Light and Surface Plasma Wave Induced Force on Nanoparticles and Nanotubes

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Abstract: Expressions for force on spherical nanoparticles and cylindrical nanotubes due to laser and surface plasma wave are obtained. It shows resonant enhancement at \( \omega = \omega_p / \sqrt{5} \) for the nanoparticle and \( \omega = \omega_p / \sqrt{2} \) for the nanotube. At frequencies lower than this resonance the ponderomotive force is along the intensity gradient of the laser while antiparallel for frequencies higher than the resonant frequency.

Key words: Nanoparticles, nanotubes, optical tweezers, surface plasma waves

INTRODUCTION

Nanotechnology has emerged as a frontline area of research during last decade with wide ranging applications (Mitin et al., 2009; Pradeep, 2007). With the development of various synthesis mechanisms nanostructures with insulating, semi-conducting and metallic properties depending on applications such as integrated, heterogeneous optoelectronics devices, nanoelectronics, biological and chemical sensing, etc., can be developed (Huang et al., 2001a; Kouklis et al., 2001; Scherer et al., 2002). The necessity of accurate and subtle manipulation of these nano sized objects requires development of new tools for manipulating and probing matter. Several techniques were developed to manipulate nanostructures employing electrical, magnetic, mechanical, fluidic effects and optical effects. The electromechanical tweezers employed electrical conducting carbon nanotubes attached to independent electrodes fabricated on pulled glass micropipettes (Smith et al., 2000). However, this technique is inherently invasive and can potentially damage the manipulated nanostructures. Magnetic forces have also been employed to trap and align nanoparticles whatever these are limited to magnetic structures only (Bentley et al., 2004). Microfluids using hydrodynamics forces to control orientation of a group of nanostructures requires a complicated pump and flow control systems and are incapable of addressing single nanoparticles (Huang et al., 2001b). Optical tweezers are a powerful tool with potential to manipulate micro/nano devices and several distinguishable advantages employs light beam or beams gradient force to trap the particle (Kim and Lieber, 1999; Khan et al., 2006). Yu et al. (2004) have demonstrated optical tweezers to trap, manipulate and rotate CuO nanorods.

The application of optical tweezers is limited by the requirements of incident power and subwavelength trapping. Surface Plasma Wave (SPW) on the other hand is a hybrid mode of light and collective electron oscillations excited at a metal/dielectric interface, is expected to overcome the limitations of conventional optical trapping (Maier and Atwater, 2005). SPW based sensors have been widely used to identify biomolecules such as DNAs, proteins and other large chain branched molecules. In Surface Enhanced Raman Spectroscopy (SERS) a laser illuminates a rough metal surface with metallic particles attached to it. A roughened optical fiber coated with silver can also be employed for SERS detection (Liu et al., 2006). It is well understood that the enhancement of optical field is caused by localized surface plasmon modes and it has been demonstrated that diverse optical effects can be enhanced near the resonantly excited metal nanoparticles. Fang et al. (2009) have demonstrated surface plasmon based tweezers and trapped colloidal silver nanoparticles without optical interactions.

In this study, we deduce expressions for the ponderomotive force on nanoparticles and nanotubes due to laser and SPW. The laser/SPW displaces the electron cloud of a nanoparticle (nanotube) with respect to the ion sphere. The ion sphere exerts a restoration force on electrons. As a result electron oscillatory velocity shows resonance at a resonant frequency. The oscillatory velocity couples with wave magnetic field to exert a ponderomotive force on the nanoparticle (nanotube).

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PONDEROMOTIVE FORCE ON A NANOPARTICLE DUE TO LASER

Consider a two-dimensional Gaussian laser beam with electric and magnetic fields:

$$\vec{E} = e^{-\Delta^2 / 2}$$
$$\vec{B} = \frac{ck}{\mu_0} A(y) - \frac{\varepsilon}{\omega} \frac{\partial A(y)}{\partial y} e^{i(\sigma t - k_0 z)}$$

impinging on a nanoparticle of radius $a$ and electron (ion) density $n_e$. Under the influence of laser electric field, the electron cloud of the particle gets displaced by an amount $\Delta \hat{x}$, with respect to ions, which are considered immobile (Fig. 1).

