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Unsteady Stagnation Point Flow and Heat Transfer over a Stretching/shrinking Sheet

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Abstract: This study investigates the problem of unsteady stagnation point flow and heat transfer over a stretching/shrinking sheet. The governing partial differential equations are converted into a system of nonlinear ordinary differential equations using a similarity transformation, before being solved numerically. Both stretching and shrinking cases are considered. It is found that dual solutions exist for the shrinking case while for the stretching case, the solution is unique. Moreover, it is found that the heat transfer rate at the surface increases as the stretching/shrinking parameter as well as the unsteadiness parameter increases.

Key words: Stagnation flow, shrinking sheet, stretching sheet, heat transfer, similarity solution

INTRODUCTION

The study of flow and heat transfer over a stretching/shrinking sheet is an important problem in many engineering processes because it has many applications in industries such as extrusion of plastic sheets, wire drawing, hot rolling and glass fiber production. Sakiadis (1961a, b) investigated the boundary layer flow on a continuously moving surface with a constant velocity. Later, this work was verified experimentally by Tsou *et al*. (1967). Following Sakiadis (1961a, b), Tsou *et al*. (1967) and Crane (1970) studied the flow over a linearly stretching sheet immersed in an ambient fluid and obtained an exact solution to the Navier-Stokes equation. Gupta and Gupta (1977) extended the work of Crane (1970) by investigating the effect of mass transfer on a stretching sheet with suction or blowing. On the other hand, the stretching boundary problem Crane (1970) was extended by Wang (1984) to a three-dimensional flow. Mahapatra and Gupta (2003) considered the stagnation flow over a stretching surface and then this problem was extended to oblique stagnation flow by Lok *et al*. (2006). Many authors such as Carragher and Crane (1982), Elbashbesy and Bazid (2000), Magyari and Keller (1999, 2000), Magyari *et al*. (2001), Liao and Pop (2004) and Nazar *et al*. (2004) investigated the stretching sheet problem with different aspects, such as uniform heat flux, permeability of the surface and unsteadiness flow and heat transfer.

Different from the stretching case, only a few works have been done on the flow induced by a shrinking sheet. Miklavcic and Wang (2006) investigated the flow over a shrinking sheet and found that the flow characteristics are different from that of the stretching case. Fang (2008) investigated the flow induced by a shrinking sheet with a power-law velocity and reported the existence of multiple solutions for certain range of the mass transfer parameter. The non-uniqueness solution of the shrinking sheet problem was also reported by Fang *et al*. (2008), when they solved the Blasius equation for the shrinking sheet. The flow characteristics induced by a shrinking sheet was also investigated by Hayat *et al*. (2007) and Sajid *et al*. (2008) and the solutions were obtained using the homotopy analysis method. The stagnation flow towards a shrinking sheet was considered by Wang (2008) where the existence of dual solutions for a certain range of the shrinking parameter was reported. He found that solutions do not exist for larger shrinking rates and may be non-unique in the two-dimensional case. This problem was then extended to a micropolar fluid by Ishak *et al*. (2010). Different from the stretching case, solutions do not exist for a shrinking impermeable sheet in an otherwise still fluid, since vorticity could not be confined in the boundary layer. However, with an added stagnation flow to contain the vorticity, similarity solutions may exist.

Different from the above-mentioned investigations, the present paper considers the problem of an unsteady

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two-dimensional stagnation-point flow and heat transfer over a stretching/shrinking sheet immersed in an incompressible viscous fluid. To the best of our knowledge, this problem has not been studied before.

MATHEMATICAL FORMULATION

Consider an unsteady stagnation point flow over a stretching/shrinking sheet immersed in an incompressible viscous fluid of ambient temperature T_∞. It is assumed that the free stream velocity is in the form, $U_\infty(x,t) = ax/(1-\lambda t)$ the sheet is stretched with velocity $U_w(x,t) = bx/(1-\lambda t)$ and the surface temperature is $T_W(x,t) = T_\infty + cx/(1-\lambda t)$. The x-axis runs along the sheet while the y-axis is measured normal to it. With these assumptions along with the boundary-layer approximations and neglecting the viscous dissipation, the governing equations are (Fang *et al*., 2011):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial U_\infty}{\partial t}+U_\infty\frac{\partial U_\infty}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

with the boundary conditions:

$$\begin{aligned} &u=U_w,\ v=0,\ T=T_w \text{ at } y=0\\ &u\to U_\infty,\ T\to T_\infty \text{ as } y\to\infty, \end{aligned} \tag{4}$$

where, u and v are the velocity components in the x and y directions, respectively, ν is the kinematic viscosity, α the thermal diffusivity and T is the fluid temperature. In order that Eq. 1-3 reduce to similarity equations, we introduce the following similarity transformation:

$$\eta=\left(\frac{U_\infty}{\nu x}\right)^{1/2} y,\ \psi=(\nu x U_\infty)^{1/2} f(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{5}$$

