one-dimensional differential transform, the function w (x, t) can be represented as:

$$w(x,t)=\sum_{i=0}^{\infty}F(i)x^i\sum_{j=0}^{\infty}G(j)t^j=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}W(i,j)x^i t^j \tag{6}$$

where, $W(i,j)=F(i)G(j)$ is called the spectrum of w (x, t).

Remark 1: The poisson function series generates a multivariate Taylor series expansion of the input expression w, with respect to the variables X, to order n, using the variable weights W.

Remark 2: The relationship introduce in (7) is the poisson series form of the input expression w (x, t) with respect to the variables x and t, to order N, using the variable weights W_k (x).

Similar on previous section, the basic definitions of two-differential reduced differential transformation are introduced as follows:

Definition 2: If w (x, t) is analytical function in the domain of interest, then the spectrum function:

$$W_k(x)=\frac{1}{k!}\Big[\frac{\partial^k}{\partial t^k}w(x,t)\Big]_{t=t_0} \tag{7}$$

is the reduced transformed function of w (x, t).

Similarly on previous sections, the lowercase w (x, t) respect the original function while the uppercase W_k (x) stand for the reduced transformed function. The differential inverse transform of W_k (x) is defined as:

$$w(x,t)=\sum_{k=0}^{\infty}W_k(x)(t-t_0)^k \tag{8}$$

Combining Eq. 7 and 8, it can be obtained that

$$w(x,t)=\sum_{k=0}^{\infty}\frac{1}{k!}\Big[\frac{\partial^k}{\partial t^k}w(x,t)\Big]_{t=t_0}(t-t_0)^k$$

From the above proposition, it can be found that the concept of the reduced two-dimensional differential transform is derived from the two-dimensional differential transform method. With Eq. 7 and 8, the fundamental mathematical operations performed by reduced two-dimensional differential transform can readily be obtained and listed in Theorem 2.

Theorem 2: Assume that $W_k(x)$, $U_k(x)$ and $V_k(x)$, are the differential transforms of the functions w (x, t), u (x, t) and v (x, t), respectively, then:

- If $w(x,t) = u(x,t) \pm v(x,t)$, then $W_k(x) = U_k(x) \pm V_k(x)$
- If $w(x,t) = cu(x,t)$ then $W_k(x) = cU_k(x)$ where $c \in R$
- If

$$w(x,t) = \frac{\partial^{r+s}}{\partial x^r \partial t^s} u(x,t),$$

then

$$W_k(x) = \frac{\partial^r}{\partial x^r} \frac{(k+s)!}{k!} U_{k+s}(x)$$

- If $w(x,t) = x^m t^n$ then

$$W_k(x) = x^m \delta(k-n) = \begin{cases} 1 & k = n \\ 0 & \text{otherwise} \end{cases}$$

- If $w(x,t) = u(x,t)\, v(x,t)$ then

$$W_k(x) = \sum_{r=0}^{k} U_r(x) V_{k-r}(x)$$

Proof: From Abazari and Borhanifar (2010), Abazari and Ganji (2011) and Abazari and Abazari (2012) and their references.

Description of the methods: Consider the nonlinear coupled Schrödinger equation (Menyuk, 1988; Chow, 2001; Chow and Lai, 2003).

$$\begin{aligned} &i\left(\frac{\partial \Phi}{\partial t} + \eta \frac{\partial \Phi}{\partial x}\right) + \frac{1}{2}\frac{\partial^2 \Phi}{\partial x^2} + (|\Phi|^2 + e|\Psi|^2)\Phi = 0, \\ &i\left(\frac{\partial \Psi}{\partial t} - \eta \frac{\partial \Psi}{\partial x}\right) + \frac{1}{2}\frac{\partial^2 \Psi}{\partial x^2} + (|\Psi|^2 + e|\Phi|^2)\Psi = 0. \end{aligned} \tag{9}$$

subject to initial conditions

$$\Phi(x,0) = \phi(x), \qquad \Psi(x,0) = \psi(x) \tag{10}$$

where, ϕ (x) and φ (x), are complex functions. For our numerical work, we decompose the complex functions ϕ and Ψ into their real and imaginary parts by writing (Borhanifar and Abazari, 2010):

$$\begin{aligned} \Phi(x,t) &= u_1(x,t) + iv_1(x,t), \\ \Psi(x,t) &= u_2(x,t) + iv_2(x,t), \end{aligned} \tag{11}$$

where, u_j and v_j, (j = 1, 2) are real functions. Therefore the coupled equation given in (9), can be written in a following form:

$$\begin{aligned} &\frac{\partial u_1}{\partial t} + \eta\frac{\partial u_1}{\partial x} + \frac{1}{2}\frac{\partial^2 v_1}{\partial x^2} + z_1 v_1 = 0, \\ &\frac{\partial v_1}{\partial t} + \eta\frac{\partial v_1}{\partial x} - \frac{1}{2}\frac{\partial^2 u_1}{\partial x^2} - z_1 u_1 = 0, \\ &\frac{\partial u_2}{\partial t} - \eta\frac{\partial u_2}{\partial x} + \frac{1}{2}\frac{\partial^2 v_2}{\partial x^2} + z_2 v_2 = 0, \\ &\frac{\partial v_2}{\partial t} - \eta\frac{\partial v_2}{\partial x} - \frac{1}{2}\frac{\partial^2 u_2}{\partial x^2} - z_2 u_2 = 0, \end{aligned} \tag{12}$$

Where:

$$\begin{aligned} z_1 &= (u_1^2 + v_1^2) + e(u_2^2 + v_2^2) \\ z_2 &= (u_2^2 + v_2^2) + e(u_1^2 + v_1^2) \end{aligned} \tag{13}$$

where, we apply the both DTM and RDTM on the coupled nonlinear Schrödinger Eq. 1. We will shown that the DTM convert the eq. (1) to a two parameters recursive equation where is the RDTM convert to a one-parameter recursive equation.