The ions attract the electron cloud and there acts a net restoration force on electron cloud. At a distance $\vec{r}$ from the center of ion charge (taken as origin) the electric field due to the ion sphere of charge density $n_i e$, can be written, using Gauss’s law, as:

$$E_r = \frac{4\pi n_i e}{3} \frac{1}{r}$$

where, $\varepsilon$ is the dielectric constant of the nanoparticle.

Similarly, the electric field due to the electron cloud at $\vec{r}$ can be written as:

$$E_r = \frac{4\pi n_e e}{3} (\vec{r} - \Delta \hat{x})$$

Using Eq. 2 and 3, the net electric field in the overlap region is:

$$E_n = \frac{4\pi n_e e}{3} \frac{1}{r} (\vec{r} - \Delta \hat{x})$$

which is uniform. Since, most of the electrons (when $\Delta$ is small) are in the overlap region, the force experienced by each electron is:

$$- e E_n = - e \frac{4\pi n_e e^2}{3\varepsilon} \Delta \hat{x}$$

The displacement of electron cloud of nanoparticle is governed by equation of motion $m(d^2\vec{r}/dt^2) = - e \vec{E}$ as follows:

$$m\frac{d^2\Delta}{dt^2} = - e E_n - e E_i$$

using $d/dt = -i\omega$ and Eq. 5 in Eq. 6, we obtain the displacement of electron cloud $\Delta$ as:

$$\Delta = -\frac{e E_i}{m(\omega^2 - \omega_0^2 / 3\varepsilon)}$$

where, $\omega_0^2 = 4m_i e^2 / m$.

The velocity of electron cloud of a nanoparticles using Eq. 7 can be written as:

$$\dot{\vec{r}} = \frac{d\Delta}{dt} = -i\omega \Delta = -\frac{e E_i i\omega}{m(\omega^2 - \omega_0^2 / 3\varepsilon)}$$

The laser exerts a ponderomotive force on the nanoparticles given by:

$$F_n = -\frac{m}{2} \vec{v} \cdot \nabla \vec{v} = -\frac{e E_i}{2c} \vec{B}^*$$

where, $^*$ denotes the complex conjugate and we have used the identity Re $\vec{A} \vec{B}^* = (1/2) Re(\vec{A} \cdot \vec{B}^* + \vec{A} \cdot \vec{B}^* \cdot \vec{B} + \vec{B}^* \cdot \vec{A})$. Using Eq. 1 and 8 in 9 we get the ponderomotive force on nanoparticle due to laser:

$$F_n = -\frac{e E_i}{2c} \text{Re} \left[ \frac{1}{m(\omega^2 - \omega_0^2 / 3\varepsilon)} \left( \frac{ck}{\omega} A_{\hat{y}} - \frac{\varepsilon}{\omega} \frac{\partial A_{\hat{y}}}{\partial y} \right) \right]$$

where, Re denotes the real part of the expression.

PONDEROMOTIVE FORCE ON ANANOTUBE DUE TO LASER

Consider the interaction of a laser with a nanotube of radius "a" and electron and ion density $n_e$. The laser
displaces the electron cloud with respect to the ion cylinder. This leaves behind the positive ions termed as ion tube. Let the net displacement of electron tube be $\Delta$ with respect to ion tube (Fig. 2).

Following the procedure given above the space charge electric field at a distance $r$ from the center of ion tube in radial direction due to the ion tube, is:

$$\vec{E}_i = \frac{2\pi n_b e}{\varepsilon} \vec{r}$$  \hspace{1cm} (11)

Similarly, the electric field at $r$ due to the electron tube is:

$$\vec{E}_e = \frac{2\pi n_e e}{\varepsilon} (\vec{r} - \Delta \hat{z})$$ \hspace{1cm} (12)

The net field in the overlap region $\vec{E}_i = \vec{E}_i + \vec{E}_e$ comes out to be:

$$\vec{E}_i = \frac{2\pi n_b e}{\varepsilon} \Delta \hat{z}$$ \hspace{1cm} (13)

