Analyzing Long and Short-run Relationships Between Comex Gold and Silver Futures

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Abstract: The objective of this research was to study the relationship between gold and silver futures in COMEX market by using daily data from 6th of May, 1991 to 6th of May, 2011. There were 5520 observations. To investigate the short run and long run relationships between gold and silver futures price, this study employed unit root test, co-integration test and error correction model. If there are short-run and long-run relationships between gold and silver futures prices, it implied that gold can be used as a predictor to forecast the silver as well. It will provide an opportunity to market participants, investors, hedgers, arbitragers and speculators. The traders can use this relation between gold and silver futures price as trading strategy in order to diversify risk in the portfolio. Speculators can use this information to predict returns and make the investment strategies. Hedgers can use these markets as substitutes against the similar type of risks in a period of time. The empirical results revealed that there is a robust positive relationship between gold and silver future prices in the long run. For the short run relationship, change in the silver futures price significantly affects the gold futures price and vice versa.

Key words: Gold and silver futures price, co-integration, co-movement, error correction model

INTRODUCTION

The portfolio management has been one of the most popular topics in finance research (e.g., Ahartor and Olopoe, 2010; Ferruz et al., 2007; Krishnasamy et al., 2006; Lye, 2011; Matalin-Saez, 2009; Ozun and Cifter, 2007). Over the past several years, researchers have focused on studying relation between gold and silver. Gold and silver were utilized in various respects such as accessories, decorations and electronic equipment etc. In term of financial investment, many people held gold and silver in stores since they considered these metals as substitutes reducing similar type of risk in the portfolio. Especially silver, it played an important role as an industrial metal for ages and it was also considered as the asset for investment. With the uncertainty of the economics and inflation, the metal commodity, including silver became the safe haven for investors. Many people were interested in investing in the silver together with investing in the gold due to the price of the silver dramatically increased and the depreciation in dollar index since year (2010). Trading gold and silver was the same as trading other financial assets as buying and selling decisions at a point of time were based on the expectations concerning uncertain future outcomes.

Although, there were many ways to trade in gold and silver, one of the popular way was trading in the futures market. The dramatic rise in prices of both metals in the past five years was affected by many reasons such as the economic crisis, inflation, international instability etc. Thus, the gold and silver futures markets attracted the considerable attention of many investors because they realized the opportunity in making the profit in the fluctuating gold and silver price.

The gold and silver were incidentally considered as a safe haven. They were used for the wide range of benefits. Many people believed that gold was an effective hedge. Airovich (1983), Aggarwal (1992), Ghosh et al. (2004) and Levin and Wright (2006) pointed out that gold can be hedge against inflation and political uncertainty in the long run. Ranson (2005) showed that the gold was the better way to hedge against inflation than inflation index bonds. Capie et al. (2005) determined that gold can also uses to hedge against the fluctuations of the US dollar in the foreign exchange market.

The relationship between the gold and silver has been studied for long time in various methods. Several studies used Co-integration technique to examine the relationship. Wahab et al. (1994) used data in cash and futures to find the co-integration between the gold and
silver. They found that there are co-integration and no arbitrage opportunities between both markets. Ciner (2001) observed daily data on the price of gold and silver futures contracts traded on the Tokyo Commodity Exchange from 1992 to 1998. The results implied that gold and silver futures markets were separate different markets. The gold futures price could not be used to predict the silver futures price. These two commodities could not be replaced by each other in financial activities.

This paper used the gold futures as a benchmark. It examined the short-run and long-run relationship between the prices of gold and silver futures contracts by using data traded on COMEX (US) to find whether gold and silver futures price had a common stochastic trend and could be modeled as co-integrated processes. If there is a short-run and long-run relationship between gold and silver futures prices, it can be said that gold can be used as a predictor to forecast the silver as well. It will provide an opportunity to market participants, investors, hedgers, arbitragers and speculators. Speculators can use this information to predict returns and make the investment strategies. Hedgers can use these markets as substitutes against the similar type of risks in a period of time.

**EMPIRICAL MODEL AND DATA**

The data used in the paper were the historical price of gold and silver futures contract from COMEX (US). Data source were from Reuters. The daily prices of the gold and silver futures were collected from 6th of May, 1991 to 6th of May, 2011. There were about 5520 sample data. Each variable are transformed by using a natural logarithm before analysis so as to avoid influence on all the empirical results by extreme data and to improve the accuracy of the tested data. This study examined 4 series of data: Gold futures price (G), Silver futures price (SF), Logarithms of gold futures (LG) and Logarithms of silver futures (LSF) by using the software named EVIEW7 to test the statistical process.