Two-dimensional DTM: According to two-dimensional differential transform operators listed in Theorem 1, the differential transform version of system given in Eq. (12), will be:

$$\begin{cases} (h+1)U_1(k,h+1) + \eta(k+1)U_1(k+1,h) + \frac{1}{2}\frac{(k+2)!}{k!}V_1(k+2,h) \\ \quad + \sum_{r=0}^{k}\sum_{s=0}^{h} Z_1(r,h-s)V_1(k-r,s) = 0, \\ (h+1)V_1(k,h+1) + \eta(k+1)V_1(k+1,h) - \frac{1}{2}\frac{(k+2)!}{k!}U_1(k+2,h) \\ \quad - \sum_{r=0}^{k}\sum_{s=0}^{h} Z_1(r,h-s)U_1(k-r,s) = 0, \\ (h+1)U_2(k,h+1) - \eta(k+1)U_2(k+1,h) + \frac{1}{2}\frac{(k+2)!}{k!}V_2(k+2,h) \\ \quad + \sum_{r=0}^{k}\sum_{s=0}^{h} Z_2(r,h-s)V_2(k-r,s) = 0, \\ (h+1)V_2(k,h+1) - \eta(k+1)V_2(k+1,h) - \frac{1}{2}\frac{(k+2)!}{k!}U_2(k+2,h) \\ \quad - \sum_{r=0}^{k}\sum_{s=0}^{h} Z_2(r,h-s)U_2(k-r,s) = 0. \end{cases} \tag{14}$$

where, U_j (k, h), V_j (k, h) and Z_j (k, h) (j = 12) are the differential transform version of u_j (x, t), v_j (x, t) and z_j (x, t) respectively.

In order to obtain the unknowns of U_j (k, h),, and V_j (k, h), k, h = 1,1,2,..., N we must construct and solve the above equations by using initial conditions (10) and substitute in Eq. 5. The following corresponding algorithm can be introduced to the case below:

DTM Algorithm:

Step 1: Choose N∈N as the degree of approximate solution

Step 2: Determine the initial value U_1 (k,0), V_1 (k,0), U_2 (k,0) and V_2 (k,0) for k = 0, 1, 2, ..., N

Step 3: Set $\overline{U}_1(x,t)=\sum_{k=0}^{N}U_1(k,0)x^k$ and $\overline{U}_2(x,t)=\sum_{k=0}^{N}U_2(k,0)x^k$

Step 4: Set $\overline{V}_1(x,t)=\sum_{k=0}^{N}V_1(k,0)x^k$ and $\overline{V}_2(x,t)=\sum_{k=0}^{N}V_2(k,0)x^k$

Step 5: For k = 0,1,2,..., N do for h = 0,1,2,..., N do:

$$(h+1)U_1(k,h+1)+\eta(k+1)U_1(k+1,h)+\frac{1}{2}\frac{(k+2)!}{k!}V_1(k+2,h)+\sum_{r=0}^{k}\sum_{s=0}^{h}Z_1(r,h-s)V_1(k-r,s)=0,$$

$$(h+1)V_1(k,h+1)+\eta(k+1)V_1(k+1,h)-\frac{1}{2}\frac{(k+2)!}{k!}U_1(k+2,h)-\sum_{r=0}^{k}\sum_{s=0}^{h}Z_1(r,h-s)U_1(k-r,s)=0,$$

$$(h+1)U_2(k,h+1)-\eta(k+1)U_2(k+1,h)+\frac{1}{2}\frac{(k+2)!}{k!}V_2(k+2,h)+\sum_{r=0}^{k}\sum_{s=0}^{h}Z_2(r,h-s)V_2(k-r,s)=0,$$

$$(h+1)V_2(k,h+1)-\eta(k+1)V_2(k+1,h)-\frac{1}{2}\frac{(k+2)!}{k!}U_2(k+2,h)-\sum_{r=0}^{k}\sum_{s=0}^{h}Z_2(r,h-s)U_2(k-r,s)=0.$$

End do

$\overline{U}_1(x,t)=\overline{U}_1(x,t)+U_1(k,h)x^kt^h$,

and $\overline{U}_2(x,t)=\overline{U}_2(x,t)+U_2(k,h)x^kt^h$.

$\overline{V}_1(x,t)=\overline{V}_1(x,t)+V_1(k,h)x^kt^h$ and

$\overline{V}_2(x,t)=\overline{V}_2(x,t)+V_2(k,h)x^kt^h$

End do

Step 6: $U_{j[N,N]}(x,t)=\overline{U}_j(x,t)$, and $V_{j[N,N]}(x,t)=\overline{V}_j(x,t)$ for j = 1,2.. .

Step 7: $\Phi_{[N,N]}=U_{1[N,N]}+iV_{1[N,N]},\Psi_{[N,N]}=U_{2[N,N]}+iV_{2[N,N]}$

Tow-dimensional RDTM: Now, according to two-dimensional reduced differential transform operators listed in Theorem 2, the coupled equation given in Eq. 12, can be written in a matrix-vector form as:

$$\frac{\partial\theta}{\partial t}+\eta A\frac{\partial\theta}{\partial x}+\frac{1}{2}B\frac{\partial^2\theta}{\partial x^2}+F(\theta)\theta=0 \quad (15)$$

Where:

$$\theta=\begin{pmatrix}u_1\\v_1\\u_2\\v_2\end{pmatrix},\ A=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix},\ B=\begin{pmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&1\\0&0&-1&0\end{pmatrix},\ F(\theta)=\begin{pmatrix}0&z_1&0&0\\-z_1&0&0&0\\0&0&0&z_2\\0&0&-z_2&0\end{pmatrix}$$