where, η is the similarity variable and ψ is the stream function defined as u = , $\partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, which identically satisfies the continuity Eq. 1. Substituting (5) into Eq. 2 and 3 yield the following nonlinear ordinary differential equations:

$$f'''+ff''+1-f'^2+A(1-f'-\frac{1}{2}\eta f'')=0 \tag{6}$$

$$\frac{1}{Pr}\theta''+f\theta'-f'\theta-A(\theta+\frac{\eta}{2}\theta')=0 \tag{7}$$

subject to the boundary conditions:

$$\begin{aligned} &f(0)=0,\ f'(0)=\varepsilon,\ \theta(0)=1\\ &f'(\eta)\to 1,\quad \theta(\eta)\to 0 \text{ as } \eta\to\infty \end{aligned} \tag{8}$$

where, primes denote the differentiation with respect to η, $Pr = \nu/\alpha$ is the Prandtl number, $A = \lambda/a$ is the unsteadiness parameter, $\epsilon = b/a$ is the stretching/shrinking parameter with $\varepsilon>0$ for stretching and $\varepsilon<0$ for shrinking.

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as:

$$C_f=\frac{\tau_w}{\rho U_\infty^2/2},\ Nu_x=\frac{xq_w}{k(T_w-T_\infty)}, \tag{9}$$

where, the surface shear stress τ_w and the surface heat flux q_w are given by:

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0},\quad q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{10}$$

with μ and κ being the dynamic viscosity and the thermal conductivity, respectively. Using the similarity variables (Eq. 5), we obtain:

$$\frac{1}{2}C_f Re_x^{1/2}=f''(0),\ Nu_x/Re_x^{1/2}=-\theta'(0), \tag{11}$$

where $Re_x = U_\infty x/\nu$ is the local Reynolds number.

RESULTS AND DISCUSSION

Equations 6 and 7 were solved numerically using a shooting method. The results are given to carry out a parametric study showing the influence of the non-dimensional parameters, namely the unsteadiness parameter A and the stretching/shrinking parameter ε, while the Prandtl number Pr is fixed at Pr = 0.7 (such as air), to conserve space. For the validation of the numerical results obtained, the case A = 0 (steady state flow) has also been considered and compared with those of Wang (2008) and Ishak *et al.* (2010). The quantitative comparisons are shown in Table 1 and found to be in a favorable agreement.

Figure 1 shows the variation of the skin friction coefficient in terms of f'' (0) as a function of ε for various

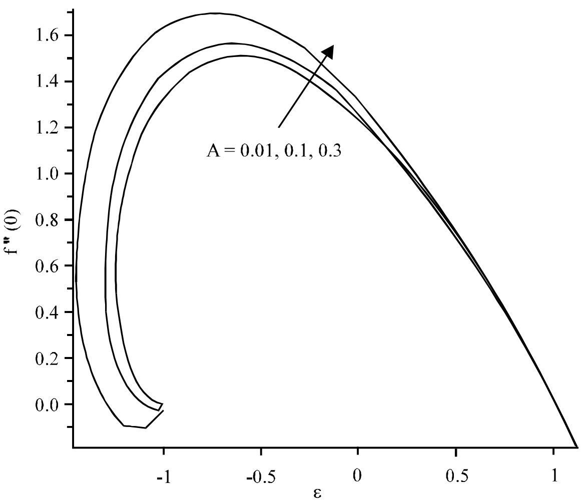

Fig. 1: Skin friction coefficient f " (0) as a function of ε for different values of A

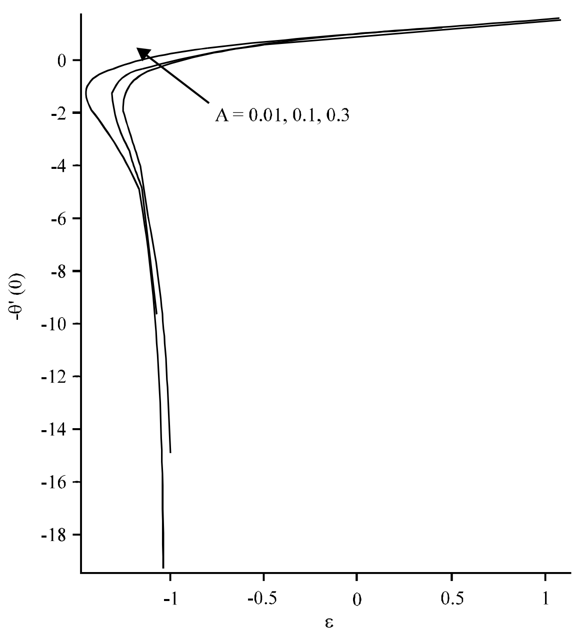

Fig. 2: Local Nusselt number -θ as a function of ε for different value of A when Pr = 0.7