This study examines time series data in order to find the relationship between the gold and silver futures. Most time series data are non-stationary variable which can be clearly seen that mean, variance and covariance keep up with time variable. Using time series data without testing the stationarity of the data, the result of the relation may cause spurious regression. The outcome of the study will be inaccuracy and unreliable. Therefore, Augmented Dickey-Fuller (ADF) test is adopted to test the stationary stochastic process. If time series data is non-stationary, Cointegration technique and Error Correction Mechanism (ECM) model are used to make the model more precise.

Time series data is a random variable that can be called stochastic process. The stochastic process consists of stationary and non-stationary stochastic process. Using non-stationary data causes the spurious regression hence the results from these data may be unreliable. To explain the non-stationary, let $Y_t$ be a stochastic time series with these properties:

- **Mean:** $E(Y_t) = \mu$
- **Variance:** $\text{Var}(Y_t) = E(Y_t - \mu)^2 = \sigma^2$
- **Covariance:** $E(Y_t - \mu) (Y_{t+h} - \mu) = \gamma_h$

Time series data will be non-stationary if mean, variance or covariance are not constant. These statistical data depend on variable t. GF and SF are also time series data thus they need to be done in stationary test.

**Unit root test:** The time series data always perform the autocorrelation which cause the spurious regression. Accordingly, in order to improve the accuracy of the research and solve the autocorrelation problem, the Augmented Dickey-Fuller (ADF) test, developed by Said and Dickey (1984), is used to make the stationary test. The ADF test can be estimated under three equations different forms including random walk, random walk with drift and random walk with drift and trend.

The ADF test of silver futures consists of estimating the following regression:

- **Random walk process:**

  $$\Delta SF_t = \delta SF_{t-1} + \sum_{i=1}^{n} \alpha_i \Delta SF_{t-i}$$

- **Random walk with drift:**

  $$\Delta SF_t = \beta_0 + \delta SF_{t-1} + \sum_{i=1}^{n} \alpha_i \Delta SF_{t-i}$$

- **Random walk with drift and deterministic trend:**

  $$\Delta SF_t = \beta_0 + \beta_1 \Delta SF_{t-1} + \sum_{i=1}^{n} \alpha_i \Delta SF_{t-i}$$

The ADF test of gold futures consists of estimating the following regression:

- **Random walk process:**

  $$\Delta GF_t = \delta GF_{t-1} + \sum_{i=1}^{n} \alpha_i \Delta GF_{t-i}$$

- **Random walk with drift:**

  $$\Delta GF_t = \beta_0 + \delta GF_{t-1} + \sum_{i=1}^{n} \alpha_i \Delta GF_{t-i}$$
Random walk with drift and deterministic trend:

\[ \Delta GF_t = \beta_t + \delta GF_{t-1} + \sum_{i=1}^{r} \sigma_i \Delta GF_{t-i} \]

where, \( \Delta GF_t = (GF_t - GF_{t-1}) \), \( \Delta GF_{t-i} = (GF_{t-i} - GF_{t-i-1}) \), \( \sigma_i \) is a white noise error term and \( \delta \) is the coefficient of lagged \( GF_{t-1} \).

At this point, the series in concerning the gold and silver futures price denoted by \( GF_t \) and \( SF_t \), respectively. The empirical analysis proceeds by testing for unit root of the series in logarithm of \( GF_t \) and \( SF_t \) (LGF and LSF). The ADF test will be conducted by including lagged value of \( \Delta GF_t \) and also \( \Delta SF_t \) until the error term \( \Delta t \) is serially uncorrelated or in practical manner using the Akaike Information Criterion (AIC). The AIC is defined as:

\[ AIC = -2(l/T) + 2(k/T) \]

where, \( k \) is the number of parameters estimated, \( l \) is the value of the log of likelihood function using \( T \) observations. The model with the lowest value of AIC is preferable.

Stationary test are considered the following hypothesis:

\[ H_0 : \delta = 0 \]
\[ H_1 : \delta \neq 0 \]

The null hypothesis of the ADF test is that a series is non-stationary. The parameter of interest in all the regression equations is \( \delta \). To investigate the hypothesis, we use the critical value of ADF test from MacKinnon (1991) comparing with t-statistic of GF, SF. If the sequence accepts the null hypothesis of unit root or \( \delta = 0 \), it is non-stationary or I(1) variable. If the sequence rejects the null hypothesis of unit root, it is stationary or I(0) variable.