Then the differential transform of system given in Eq. 15, will be:

$$(k+1)\Theta_{k+1}(x)=-\eta A\frac{d}{dx}\Theta_k(x)-\frac{1}{2}B\frac{d^2}{dx^2}\Theta_k(x)-\sum_{r=0}^{k}F(\Theta_r(x))\Theta_{k-r}(x) \quad (16)$$

where, $\Theta=[U_1,V_1,U_2,V_2]^T$ is the reduced differential transform of $\theta=[u_1,v_1,u_2,v_2]^T$ and for $\ell=1,2$,

$$\frac{d^\ell}{dx^\ell}\Theta_k(x)=\begin{pmatrix}\frac{d^\ell}{dx^\ell}U_{1,k}(x)\\\frac{d^\ell}{dx^\ell}V_{1,k}(x)\\\frac{d^\ell}{dx^\ell}U_{2,k}(x)\\\frac{d^\ell}{dx^\ell}V_{2,k}(x)\end{pmatrix},\ F(\Theta_r(x))=\begin{pmatrix}0&Z_{1,r}&0&0\\-Z_{1,r}&0&0&0\\0&0&0&Z_{2,r}\\0&0&-Z_{2,r}&0\end{pmatrix},$$

therefore, for k = 0,1,2,..., N Equation 16, can be rewritten as follows iteration method

$$\Theta_{k+1}(x)=\frac{-1}{k+1}\left(\eta A\frac{d}{dx}\Theta_k(x)+\frac{1}{2}B\frac{d^2}{dx^2}\Theta_k(x)+\sum_{r=0}^{k}F(\Theta_r(x))\Theta_{k-r}(x)\right) \quad (17)$$

with initial condition $\Theta_0(x)=(U_{1,0}(x),V_{1,0}(x),U_{2,0}(x),V_{2,0}(x))^T$ where, $U_{1,0}(x)=u_1(x,0),V_{1,0}(x)=v_1(x,0),U_{2,0}(x)=u_2(x,0)$ and $V_{2,0}(x)=v_2(x,0)$ In order to obtain the unknowns of $\Theta_k(x)=(U_{1,k}(x),V_{1,k}(x),U_{2,k}(x),V_{2,k}(x))^T$ for $k=0,1,2,...,N$ we must construct iteration eq. 17 and by substituting in the following series, we can obtain the N-th term approximation series form of exact solutions:

$$\Theta_N(x,t)=\Theta_0(x)+\Theta_1(x)t+\Theta_2(x)t^2+...+\Theta_N(x)t^N \quad (18)$$

Therefore, the corresponding algorithm can be introduced to the case below:

RDTM Algorithm:

Step 1: Choose N∈N as the degree of approximate solution

Step 2: Determine the initial value $\Theta_0(x)$ from initial conditions (10)

Step 3: Set $\overline{\Theta}(x,t)=\Theta_0(x)$

Step 4: For $k=0,1,2,..,N$ do

$$\Theta_{k+1}(x)=\frac{-1}{k+1}\left(\eta A\frac{d}{dx}\Theta_k(x)+\frac{1}{2}B\frac{d^2}{dx^2}\Theta_k(x)+\sum_{r=0}^{k}F(\Theta_r(x))\Theta_{k-r}(x)\right)$$

$\overline{\Theta}(x,t)=\overline{\Theta}(x,t)+\Theta_{k+1}(x)t^{k+1}$

End do

Step 5: $\Theta_N(x,t)=\overline{\Theta}(x,t)$

Step 6: $\Phi_{[N]}=\Theta_{[N](1)}+i\Theta_{[N](2)},\Psi_{[N]}=\Theta_{[N](3)}+i\Theta_{[N](4)}$

Numerical Examples: To compare the numerical results of DTM and RDTM, we consider the coupled nonlinear Schrödinger Eq. 1 with exact wave solution (2), when $e=\frac{2}{3}, \alpha=1, \nu=1$ and $\eta=\frac{1}{2}$ (Borhanifar and Abazari, 2010)

$$i\left(\frac{\partial\Phi}{\partial t}+\frac{1}{2}\frac{\partial\Phi}{\partial x}\right)+\frac{1}{2}\frac{\partial^2\Phi}{\partial x^2}+\left(|\Phi|^2+\frac{2}{3}|\Psi|^2\right)\Phi=0,$$
$$i\left(\frac{\partial\Psi}{\partial t}-\frac{1}{2}\frac{\partial\Psi}{\partial x}\right)+\frac{1}{2}\frac{\partial^2\Psi}{\partial x^2}+\left(|\Psi|^2+\frac{2}{3}|\Phi|^2\right)\Psi=0. \quad (19)$$

subject to the initial conditions

$$\Phi(x,0)=\frac{1}{5}\sqrt{30}\,\mathrm{sech}(\sqrt{2}x)\exp(\frac{i}{2}x),$$
$$\Psi(x,0)=\frac{1}{5}\sqrt{30}\,\mathrm{sech}(\sqrt{2}x)\exp(\frac{3i}{2}x). \quad (20)$$

From the initial conditions (20) and according to Eq. 11, we get:

$$u_1(x,0)=\frac{1}{5}\sqrt{30}\,\mathrm{sech}(\sqrt{2}x)\cos(\frac{1}{2}x),$$
$$v_1(x,0)=\frac{1}{5}\sqrt{30}\,\mathrm{sech}(\sqrt{2}x)\sin(\frac{1}{2}x),$$
$$u_2(x,0)=\frac{1}{5}\sqrt{30}\,\mathrm{sech}(\sqrt{2}x)\cos(\frac{3}{2}x),$$
$$v_2(x,0)=\frac{1}{5}\sqrt{30}\,\mathrm{sech}(\sqrt{2}x)\sin(\frac{3}{2}x). \quad (21)$$

In these kind of initial conditions, we first obtain the DTM recurrence relation and secondly, the RDTM version of equation and solve both recurrence relations by programming in MATLAB environment. The results of the test example show that the RDTM results are more powerful than DTM results. These numerical results are given in Table 1- 4.

Numerical results of DTM: For finite values of $k=0,1,2,..,N$ the differential transform version of initial conditions (21) obtain from following recursing relationship

$$\sum_{\ell=0}^{k}\left\{\left(\frac{(\sqrt{2})^\ell}{\ell!}+\frac{(-\sqrt{2})^\ell}{\ell!}\right)U_1(k-\ell,0)\right\}-\frac{2\sqrt{30}}{5}\left(\frac{(1/2)^k}{k!}\cos(\frac{k\pi}{2})\right)=0,$$
$$\sum_{\ell=0}^{k}\left\{\left(\frac{(\sqrt{2})^\ell}{\ell!}+\frac{(-\sqrt{2})^\ell}{\ell!}\right)V_1(k-\ell,0)\right\}-\frac{2\sqrt{30}}{5}\left(\frac{(1/2)^k}{k!}\sin(\frac{k\pi}{2})\right)=0,$$
$$\sum_{\ell=0}^{k}\left\{\left(\frac{(\sqrt{2})^\ell}{\ell!}+\frac{(-\sqrt{2})^\ell}{\ell!}\right)U_2(k-\ell,0)\right\}-\frac{2\sqrt{30}}{5}\left(\frac{(3/2)^k}{k!}\cos(\frac{k\pi}{2})\right)=0,$$
$$\sum_{\ell=0}^{k}\left\{\left(\frac{(\sqrt{2})^\ell}{\ell!}+\frac{(-\sqrt{2})^\ell}{\ell!}\right)V_2(k-\ell,0)\right\}-\frac{2\sqrt{30}}{5}\left(\frac{(3/2)^k}{k!}\sin(\frac{k\pi}{2})\right)=0, \quad (22)$$