Table 1: Values of f" for different values of ε

ε	Wang (2008)	Ishak *et al.* (2010)	Present results A = 0	Present results A = 0.1
-0.5	1.49567	1.495670	1.495672	1.549006
-0.1			1.308602	1.340960
0	1.232588	1.232588	1.232588	1.260923
0.1	1.14656	1.146561	1.146561	1.171193
0.2	1.05113	1.051130	1.051130	1.072329
0.5	0.71330	0.713295	0.713295	0.725457
1	0	0	0	0
2	-1.88731	-1.887307	-1.887307	-1.905564
4		-7.086378	-7.086378	-7.130017

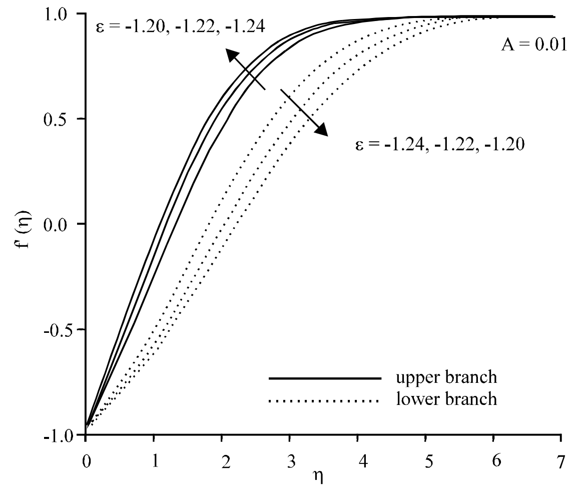

Fig. 3: Velocity profiles f ' (η) when A = 0.01

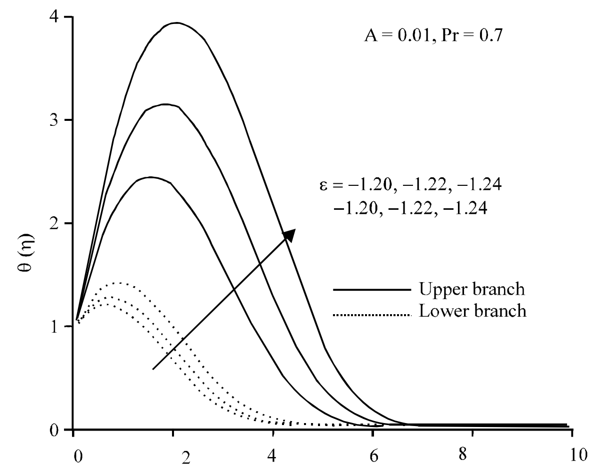

Fig. 4: Temperature profiles θ (η) when A = 0.01 and Pr = 0.7

values of A. It is seen that the range of ε for which the solution exists increases as A increases. Thus, the solution domain is widen for the unsteady flow. For a particular value of A, the solution exists up to a critical value of ε, which depends on A. Based on our computations, ε_c = 1.2465, 1.2536, -1.3118 and -1.4520 for ε = , 0.01, 0.1 and 0.3, respectively. For a particular value of ε, the skin friction coefficient is higher for higher values of A. This results in increasing manner of the local Nusselt number which represents the heat transfer rate at the surface, as presented in Fig. 2. Moreover, Fig. 2 shows that the heat transfer rate at the surface increases as the stretching/shrinking parameter ε increases. Figure 1 shows that the lower solution branch is attracted to (ε f"(0) = (1,0)), while the local Nusselt number is negative and unbounded as ε1. As discussed by previous authors (Merkin, 1994; Weidman *et al.*, 2006; Harris *et al.*, 2009; Postelnicu and Pop, 2011), we expect that the lower solution branch is unstable and not physically realizable. Although, such solutions are deprived of physical significance they are nevertheless of interest as the differential equations are concerned. Similar equations may reappear in other situations where the corresponding solutions have more realistic meaning (Ridha, 1996).

Figure 3 and 4 show the velocity and temperature profiles for selected values of parameters respectively. It is seen that there are two different profiles for particular values of ε (as shown in the figures) which support the existence of dual solutions presented in Fig. 1 and 2. Moreover, the velocity and temperature profiles satisfy the far field boundary conditions (8) asymptotically, which support the validity of the numerical results obtained.

CONCLUSIONS

A numerical study was performed to investigate the flow and heat transfer characteristics of unsteady two-dimensional stagnation point flow over a stretching/shrinking sheet. The similarity transformation reduced the partial differential equations into a system of nonlinear ordinary differential equations, which was solved numerically by a shooting method. The effects of the unsteadiness parameter A and the stretching/shrinking parameter ε were obtained and discussed. Both stretching and shrinking cases were considered. It was found that dual solutions exist for the shrinking case while for the stretching case, the solution is unique. The unsteady parameter A widen the range of ε for which the solution exists. For the upper branch solution, which we expect to be the physically relevant solution, the heat transfer rate at the surface increases as *A* as well as ε well increases.

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