Cointegration technique: Suppose we subject LGF and LSF, to unit root analysis and find that they are non-stationary. These data are individually I(1) and have stochastic trends. To run the regression between LGF and LSF, could give us a spurious regression where there might be a high R² and significant t-statistic, but the results are without any economic meanings. Thus, cointegration technique is used instead. To be specific, we run the regression of LSF on LGF in order to find the regression residuals:

\[ LSF_t = \alpha + \beta LGF_t + \epsilon_t \]

The regression residuals in order to test the cointegration analysis; therefore, we examine \( \epsilon_t \) to unit root test.
where, $\varepsilon_i$ is a white noise error term, $e_{i-1}$ is the lagged value of the error term and $a_i$ is a speed of adjustment from disequilibrium to equilibrium or the long run relationship (Enders, 2009).

In this study we adopt (Enders, 2009) model, this model can be divided in ECM forward and ECM backward, which can be seen by the model below, respectively:

\[
\Delta \text{LSF}_t - a_{1,t} + 1 + \sum_{i=1}^{p} \psi_{\Delta \text{LSF},i} - 1 + \sum_{i=1}^{p} \Phi_{\Delta \text{LSF},i} \varepsilon_i
\]

\[
\Delta \text{LGF}_t - a_{1,t} + 1 + \sum_{i=1}^{p} \psi_{\Delta \text{LGF},i} - 1 + \sum_{i=1}^{p} \Phi_{\Delta \text{LGF},i} \varepsilon_i
\]

where, $e_i$ is cointegration error, $\varepsilon_i$ and $\varepsilon_i$ are white noise disturbance terms.

In this model, we can use standard OLS to estimate the parameters since each variables are now stationary assuming that \text{LSF}_i and \text{LGF}_i are I(1) and they are cointegrated. The lagged terms of $\Delta \text{LSF}_i$ and $\Delta \text{LGF}_i$ will be included in each equation to yield the serially uncorrelated residuals. These lag lengths can be determined by AIC value as in the unit root test. Error correction mechanism test is considered in the following hypothesis:

\[H_0: a_i, a'_{i} = 0 \quad H_1: \Phi_i, \Phi = 0\]

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The important of the parameter $a_i$ is to indicate the speed of adjustment from deviation in short run to the long run equilibrium relationship. For the parameter $\Phi_i$, it indicated the short term relationship between these two variables. If the result rejects the null hypothesis or variable $\Phi$ is significantly different from zero, we can conclude that variable LSF and LGF are cointegrated and have a short term relationship. On the other hand, if the result accepts the null hypothesis, we can conclude that variable LSF, and LGF do not have a short term relationship.

**RESULTS**

We selected the ADF and used Schwarz Info Criterion for the Unit root test method. As shown in the Table 1 and 2, all variables contained unit root and they were non-stationary at level. However, they were stationary at the first difference or I(1) as shown in Table 1 and 2.

From the unit root test, we found that both of gold and silver futures price were stationary at I(1). This also meant that there was a possibility of the existence of long-term equilibrium among the gold and silver futures. As a result, we applied the cointegration technique in order to find the long run relationship between these two variables. We considered the logarithms of gold and silver futures data in co-integration test. We conducted regression of LSF, on LGF, and regression of LGF, on LSF, using LS-Least Squares (NLS and ARMA) method. Then we conducted the stationary tests of the residual error from regression (e_{t}^{LSF}, e_{t}^{LGF}). The model for testing the regression residual was random walk process (no intercept and trend). According to Table 4 and 5, ADF test statistics were compared with Test critical values (MacKinnon, 1991) in Table 3. Both residual error term of $\Delta \varepsilon^{LSF}$ and $\Delta \varepsilon^{LGF}$ were stationary at 95% confident interval. We concluded that both of gold and silver futures price had co-integration or long term relationship.

