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Performance Measurement of Two-Stage Production Systems with Undesirable Factors by Data Envelopment Analysis

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Abstract: Data Envelopment Analysis (DEA) is a non-parametric method for evaluating the efficiency of peer Decision Making Units (DMUs) with multiple inputs and outputs. During the last few years, a great number of DEA studies have focused on two-stage production systems where all outputs from the first stage are intermediate products that make up the inputs to the second stage. In these production processes, however, both desirable and undesirable factors may be present. Obviously, undesirable factors in production process should be reduced to improve the performance. In this study, a two-stage DEA model was proposed to improve the performance of stages by increasing undesirable inputs and decreasing undesirable outputs. A real numerical example was used to illustrate the model.

Key words: Data envelopment analysis, two-stage, intermediate product, additive form, undesirable factors, efficiency

INTRODUCTION

Data Envelopment Analysis (DEA), as reported by Charnes *et al.* (1978), is a data oriented non-parametric method which is used to assess the efficiency of peer Decision Making Units (DMUs) with multiple inputs and outputs. DEA has been widely applied in a variety of fields, including banking (Seiford and Zhu, 1999; Amirteimoori, 2008), agriculture (Ghorbani *et al.*, 2009; Armagan, 2008; Laha and Kuri, 2011), education (Rayeni *et al.*, 2010; Rayeni and Saljooghi, 2010), airlines (Zandieh *et al.*, 2009; Coli *et al.*, 2011) and hospitals (Taher and Malek, 2009) among many others.

Generally, DEA enhances the performance of inefficient DMUs by reducing the inputs or increasing the outputs. However, sometimes DMUs involve both desirable and undesirable factors. DEA literature has suggested several models which can be used to measure the efficiency of DMUs with undesirable factors. For instance models proposed by Fare *et al.* (1989), Seiford and Zhu (2002), Vencheh *et al.* (2005) and Amirteimoori *et al.* (2006) among others can be observed in the literature. During the last few years, the focus of some studies have been on two-stage processes connected in series in which all outputs of the first stage are intermediate products which constitute the inputs of the second stage. For example, two-stage DEA approaches have been developed to evaluate the efficiency of the profitability and marketability of US commercial banks (Seiford and Zhu, 1999), the best 500 companies (Zhu, 2000), the Major League Baseball (Sexton and Lewis, 2003). Chen and Zhu (2004) introduced a linear DEA model, in which the efficiency of each stage is defined on its own production possibility set. Later, Kao and Hwang (2008) developed a new approach that the overall efficiency of the system is defined as the product of the efficiencies of two individual stages. This approach can be only applied under the assumption of Constant Returns to Scale (CRS). Chen *et al.* (2009) modified this model to be applied in both CRS and Variable Returns to Scale (VRS) assumptions, by using an additive form. Recently, Wang and Chin (2010) generalized Chen *et al.* (2009) model by considering the relative importance weight of two stages. None of these studies have considered the undesirable factors that may be exist in two-stage processes. Based on Chen *et al.* (2009) two-stage DEA model, a DEA model for measuring the efficiency of two-stage processes with desirable and undesirable factors would be proposed in the present research. According to the suggested approach, undesirable outputs are decreased and undesirable inputs are increased to improve the performance of two-stage processes. An empirical example of 21 firms in the banking industry in Iran is used to illustrate the model.

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CHEN *et al.*'S TWO-STAGE DEA MODEL

Figure 1 shows a graphical representation of a two-stage process. Consider n two-stage processes, DMU_j (j = 1,..., n), with m inputs, p intermediate products and s outputs. Let x_{ij} (i = 1,..., m) be the inputs of the first stage and y_{rj} (r = 1,..., s) the outputs of the second stage of the DMU_j. Also, let z_{dj} (d = 1,..., p) be the intermediate products of DMU_j that are the outputs of first stage as well as the inputs of second stage. The efficiencies of DMU_j in the first and the second stage are defined as:

$$E_1^* = \frac{\sum_{d=1}^{p} \eta_d z_{do}}{\sum_{i=1}^{m} v_i x_{io}} \quad \text{and} \quad E_2^* = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{d=1}^{p} \tilde{\eta}_d z_{do}},$$

where, v_i (i = 1,..., m) and η_d (d = 1,..., p) are the multipliers associated with inputs and outputs in the first stage and:

$$\tilde{\eta}_d \ (d=1,\ldots,p) \ \text{and} \ u_r \ (r=1,\ldots,s)$$

are the multipliers associated with inputs and outputs in the second stage.