from the recursive Eq. 22, the following initial values can be obtained easily

$$U_1(k,0)=0, \quad \text{for} \quad k=1,3,5,7,...$$
$$U_1(0,0)=\frac{\sqrt{30}}{5}, U_1(2,0)=-\frac{9\sqrt{30}}{40}, U_1(4,0)=\frac{123\sqrt{30}}{640}, U_1(6,0)=-\frac{4017\sqrt{30}}{25600},...$$
$$V_1(k,0)=0, \quad \text{for} \quad k=0,2,4,6,...$$
$$V_1(1,0)=\frac{\sqrt{30}}{10}, V_1(3,0)=-\frac{5\sqrt{30}}{48}, V_1(5,0)=\frac{1681\sqrt{30}}{19200}, V_1(7,0)=-\frac{229993\sqrt{30}}{3525600},...$$
$$U_2(k,0)=0, \quad \text{for} \quad k=1,3,5,7,...$$
$$U_2(0,0)=\frac{\sqrt{30}}{5}, U_2(2,0)=-\frac{17\sqrt{30}}{40}, U_2(4,0)=\frac{833\sqrt{30}}{1920}, U_2(6,0)=-\frac{84881\sqrt{30}}{230400},...$$
$$V_2(k,0)=0, \quad \text{for} \quad k=0,2,4,6,...$$
$$V_2(1,0)=\frac{3\sqrt{30}}{10}, V_2(3,0)=-\frac{33\sqrt{30}}{80}, V_2(5,0)=\frac{2401\sqrt{30}}{6400}, V_2(7,0)=-\frac{333761\sqrt{30}}{1075200},... \quad (23)$$

Table 1: The error of real and imaginary parts of Φ (x, t)respect to first 3-terms approximation solutions obtained by DTM and RDTM at some points in the intervals $0\le x\le 5$ and $0\le t\le 1$

x	t	DTM $\lvert u_1-U_{1[3,3]}\rvert$	RDTM $\lvert u_1-\Theta_3(1)\rvert$	DTM $\lvert v_1-V_{1[3,3]}\rvert$	RDTM $\lvert v_1-\Theta_3(2)\rvert$	DTM $\lvert\Phi\rvert-\lvert\Phi_{[3,3]}\rvert$	RDTM $\lvert\Phi\rvert-\lvert\Phi_{[3]}\rvert$
0.5	0.3	2.63768977e-02	2.64145594e-04	1.36242513e-02	4.04877457e-03	+1.78531397e-02	-1.48287545e-03
	0.7	2.77028411e-01	6.61040060e-02	1.51374092e-01	8.93104950e-02	-3.11428583e-01	-1.13901012e-02
	1.0	6.24698848e-01	3.91193327e-01	3.61851043e-01	2.37121914e-01	-6.97843753e-01	-5.02008718e-02
1	0.3	5.71929584e-01	1.49306536e-03	5.04232067e-02	1.78643012e-03	+3.11355079e-01	-2.28770499e-03
	0.7	3.59545057e+00	3.96238894e-02	7.01703175e-01	7.13431311e-02	-3.44981552e+00	-8.10066920e-02
	1.0	1.21399311e+01	1.18027357e-01	1.92286239e+00	3.26157646e-01	-1.18484528e+01	-3.45576553e-01
2	0.3	6.72667862e+00	2.42415004e-04	2.48025575e+00	4.95180906e-05	-6.84266954e+00	-4.48841452e-05
	0.7	2.49706826e+01	8.01430577e-03	1.76328554e+00	3.79617250e-04	-2.47639732e+01	-7.82665101e-04
	0.1	9.78690024e+01	3.51112951e-02	8.14780875e+00	4.38663389e-03	-9.77204585e+01	-3.70906865e-03
3	0.3	2.30226938e+01	6.55827865e-05	1.11639368e+01	4.97379966e-06	-2.55228152e+01	+2.65152551e-06
	0.7	7.74520193e+01	2.17385641e-03	2.71640326e-01	3.08245000e-04	-7.73373330e+01	+4.64446250e-04
	1.0	3.20070068e+02	9.82648484e-03	1.61558221e+01	1.97502426e-03	-3.20286232e+02	+3.14092004e-03
4	0.3	5.32408037e+01	1.36075929e-05	2.92566959e+01	8.59586876e-06	-6.07394299e+01	+8.46480853e-07
	0.7	1.75911378e+02	4.37346122e-04	7.57084988e+00	3.14861753e-04	-1.76037367e+02	+1.22837464e-04
	1.0	7.43648461e+02	1.92795602e-03	2.48362948e+01	1.54537306e-03	-7.44004776e+02	+8.26961252e-04
5	0.3	1.01411902e+02	1.90456235e-06	5.98890994e+01	3.41998487e-06	-1.17774263e+02	+2.08700175e-07
	0.7	3.34931435e+02	5.67602169e-05	2.21852586e+01	1.18164019e-04	-3.35655419e+02	+3.00068809e-05
	1.0	1.43323384e+03	2.32145004e-04	3.35026373e+01	5.54584077e-04	-1.43361006e+03	+2.01963199e-04

Table 2: The error of real and imaginary parts of Φ (x, t) respect to first 6-terms approximation solutions obtained by DTM and RDTM at some points in the intervals 0≤x≤5 and 0≤t≤1