The restoration force on each electron in the overlap region can be written as:

$$e \vec{E}_i = \frac{2\pi n_b e^2}{\varepsilon} \Delta \hat{z}$$ \hspace{1cm} (14)

Following, the procedure adopted for a nanoparticle, the velocity of electron cloud of a nanotube can be obtained as:

$$\vec{v} = \frac{d\Delta}{dt} = -i\omega \Delta = \frac{e\vec{E}_i i\omega}{m(\omega^2 - \omega_0^2 / 2\varepsilon)}$$ \hspace{1cm} (15)

and the ponderomotive force on the nanotube using Eq. 15 in 9 turns out to be:

$$\vec{F}_p = -\frac{e^2 A_0^2 \vec{E} \cdot \vec{E}^*}{2m(\omega^2 - \omega_0^2 / 2\varepsilon)}$$ \hspace{1cm} (16)

**PONDEROMOTIVE FORCE ON A NANOPARTICLE AND A NANOTUBE PLACED OVER AN OPTICAL FIBER**

Consider an optical fiber of radius $a'$ and permittivity of core $\varepsilon_c$. A portion of the cladding of the fiber is removed and a nanoparticle is positioned in this region (Fig. 3).

A laser propagates through this structure in azimuthally symmetric TM mode with $t-z$ variations as $\exp(-i(\omega t - k_z z))$. The field variation in the radial direction is governed by the wave equation:

$$\frac{\partial^2 E_{t \phi}}{\partial r^2} + 1 \frac{\partial E_{t \phi}}{\partial r} \left( \frac{\alpha'}{\varepsilon_c} - k_z^2 \right) E_{t \phi} = 0$$ \hspace{1cm} (17)

where, $\alpha = \varepsilon_c$ for $r < a$ and $\alpha = 1$ for $r > a$.

The well-behaved solutions of Eq. 17 in the region $r > a$ can be written following (Liu et al., 2006):

$$E_{t \phi} = A \left[ J_0(k_c r) \hat{z} \frac{\partial}{\partial k_c} L_0(k_c r) \right] e^{-i(k_c r)}$$ \hspace{1cm} (18)

where, $k_c = (\omega^2 / c^2 - k_z^2)^{1/2}$, $\varepsilon_c$ is the permittivity of core (glass) of the fiber and prime over $I_0$ denotes differentiation with respect to argument and $A$ is a constant.

Using Faraday's law, $\nabla \times \overrightarrow{E} = -(1/c)(\partial \overrightarrow{B} / \partial t)$, the magnetic field of the wave can be written as:

$$\vec{B} = \frac{e k_c}{k_0} A J_0(k_c r) e^{-i(k_c r)}$$ \hspace{1cm} (19)

The electron cloud of nanoparticle gets displaced under the influence of the wave electric field. Here, we can write the velocity of electron cloud as:

![Fig. 2: Displacement of electron cloud of a nanotube](image)

![Fig. 3: Schematic for interaction of a nanoparticle/ nanotube with laser guided through an optical fiber](image)
Using Eq. 19 and 20, the ponderomotive force on the nanoparticle placed over an optical fiber comes out to be:

\[ \vec{F}_{p} = \frac{\epsilon k \cdot \vec{A} \left( L_{p}(r, \omega) \right) \mid \vec{k}_{r} \cdot \vec{k}_{r}^{*}}{2m_{e} c (\omega^{2} - \omega_{p}^{2} / 3 \epsilon)} \]  

(21)

We now replace the nanoparticle by a nanotube; the ponderomotive force on the nanotube following Section III can be obtained. The velocity of electron cloud of a nanotube under the electric field of the wave can be written as:

\[ \vec{v}_{e} = \frac{e \vec{E} \cdot \omega}{m(\omega^{2} - \omega_{p}^{2} / 3 \epsilon)} \]  

(22)

Using Eq. 19 and 22, the ponderomotive force on the nanotube comes out to be:

\[ \vec{F}_{p} = \frac{\epsilon k \cdot \vec{A} \left( L_{p}(r, \omega) \right) \mid \vec{k}_{r} \cdot \vec{k}_{r}^{*}}{2m_{e} c (\omega^{2} - \omega_{p}^{2} / 3 \epsilon)} \]  