Based on the efficiencies of DMU_j in the two individual stages, Chen *et al.* (2009) proposed the overall efficiency of DMU_j in the whole process as the weighted sum of the two individual efficiencies, namely, $E^* = w_1.E^*_1+w_2.E^*_2$, where w_1 and w_2 are user-specified weights satisfying $w_1+w_2 = 1$.

These weights are proposed to reflect the relative importance or contribution of two stages, in measuring the overall efficiency of the whole process. Their model for measuring the overall efficiency of DMU_o under the assumption of Constant Returns to Scale (CRS) can be shown as follows:

$$\begin{aligned} E^* = \max \quad & w_1.\frac{\sum_{d=1}^{p} \eta_d z_{do}}{\sum_{i=1}^{m} v_i x_{io}} + w_2.\frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{d=1}^{p} \tilde{\eta}_d z_{do}} \\ \text{s.t.} \quad & \frac{\sum_{d=1}^{p} \eta_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \\ & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{p} \eta_d z_{dj}} \le 1, \\ & u_r, v_i, \eta_d \ge 0, \ i = 1,\ldots,m, \ r=1,\ldots,s, d=1,\ldots,p. \end{aligned} \tag{1}$$

Due to the series relationship between the two stages, they assumed that $\eta_d = \tilde{\eta}_d \ (d=1,\ldots,p)$.

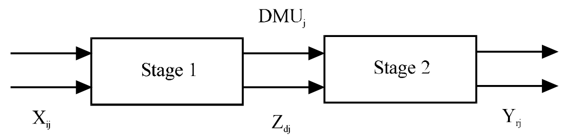

Fig. 1: Two stage process

By setting:

$$w_1 = \frac{\sum_{i=1}^{m} v_i x_{io}}{\sum_{i=1}^{m} v_i x_{io} + \sum_{d=1}^{p} \eta_d z_{do}} \quad \text{and} \quad w_2 = \frac{\sum_{d=1}^{p} \eta_d z_{do}}{\sum_{i=1}^{m} v_i x_{io} + \sum_{d=1}^{p} \eta_d z_{do}} \tag{2}$$

which, reflect the relative sizes of the two stages. By substituting (2) into (1) and using (Charnes and Cooper, 1962) transformation, model Eq. 1 becomes the following Linear Programming (LP) program:

$$\begin{aligned} \theta_o^* = \max \quad & \sum_{r=1}^{s} u_r y_{ro} + \sum_{d=1}^{p} \eta_d z_{do} \\ \text{s.t.} \quad & \sum_{i=1}^{m} v_i x_{io} + \sum_{d=1}^{p} \eta_d z_{do} = 1, \\ & \sum_{d=1}^{p} \eta_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \ j=1,\ldots,n, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{p} \eta_d z_{dj} \le 0, j=1,\ldots,n, \\ & v_i, u_r, \eta_d \ge 0, \ i=1,\ldots,m, \ r=1,\ldots,s, d=1,\ldots,p \end{aligned} \tag{3}$$

Note that from model (3), it is observed that the overall efficiency of DMU_o is measured by inputs x_{io} (i = 1,..., m), intermediate measures z_{do} (d = 1,..., p) and outputs y_{ro} (r = 1,..., s) where the intermediate products serve as both inputs and outputs of DMU_o at the same time.

Note that Chen *et al.* (2009) approach can be applied under the Variable Returns to Scale (VRS). By following the method proposed for CRS condition, the overall efficiency of DMU_o under VRS assumption can be calculated using the following linear model:

$$\begin{aligned} E_o^* = \max \quad & \sum_{r=1}^{s} u_r y_{ro} + u^1 + \sum_{d=1}^{p} \eta_d z_{do} + u^2 \\ \text{s.t.} \quad & \sum_{i=1}^{m} v_i x_{io} + \sum_{d=1}^{p} \eta_d z_{do} = 1, \\ & \sum_{d=1}^{p} \eta_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} + u^1 \le 0, \ j=1,\ldots,n, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{p} \eta_d z_{dj} + u^2 \le 0, j=1,\ldots,n, \\ & v_i, u_r, \eta_d \ge 0, \ i=1,\ldots,m, \ r=1,\ldots,s, d=1,\ldots,p, \\ & u^1 \ \text{and} \ u^2 \ \text{free in sign} \end{aligned} \tag{4}$$

A TWO-STAGE DEA MODEL WITH UNDESIRABLE FACTORS

Now, suppose a two-stage process with undesirable factors. Let there are n DMUs with two-stage structure, where each DMU_j (j = 1,…, n) in the first stage utilizes m_1 desirable inputs x^D_{ij} (i = 1,…, m_1) and m_2 undesirable inputs x^U_{ij} (i = 1,…, m_2) with m_1+m_2 = m to produce p_1 desirable intermediate products z^D_{dj} (d = 1,…, p_1) and p_2 undesirable inputs z^U_{dj} (d = 1,…, p_1) with p_1+p_2 = p. These intermediate products are used in the second stage to produce s_1 desirable outputs y^D_{rj} (r = 1,…, s_1) and s_2 undesirable outputs y^U_{rj} (r = 1,…, s_1) with s_2+s_2 = s.