		DTM	RDTM	DTM	RDTM	DTM	RDTM
x	t	$\lvert u_1-U_{1[3,3]}\rvert$	$\lvert u_1-\Theta_6(1)\rvert$	$\lvert v_1-V_{1[6,6]}\rvert$	$\lvert v_1-\Theta_6(2)\rvert$	$\lvert\Phi\rvert-\lvert\Phi_{[6,6]}\rvert$	$\lvert\Phi\rvert-\lvert\Phi_{[6]}\rvert$
0.5	0.3	5.74631736e-03	3.03943504e-05	1.29756444e-03	4.78965102e-05	-5.75558894e-03	-7.24050730e-06
	0.7	3.72941504e-01	2.12057492e-02	1.06347040e-02	1.02789218e-02	-3.14192363e-01	-1.00808984e-02
	1.0	3.44705389e+00	2.97339584e-01	1.69293055e-01	4.65436592e-02	-3.22148274e+00	-1.86889067e-01
1	0.3	5.28568050e-01	1.34323445e-05	2.28846016e-01	5.58063163e-06	-5.63591783e-01	+1.39224812e-05
	0.7	2.97626957e+01	5.65301706e-03	9.31696028e-01	4.48308515e-03	-2.94028439e+01	+6.95739659e-03
	1.0	2.86956576e+02	6.45681747e-02	1.40980246e+01	7.54877247e-02	-2.86729112e+02	+9.56198135e-02
2	0.3	3.04439619e+01	1.91937322e-07	2.33442161e+01	5.59685678e-07	-3.83367420e+01	-4.47292893e-07
	0.7	1.90998134e+03	1.11283540e-04	8.78866735e+00	2.50753325e-04	-1.90970533e+03	-2.33769465e-04
	0.1	1.93186040e+04	1.88330365e-03	3.44921036e+02	3.43993652e-03	-1.93211617e+04	-3.53975005e-03
3	0.3	2.78610613e+02	9.09770789e-10	2.84847427e+02	3.42201772e-08	-3.98431547e+02	-3.38814739e-08
	0.7	2.10851536e+04	7.81931501e-07	3.94286195e+02	1.42273799e-05	-2.10887234e+04	-1.35619291e-05
	1.0	2.17052640e+05	2.50660543e-05	9.29804610e+02	1.86612147e-04	-2.17054435e+05	-1.71894301e-04
4	0.3	1.31577981e+03	1.77931753e-09	1.61155967e+03	7.22971725e-09	-2.08047239e+03	-6.92699326e-09
	0.7	1.15833925e+05	8.37596987e-07	3.03067760e+03	2.90572605e-06	-1.15873529e+05	-2.51948370e-06
	1.0	1.20378645e+06	1.20370687e-05	3.42476612e+03	3.70472405e-05	-1.20379127e+06	-2.87756523e-05
5	0.3	4.40040864e+03	1.19542961e-09	6.11468139e+03	1.34675279e-09	-7.53345037e+03	-1.66503334e-09
	0.7	4.34953593e+05	5.04521420e-07	1.34027255e+04	5.28650293e-07	-4.35160031e+05	-6.01213622e-07
	1.0	4.54725891e+06	6.69446997e-06	3.27965335e+04	6.60735541e-06	-4.54737716e+06	-6.80593068e-06

Table 3: The error of real and imaginary parts of Ψ(x, t) respect to first 3-terms approximation solutions obtained by DTM and RDTM at some points in the intervals 0≤x≤5 and 0≤t≤1

		DTM	RDTM	DTM	RDTM	DTM	RDTM
x	t	$\lvert u_2-U_{2[3,3]}\rvert$	$\lvert u_2-\Theta_{[3]}(3)\rvert$	$\lvert v_2-V_{2[3,3]}\rvert$	$\lvert v_2-\Theta_{[4]}(3)\rvert$	$\lvert\Psi\rvert-\lvert\Psi_{[3,3]}\rvert$	$\lvert\Psi\rvert-\lvert\Psi_{[3]}\rvert$
0.5	0.3	9.63262304e-01	9.11269935e-03	5.10516868e-01	3.94868247e-03	-4.58771955e-01	-6.80698196e-03
	0.7	7.55130741e-01	3.05005671e-02	3.58487495e-01	1.05522773e-02	-3.87441944e-01	-2.47536913e-02
	1.0	3.95937185e-01	7.80231283e-02	1.53176165e-01	2.01837224e-02	-2.15663676e-01	-6.75018259e-02
1	0.3	1.36262385e+00	7.63376142e-03	8.55105455e-01	3.01536870e-03	-9.99769415e-01	-6.28175837e-03
	0.7	1.06834081e+00	2.55344295e-02	6.56609313e-01	8.01146739e-03	-7.74147312e-01	-2.24085940e-02
	1.0	5.44167176e-01	6.53951431e-02	3.84602986e-01	1.51504041e-02	-3.50569978e-01	-6.01617712e-02
2	0.3	9.52156586e+00	1.23400055e-03	1.41692007e+01	8.25625299e-04	-1.70417298e+01	-1.46379366e-03
	0.7	1.52292599e+01	3.43973253e-02	7.02510712e+00	3.48160949e-02	-1.64460146e+01	-4.58869025e-02
	0.1	7.05173196e+01	1.22966137e-01	5.56432061e+00	1.79842756e-01	-7.02774973e+01	-1.74483501e-01
3	0.3	2.48501013e+01	1.58259053e-04	5.43362239e+01	3.14101315e-04	-5.97352193e+01	-3.49346714e-04
	0.7	5.77375279e+01	6.96608684e-03	3.34706612e+01	9.21053502e-03	-6.67374740e+01	-1.05196789e-02
	1.0	2.45462018e+02	3.69377273e-02	2.75258956e+00	3.58551808e-02	-2.45477032e+02	-3.81293793e-02
4	0.3	4.87661201e+01	7.88769240e-05	1.33458592e+02	3.26730712e-05	-1.42070576e+02	-8.48318442e-05
	0.7	1.42575470e+02	2.35622792e-03	8.74311366e+01	1.51742371e-03	-1.67227452e+02	-2.54785070e-03
	1.0	5.83359656e+02	9.36851728e-03	1.05868955e+01	8.26716151e-03	-5.83423007e+02	-9.20297109e-03
5	0.3	8.31199489e+01	6.56264525e-06	2.64308996e+02	1.96896016e-05	-2.77065376e+02	-2.06225957e-05
	0.7	2.83475855e+02	3.27269040e-04	1.78683884e+02	5.97525477e-04	-3.35081756e+02	-6.19285888e-04
	1.0	1.13662953e+03	1.84263328e-03	4.04343672e+01	2.41458298e-03	-1.13733320e+03	-2.23642992e-03