(23)

**PONDEROMOTIVE FORCE ON A NANOPARTICLE AND A NANOTUBE DUE TO SURFACE PLASMA WAVE**

We now consider an optical fiber, coated with metal in the region a<r<b of effective permittivity \( \varepsilon_{e} \). A nanoparticle is placed over the fiber (Fig. 4). The field of the surface plasma wave in the region a<r<b (Eq. 17), can be written using the boundary conditions for continuity of \( \vec{E} \) and \( \varepsilon \vec{E} \) at \( r=a \):

\[ \vec{E}_{p} = \frac{A k \cdot \vec{e} \left( L_{p}(r, \omega) \right) \mid \vec{k}_{r} \cdot \vec{k}_{r}^{*}}{L_{p}(r, \omega) - L_{p}(r, \omega) K_{0}(r, \omega) + K_{0}(r, \omega) K_{0}(r, \omega)} \]  

(24)

where, \( \alpha_{e} = [k_{r}^{2} - \omega^{2} / \varepsilon_{e}^{2}]^{1/2} \).

Using Faraday’s law, \( \vec{E} = -(1/c)(\partial \vec{B} / \partial t) \), the magnetic field of the wave can be written as:

\[ \vec{B}_{p} = \frac{A k \cdot \vec{e} \left( L_{p}(r, \omega) \right) \mid \vec{k}_{r} \cdot \vec{k}_{r}^{*}}{L_{p}(r, \omega) - L_{p}(r, \omega) K_{0}(r, \omega) + K_{0}(r, \omega) K_{0}(r, \omega)} \]  

(25)

![Fig. 4: Schematic for interaction of a nanoparticle/nanotube with surface plasma wave](image)

The electron cloud of nanoparticle gets displaced under the influence of the wave electric field. Here, we can write the velocity of electron cloud as:

\[ \vec{v}_{e} = \frac{e \vec{E} \cdot \omega}{m(\omega^{2} - \omega_{p}^{2} / 3 \epsilon)} \]  

(26)

Using Eq. 25 and 26, the ponderomotive force on the nanotube due to surface plasma wave comes out to be:

\[ \vec{F}_{p} = \frac{A k \cdot \vec{e} \left( L_{p}(r, \omega) \right) \mid \vec{k}_{r} \cdot \vec{k}_{r}^{*}}{L_{p}(r, \omega) - L_{p}(r, \omega) K_{0}(r, \omega) + K_{0}(r, \omega) K_{0}(r, \omega)} \]  

(27)

We now replace the nanoparticle by a nanotube. The ponderomotive force on the nanotube due to surface plasma wave can be obtained following Section IV. The velocity of electron cloud of a nanotube under the electric field of the wave can be written as:

\[ \vec{v}_{e} = \frac{e \vec{E} \cdot \omega}{m(\omega^{2} - \omega_{p}^{2} / 3 \epsilon)} \]  

(28)

Using Eq. 25 and 28, the ponderomotive force on the nanotube comes out to be:

\[ \vec{F}_{p} = \frac{A k \cdot \vec{e} \left( L_{p}(r, \omega) \right) \mid \vec{k}_{r} \cdot \vec{k}_{r}^{*}}{L_{p}(r, \omega) - L_{p}(r, \omega) K_{0}(r, \omega) + K_{0}(r, \omega) K_{0}(r, \omega)} \]  

(29)

For \( \omega < \omega_{p} \), the nanotube is pushed away from the optical fiber whereas for \( \omega > \omega_{p} \), the nanotube is attracted towards the optical fiber.
CONCLUSIONS

The ponderomotive force of a laser can play an important role in optical tweezers. Depending upon the frequency of the wave one can push or pull the nanoparticle or a nanotube. Optical fiber plays an important role in precisely targeting the nanoparticle or nanotube as well as in guiding the laser. A metal coated optical fiber can support a surface plasma wave, which in turn can also be employed in pushing or pulling of nanostructures.

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REFERENCES