Improving the performance of two-stage processes through reducing undesirable outputs and increasing undesirable inputs is the aim. In order to achieve this goal, the following two-stage DEA model under VRS assumption would be proposed on the basis of Chen *et al*. (2009) two-stage model:

$$\begin{aligned}
E_o^* = \max\ & \sum_{r=1}^{s_1} u_r^D y_{ro}^D - \sum_{r=1}^{s_2} u_r^U y_{ro}^U + u^2 + \sum_{d=1}^{p_1} \eta_d^D z_{do}^D - \sum_{d=1}^{p_2} \eta_d^U z_{do}^U + u^1 \\
\text{s.t.}\ & \sum_{i=1}^{m_1} v_i^D x_{io}^D - \sum_{i=1}^{m_2} v_i^U x_{io}^U + \sum_{d=1}^{p_1} \eta_d^D z_{do}^D - \sum_{d=1}^{p_2} \eta_d^U z_{do}^U = 1, \\
& \left[\sum_{d=1}^{p_1} \eta_d^D z_{dj}^D - \sum_{d=1}^{p_2} \eta_d^U z_{dj}^U\right] - \left[\sum_{i=1}^{m_1} v_i^D x_{ij}^D - \sum_{i=1}^{m_2} v_i^U x_{ij}^U\right] + u^1 \le 0,\ j = 1,\dots,n, \\
& \left[\sum_{r=1}^{s_1} u_r^D y_{rj}^D - \sum_{r=1}^{s_2} u_r^U y_{rj}^U\right] - \left[\sum_{d=1}^{p_1} \eta_d^D z_{dj}^D - \sum_{d=1}^{p_2} \eta_d^U z_{dj}^U\right] + u^2 \le 0,\ j = 1,\dots,n, \\
& v_i^D, v_i^U, u_r^D, u_r^U, \eta_d^D, \eta_d^U \ge 0,\ \forall i,r,d, \\
& u^1 \text{ and } u^2 \text{ free in sign}
\end{aligned} \tag{5}$$

where, v_i^D (i = 1,…, m_1), v_i^U (i = 1,…, m_2), u_r^D (r = 1,…, s_1), u_r^U (r = 1,…, s_2), η^D_d (d = 1,…, p_1) and η^U_d (d = 1,…, p_2) are the multipliers associated with desirable and undesirable inputs, outputs and intermediate products, respectively. Note that if $u^1 = u^2 = 0$ then model (5) becomes for CRS cases.

In order to evaluate the overall efficiency scores of the whole two-stage processes, this model will be solved for n times, once for each DMU. On optimality, the efficiency scores of two stages of each DMU_o (o = 1,…,n), can be calculated as follows:

$$E_o^{1*} = \frac{\sum_{d=1}^{p_1} \eta_d^{D*} z_{do}^D - \sum_{d=1}^{p_2} \eta_d^{U*} z_{do}^U + u^1}{\sum_{i=1}^{m_1} v_i^{D*} x_{io}^D - \sum_{i=1}^{m_2} v_i^{U*} x_{io}^U},\qquad E_o^{2*} = \frac{\sum_{r=1}^{s_1} u_r^{D*} y_{ro}^D - \sum_{r=1}^{s_2} u_r^{U*} y_{ro}^U + u^2}{\sum_{d=1}^{p_1} \eta_d^{D*} z_{do}^D - \sum_{d=1}^{p_2} \eta_d^{U*} z_{do}^U} \tag{6}$$

where, $v_i^{D*}, v_i^{U*}, u_r^{D*}, u_r^{U*}, \eta_r^{D*}$ and η_d^{U*} are the optimal multipliers calculated from model (5). Using the model (5), the overall efficiency of the DMU_o can be evaluated in such a way that the operations of its two stages are taken into account. Also, recognizing the inefficient sub-processes and making later improvements can be done through Eq. 6.