Table 4: The error of real and imaginary parts of Ψ(x, t) respect to first 6-terms approximation solutions obtained by DTM and RDTM at some points in the intervals 0≤x≤5 and 0≤t≤1

		DTM	RDTM	DTM	RDTM	DTM	RDTM
x	t	$\lvert u_2-U_{2[6,6]}\rvert$	$\lvert u_2-\Theta_{[6]}(3)\rvert$	$\lvert v_2-V_{2[6,6]}\rvert$	$\lvert v_2-\Theta_{[6]}(4)\rvert$	$\lvert\Psi\rvert-\lvert\Psi_{[6,6]}\rvert$	$\lvert\Psi\rvert-\lvert\Psi_{[6]}\rvert$
0.5	0.3	8.10982284e-03	2.97484395e-04	8.83638754e-03	3.22612055e-05	-1.06294324e-02	-1.07215606e-04
	0.7	1.76513816e-02	2.22280133e-03	6.63506434e-03	4.48483798e-04	-8.00146954e-03	-6.06795835e-04
	1.0	5.27473912e-02	1.03993722e-02	1.06034833e-02	3.12028192e-03	-5.69595131e-03	-1.94025315e-03
1	0.3	4.30784173e-01	4.80159739e-05	1.14367322e+00	4.33169689e-05	-1.06136406e+00	+6.08545693e-05
	0.7	1.17140188e+00	3.95750575e-04	8.80450719e-01	3.12077821e-04	-8.49080817e-01	+4.91411558e-04
	1.0	3.47158885e+00	2.06177144e-03	6.96938436e-01	1.40026276e-03	-2.20966340e+00	+2.47788896e-03
2	0.3	6.77893570e+00	3.32250279e-06	9.49977874e+01	7.11442059e-08	-9.49759441e+01	+3.06157452e-06
	0.7	1.75066141e+03	1.40293698e-03	3.33827680e+01	1.91479685e-04	-1.75050154e+03	+7.20372689e-04
	0.1	1.72115571e+04	1.84120426e-02	9.42901084e+02	4.59652580e-03	-1.72369165e+04	+1.19844969e-03
3	0.3	2.93058572e+02	2.28477324e-08	1.06545881e+03	5.80841813e-07	-1.10493148e+03	+5.53441326e-07
	0.7	1.93099366e+04	3.40537417e-05	4.18321771e+02	2.32547568e-04	-1.93144589e+04	+1.35592508e-04
	1.0	1.94438848e+05	6.93749866e-04	3.09509098e+03	2.92369826e-03	-1.94463480e+05	+3.88365208e-04
4	0.3	2.25809438e+03	1.38052514e-07	5.76030829e+03	2.99393094e-09	-6.18708198e+03	+1.31876745e-07
	0.7	1.05939899e+05	5.53498696e-05	4.72210853e+03	4.73228268e-06	-1.06045077e+05	+3.25337552e-05
	1.0	1.08066642e+06	6.95811945e-04	5.06569509e+03	1.21446607e-04	-1.08067826e+06	+9.90265884e-05
5	0.3	9.94474529e+03	3.07581243e-09	2.12467114e+04	3.33829440e-08	-2.34589152e+04	+3.20230831e-08
	0.7	3.97422771e+05	2.04355771e-07	2.35008946e+04	1.34812361e-05	-3.98116998e+05	+7.90343105e-06
	1.0	4.08653970e+06	1.75643132e-05	7.09328529e+04	1.70470209e-04	-4.08715525e+06	+2.41442208e-05

then by utilize the differential transform of initial condition (23) in recursive method (14), the three term DTM approximation solution, $U_{1[3,3]}(x,t), V_{1[3,3]}(x,t), U_{2[3,3]}(x,t)$ and $V_{2[3,3]}(x,t)$ in a series form obtain as follow:

$$
\begin{aligned}
U_{1[3,3]}(x,t) &= \sqrt{30}\left\{(\frac{1}{5}-\frac{9}{40}x^2)+(\frac{27}{80}x-\frac{417}{640}x^3)t+(-\frac{153}{640}+\frac{4833}{5120}x^2)t^2 \right.\\
&\left. +(-\frac{1389}{2048}x+\frac{214227}{81920}x^3)t^3\right\},\\
V_{1[3,3]}(x,t) &= \sqrt{30}\left\{(\frac{1}{10}x-\frac{5}{48}x^3)+(\frac{1}{8}+\frac{19}{320}x^2)t+(\frac{167}{1280}x+\frac{2353}{30720}x^3)t^2 \right.\\
&\left. +(-\frac{409}{3072}+\frac{6817}{24576}x^2)t^3\right\},\\
U_{2[3,3]}(x,t) &= \sqrt{30}\left\{(\frac{1}{5}-\frac{17}{40}x^2)+(-\frac{13}{80}x-\frac{659}{1920}x^3)t+(-\frac{353}{640}+\frac{4337}{5120}x^2)t^2 \right.\\
&\left. +(-\frac{2935}{6144}x+\frac{1435763}{737280}x^3)t^3\right\},\\
V_{2[3,3]}(x,t) &= \sqrt{30}\left\{(\frac{3}{10}x-\frac{33}{80}x^3)+(\frac{3}{8}-\frac{63}{320}x^2)t+(-\frac{99}{1280}x+\frac{2497}{10240}x^3)t^2\{ \right.\\
&\left. +(-\frac{609}{1024}+\frac{5809}{8192}x^2)t^3\right\}.
\end{aligned}
\tag{24}
$$

which are the same as the partial sum of the Taylor series form of real and imaginary parts of traveling wave solution (1.2), when $e=\frac{2}{3}, \alpha=1, \nu=1$

Numerical results of RDTM: Similarity, we consider the Eq. (1) subject to vector form of initial conditions:

$$
\Theta_0(x)=\begin{pmatrix} U_{1[0]}(x)\\ V_{1[0]}(x)\\ U_{2[0]}(x)\\ V_{2[0]}(x)\end{pmatrix}=\begin{pmatrix} \frac{\sqrt{30}}{5}\operatorname{sech}(\sqrt{2}x)\cos(\frac{1}{2}x)\\ \frac{\sqrt{30}}{5}\operatorname{sech}(\sqrt{2}x)\sin(\frac{1}{2}x)\\ \frac{\sqrt{30}}{5}\operatorname{sech}(\sqrt{2}x)\cos(\frac{3}{2}x)\\ \frac{\sqrt{30}}{5}\operatorname{sech}(\sqrt{2}x)\sin(\frac{3}{2}x)\end{pmatrix}
\tag{25}
$$

Then by utilize the initial value (25) in recursive method (17) for $k=0,1,2,$ the first three terms of $\Theta_k(x)$ obtain as follow:

$$
\begin{aligned}
\Theta_1(x) &= \frac{-1}{2}(\frac{1}{2}A\frac{d}{dx}\Theta_0(x)+\frac{1}{2}B\frac{d^2}{dx^2}\Theta_0(x)+F(\Theta_0(x))\Theta_0(x))\\
&= (U_{1[1]}(x), V_{1[1]}(x), U_{2[1]}(x), V_{2[1]}(x))^T,
\end{aligned}
\tag{26}
$$

Where:

$$
\begin{cases}
U_{1[1]}(x)=\dfrac{-\sqrt{30}e^{\sqrt{2}x}}{e^{2\sqrt{2}x}+1}\{\dfrac{1}{4}\sin(\dfrac{x}{2})-\dfrac{2\sqrt{2}}{5}\dfrac{(e^{2\sqrt{2}x}-1)}{(e^{2\sqrt{2}x}+1)}\cos(\dfrac{x}{2})\},\\
V_{1[1]}(x)=\dfrac{\sqrt{30}e^{\sqrt{2}x}}{e^{2\sqrt{2}x}+1}\{\dfrac{1}{4}\cos(\dfrac{x}{2})+\dfrac{2\sqrt{2}}{5}\dfrac{(e^{2\sqrt{2}x}-1)}{(e^{2\sqrt{2}x}+1)}\sin(\dfrac{x}{2})\},\\
U_{2[1]}(x)=\dfrac{-\sqrt{30}e^{\sqrt{2}x}}{e^{2\sqrt{2}x}+1}\{\dfrac{3}{4}\sin(\dfrac{3}{2}x)-\dfrac{2\sqrt{2}}{5}\dfrac{(e^{2\sqrt{2}x}-1)}{(e^{2\sqrt{2}x}+1)}\cos(\dfrac{3}{2}x)\},\\
V_{2[1]}(x)=\dfrac{\sqrt{30}e^{\sqrt{2}x}}{e^{2\sqrt{2}x}+1}\{\dfrac{3}{4}\cos(\dfrac{3}{2}x)+\dfrac{2\sqrt{2}}{5}\dfrac{(e^{2\sqrt{2}x}-1)}{(e^{2\sqrt{2}x}+1)}\sin(\dfrac{3}{2}x)\},
\end{cases}
\tag{27}
$$

and

$$
\begin{aligned}
\Theta_2(x) &= \frac{-1}{3}(\frac{1}{2}A\frac{d}{dx}\Theta_1(x)+\frac{1}{2}B\frac{d^2}{dx^2}\Theta_1(x)+\sum_{r=0}^{1}F(\Theta_r(x))\Theta_{1-r}(x))\\
&= (U_{1[2]}(x), V_{1[2]}(x), U_{2[2]}(x), V_{2[2]}(x))^T,
\end{aligned}
\tag{28}
$$

Where:

$$
\begin{cases}
U_{1[2]}(x)=\dfrac{-\sqrt{30}e^{\sqrt{2}x}}{(e^{2\sqrt{2}x}+1)^2}\{\dfrac{\sqrt{2}(e^{2\sqrt{2}x}-1)}{4}\sin(\dfrac{x}{2})-\dfrac{(103e^{4\sqrt{2}x}-818e^{2\sqrt{2}x}+103)}{320(e^{2\sqrt{2}x}+1)}\cos(\dfrac{x}{2})\},\\
V_{1[2]}(x)=\dfrac{\sqrt{30}e^{\sqrt{2}x}}{(e^{2\sqrt{2}x}+1)^2}\{\dfrac{\sqrt{2}(e^{2\sqrt{2}x}-1)}{4}\cos(\dfrac{x}{2})+\dfrac{(103e^{4\sqrt{2}x}-818e^{2\sqrt{2}x}+103)}{320(e^{2\sqrt{2}x}+1)}\sin(\dfrac{x}{2})\},\\
U_{2[2]}(x)=\dfrac{-\sqrt{30}e^{\sqrt{2}x}}{(e^{2\sqrt{2}x}+1)^2}\{\dfrac{3\sqrt{2}(e^{2\sqrt{2}x}-1)}{4}\sin(\dfrac{3}{2}x)+\dfrac{(97e^{4\sqrt{2}x}+1218e^{2\sqrt{2}x}+97)}{320(e^{2\sqrt{2}x}+1)}\cos(\dfrac{3}{2}x)\},\\
V_{2[2]}(x)=\dfrac{\sqrt{30}e^{\sqrt{2}x}}{(e^{2\sqrt{2}x}+1)^2}\{\dfrac{3\sqrt{2}(e^{2\sqrt{2}x}-1)}{4}\cos(\dfrac{3}{2}x)-\dfrac{(97e^{4\sqrt{2}x}+1218e^{2\sqrt{2}x}+97)}{320(e^{2\sqrt{2}x}+1)}\sin(\dfrac{3}{2}x)\},
\end{cases}
\tag{29}
$$

and

$$
\begin{aligned}
\Theta_3(x) &= \frac{-1}{4}(\frac{1}{2}A\frac{d}{dx}\Theta_2(x)+\frac{1}{2}B\frac{d^2}{dx^2}\Theta_2(x)+\sum_{r=0}^{2}F(\Theta_r(x))\Theta_{2-r}(x))\\
&= (U_{1[3]}(x), V_{1[3]}(x), U_{2[3]}(x), V_{2[3]}(x))^T,
\end{aligned}
\tag{30}
$$