NUMERICAL EXAMPLE

In this study, for a group of real data derived from 21 branches of Commercial Bank situated in 10 cities of one of the provinces in Iran; proposed model would be used. These data have been adopted from banking activities of mentioned branches during 2009. Production process in any of these banks has been divided into two stages: Absorbing resources and spending resources. Inputs of the first stage include the number of personnel (x_1), expenditures (x_2) and depreciation (x_3). Intermediate products (or output in the first stage) consist of total resources (z_1). Outputs of the second stage include income (y_1), usages (y_2) and receivables (y_3). Data set derived from 21 branches of the bank has been provided in Table 1. In this data set all factors are desirables and only output y_3 is undesirable.

By applying the CRS version of model (5) and Eq. 6, the CRS efficiency scores of the whole process and two stages of the 21 banks are calculated. The results are shown in Table 2 under the heading CRS efficiency scores. It is worth mentioning that under the assumption

Table 1: Data set of 21 bank branches in Iran

DMU	x_1	x_2	x_3	z_1	y_1	y_2	y_3
1	17	150	1839	89781	2673	51114	11796
2	11	97	1914	72149	3153	57537	17861
3	7	61	1511	42654	2354	52485	14867
4	12	105	1962	97812	4782	67298	10383
5	14	123	1430	77031	1881	43487	15118
6	14	123	1409	75923	2261	41442	11947
7	9	79	1478	47763	2028	43262	16423
8	5	44	1500	45732	756	14237	3772
9	6	52	1153	55222	863	41062	31647
10	6	52	2429	53323	2469	37418	4986
11	8	70	2076	69734	2433	57883	18700
12	9	79	1652	49153	2364	47139	15773
13	8	70	2100	92365	5663	55543	7705
14	7	61	1944	64235	1361	22347	3752
15	9	79	1528	89104	2681	45717	4875
16	7	61	1728	42012	2814	73925	30614
17	7	61	2008	69360	2240	27246	4584
18	7	61	1670	51438	2293	26531	4977
19	6	52	1578	39948	1151	20223	4495
20	7	61	1514	154284	1518	43928	9464
21	7	61	1594	61101	1855	25718	4953

Table 2: Efficiency scores

	CRS efficiency scores			VRS efficiency scores		
DMU	Whole process	Stage 1	Stage 2	Whole process	Stage 1	Stage 2
1	0.513	0.479	0.585	0.681	0.695	0.598
2	0.463	0.370	0.264	0.646	0.635	0.727
3	0.434	0.277	1.000	0.821	0.820	1.000
4	0.644	0.489	0.961	0.718	0.667	1.000
5	0.485	0.529	O.213	0.808	0.862	0.533
6	0.520	0.529	0.502	0.824	0.872	0.583
7	0.398	0.317	0.651	0.779	0.779	0.664
8	0.378	0.415	0.290	1.000	1.000	1.000
9	0.455	0.470	0.422	0.995	1.000	0.112
10	0.577	0.405	1.000	0.887	0.869	1.000
11	0.457	0.395	0.613	0.690	0.697	0.616
12	0.402	0.292	0.777	0.702	0.684	0.859
13	0.688	0.524	1.000	0.779	0.732	1.000
14	0.425	0.416	0.446	0.801	0.769	1.000
15	0.728	0.572	1.000	0.864	0.835	1.000
16	0.428	0.272	1.000	0.781	0.766	1.000
17	0.485	0.450	0.562	0.811	0.782	0.978
18	0.433	0.333	0.732	0.807	0.791	0.990
19	0.361	0.748	0.685	0.889	0.882	1.000
20	0.655	1.000	0.310	0.852	1.000	0.312
21	0.437	0.396	0.541	0.815	0.820	0.764

of CRS only DMU 20 is efficient in the first stage while DMUs 3, 10, 13, 15 and 16 are efficient in the second stage. Because none of the banks perform efficiently in both stages, none of them perform efficiently for the whole process.

Finally, the VRS efficiency scores of the whole process and two stages of the 21 banks, evaluated by model (5) and Eq. 6, are reported in Table 2 under the heading VRS efficiency scores. It can be seen that DMUs 8, 9 and 20 are efficient in the first stage and DMUs 3, 4, 8, 10, 13, 14, 15, 16 and 19 are efficient in the second stage. As a result, DMU 8 is known as the only VRS overall efficient DMU.

CONCLUSION

It has been realized that DMUs may have a two-stage structure in some specific applications, where the first stage utilizes inputs to generate outputs that become the inputs of the second stage and the second stage the employs the first stage outputs to produce its own outputs. The first stage outputs are called intermediate products. Based on Chen *et al.* (2009) model, the current paper provides a two-stage DEA model in dealing with desirable and undesirable factors in data envelopment analysis (DEA). An empirical example has been examined using the proposed model.

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