Where:

$$
\begin{cases}
U_{1[3]}(x)=\dfrac{-\sqrt{30}e^{\sqrt{2}x}}{(e^{2\sqrt{2}x}+1)^3}\{\dfrac{(359e^{4\sqrt{2}x}-2354e^{2\sqrt{2}x}+359)}{1536}\sin(\dfrac{x}{2})\\
\qquad -\dfrac{\sqrt{2}}{960}\dfrac{(e^{2\sqrt{2}x}-1)(53e^{4\sqrt{2}x}-2966e^{2\sqrt{2}x}+53)}{(e^{2\sqrt{2}x}+1)}\cos(\dfrac{x}{2})\},\\
V_{1[3]}(x)=\dfrac{\sqrt{30}e^{\sqrt{2}x}}{(e^{2\sqrt{2}x}+1)^3}\{\dfrac{(359e^{4\sqrt{2}x}-2354e^{2\sqrt{2}x}+359)}{1536}\cos(\dfrac{x}{2})\\
\qquad +\dfrac{\sqrt{2}}{960}\dfrac{(e^{2\sqrt{2}x}-1)(53e^{4\sqrt{2}x}-2966e^{2\sqrt{2}x}+53)}{(e^{2\sqrt{2}x}+1)}\sin(\dfrac{x}{2})\},\\
U_{2[3]}(x)=\dfrac{-\sqrt{30}e^{\sqrt{2}x}}{(e^{2\sqrt{2}x}+1)^3}\{\dfrac{3(53e^{4\sqrt{2}x}-916e^{2\sqrt{2}x}+53)}{1536}\sin(\dfrac{3}{2}x)\\
\qquad +\dfrac{\sqrt{2}}{960}\dfrac{(e^{2\sqrt{2}x}-1)(547e^{4\sqrt{2}x}+4116e^{2\sqrt{2}x}+547)}{(e^{2\sqrt{2}x}+1)}\cos(\dfrac{3}{2}x)\},\\
V_{2[3]}(x)=\dfrac{\sqrt{30}e^{\sqrt{2}x}}{(e^{2\sqrt{2}x}+1)^3}\{\dfrac{3(53e^{4\sqrt{2}x}-918e^{2\sqrt{2}x}+53)}{512}\cos(\dfrac{3}{2}x)\\
\qquad -\dfrac{\sqrt{2}}{960}\dfrac{(e^{2\sqrt{2}x}-1)(547e^{4\sqrt{2}x}+4166e^{2\sqrt{2}x}+547)}{(e^{2\sqrt{2}x}+1)}\sin(\dfrac{3}{2}x)\}.
\end{cases}
\tag{31}
$$

In the same manner, the rest of components can be obtained using the recurrence relation (26). Substituted the obtained quantities (27), (28) and (29) in the approximation series form solution (18), the following 3-th approximation solution in the Poisson series form can be obtained

$$
\Theta_{[3]}(x,t)=\begin{pmatrix}U_{1[0]}(x)\\V_{1[0]}(x)\\U_{2[0]}(x)\\V_{2[0]}(x)\end{pmatrix}+\begin{pmatrix}U_{1[1]}(x)\\V_{1[1]}(x)\\U_{2[1]}(x)\\V_{2[1]}(x)\end{pmatrix}t+\begin{pmatrix}U_{1[2]}(x)\\V_{1[2]}(x)\\U_{2[2]}(x)\\V_{2[2]}(x)\end{pmatrix}t^2+\begin{pmatrix}U_{1[3]}(x)\\V_{1[3]}(x)\\U_{2[3]}(x)\\V_{2[3]}(x)\end{pmatrix}t^3.
\tag{32}
$$

where, $U_{\ell[m]}(x), V_{\ell[m]}(x)$ for $\ell = 1,2$ and $m = 0,1,2,3$ are listed in (27), (29) and (31). The closed form solution of (32) can be obtain as follow:

$$\theta(x,t) = \begin{pmatrix} u_1(x,t) \\ v_1(x,t) \\ u_2(x,t) \\ v_2(x,t) \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{30}}{5}\operatorname{sech}(\sqrt{2}(x-t))\cos(\frac{1}{2}(x+\frac{5}{4}t)) \\ \frac{\sqrt{30}}{5}\operatorname{sech}(\sqrt{2}(x-t))\sin(\frac{1}{2}(x+\frac{5}{4}t)) \\ \frac{\sqrt{30}}{5}\operatorname{sech}(\sqrt{2}(x-t))\cos(\frac{3}{2}(x+\frac{5}{4}t)) \\ \frac{\sqrt{30}}{5}\operatorname{sech}(\sqrt{2}(x-t))\sin(\frac{3}{2}(x+\frac{5}{4}t)) \end{pmatrix} \quad (33)$$

and substitute in (11), the traveling wave solution of coupled nonlinear Schrödinger eq. 1, obtained as follow:

$$\Phi(x,y) = u_1(x,t) + iv_1(x,t) = \frac{1}{5}\sqrt{30}\operatorname{sech}(\sqrt{2}(x-t))\exp(\frac{i}{2}(x+\frac{5}{4}t)),$$
$$\Psi(x,y) = u_2(x,t) + iv_2(x,t) = \frac{1}{5}\sqrt{30}\operatorname{sech}(\sqrt{2}(x-t))\exp(\frac{3i}{2}(x+\frac{5}{4}t)). \quad (34)$$

which are the same as the closed form solutions of exact solutions.

CONCLUSIONS

In this study, we presented the definition and operation of both two-dimensional Differential Transformation Method (DTM) and their reduced form, reduced-DTM (RDTM). These methods used for finding soliton solutions of nonlinear coupled Schrodinger equation. These methods have been applied directly without using bilinear forms, Wronskien, or inverse scattering method. It is worth pointing out that the both DTM and RDTM have convergence for the solutions, actually, the accuracy of the series solution increases when the number of terms in the series solution is increased. From computational precess of DTM and RDTM, we find that the RDTM is more easier to apply. In the other word, it is seem that DTM have very complicated computational precess rather than RDTM. The RDTM reduces the computational difficulties of the DTM and all the calculations can be made with simple manipulations MATLAB. Actually, as a special advantage of RDTM rather than DTM, the reduced differential transform recursive equations produce exactly all the Poisson series coefficients of solutions, whereas the differential transform recursive equations produce exactly all the Taylor series coefficients of solutions.

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