The Granger representation theorem asserted that the short run dynamic equilibrium of any two co-integrated time series data could be described by the Error Correction Model (ECM). Since we obtained the results, where LGF, and LSF, are co-integrated, the ECM of these

<table>
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<th>Variables</th>
<th>Intercept</th>
<th>Lag</th>
<th>Trend and intercept</th>
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two variables could be constructed. The lag lengths were determined by AIC value as in unit root test. We estimated the error correction by using LS-Least Squares (NLS and ARMA). The lagged term in each equation was chosen on the basis of lowest AIC value which in this case equal to zero lag for both ΔLSF, and ΔLGF. Table 6 and 7 below demonstrated the results of the models employing co-integration error, which were defined as error correction forward and backward model. The parameter of the co-integration error (εₜ) was significant at 5% level for equation that ΔLGF, was a dependent variable in ECM (t-statistic is -2.58), while it was insignificant for equation that ΔLSF, was a dependent variable. The coefficients of co-integration error implied the response of the previous period’s deviation into the long run relationship. Our results appear to support the finding in Wahab et al. (1994) which used data in cash and futures to find the co-integration between the gold and silver. They found that there are co-integration and no arbitrage opportunities between both markets. However our results are contrast sharply to Ciner (2001) which observed that The gold futures price could not be used to predict the silver futures price.

**DISCUSSION**

The daily prices of gold and silver futures are used in this study. All of the data are collected in time series data. Using time series data may cause the spurious regression thus unit root test (ADF test) is adopted in the stationary test in order to avoid this problem. After doing the unit root test, both of gold and silver futures prices are non-stationary at level but they are stationary at first difference level. These non stationary parameters at level can be estimated by using cointegration test to find the long term equilibrium relationship between the parameters. Although both parameters have long term relationship, the price may be fluctuated from the equilibrium in long term. However, the direction of the price should move to the equilibrium. Thus, we consider the short term relationship between the parameters by using Error Correction Mechanism (ECM) in order to examine how fast the gold and silver futures price move to the equilibrium.

We can produce the cointegration equation to forecast long run relationship as follow:

\[
\text{LSF}_t = -4.863745 + 1.113532 \times \text{LGF}_t
\]

\[
\text{LGF}_t = 4.552819 + 0.801485 \times \text{LSF}_t
\]

The silver futures price move in the positive direction with the gold futures price by changing in gold futures price in one percent make the silver futures price increase 1.11 percent. On the other hand, changing in silver futures price in one percent make the gold futures price increase...
0.80%. We summarize that the gold and silver futures price from the past to present moving in the same direction for the long term equilibrium relationship.

We can produce the error correction mechanism equation to examine the short run relationship as follow:

\[
\Delta \text{LSF}_t = 0.000109 - 0.0000947 \text{e}_{t-1} + 0.396784 \Delta \text{LGF}_{t-1}, \\
\Delta \text{LGF}_t = 0.00000672 - 0.00251 \text{e}_{t-1} + 1.274607 \Delta \text{LSF}_{t-1}
\]

For a dependent variable \(\Delta \text{LGF}_t\), the coefficient of the cointegration error (\(e_{t-1}\)) is -0.00251, which is significant at 5% level. When there is the situation that makes the gold futures price out of the equilibrium in the long term relationship, the gold futures price will be back in the equilibrium with the speed of -0.00251. For a dependent variable \(\Delta \text{LSF}_t\), the coefficient of the cointegration error (\(e_{t-1}\)) is: -0.000947, which is not significant at 5% level.

For a dependent variable \(\Delta \text{LGF}_t\) and \(\Delta \text{LSF}_t\), the coefficients are 0.396784 and 1.274607, which are significant at 5% level. We can conclude that changing in gold futures price and changing in silver futures price are affected each other. According to the equation above, the coefficient of \(\Delta \text{LSF}_{t-1}\) and \(\Delta \text{LGF}_{t-1}\) are positive thus, the gold and silver futures prices move together in the same direction. It can imply that changing in silver futures prices one percent can make gold futures prices change 1.274607% and changing in gold futures prices one percent can make silver futures prices change 0.396784%. In summary, varying in silver futures price can make gold futures price change in the short term relationship and also in varying in gold futures price can make silver futures price change in the short term relationship.

**CONCLUSION**

Today gold and silver become the metals, which play a significant role in the commodity market. We can make a good decision if we understand the reasons for the recent, sudden changes in the value of precious metals, gold and silver and their mutual relationship. This paper has examined throughout the long term and short term linkages between the prices of the gold and silver futures contracts traded on the COMEX (US). The cointegration and error correction mechanism are applied in this research. The results support many studies that the evidence of cointegration is found in the gold and silver futures. The price of gold and silver futures contract shows the long term relationship in the positive direction for a long time. For the short term relationship, ECM model shows that changing in price of silver futures and gold futures have short run relationship between each other.

It can be implied that these two markets can be regarded as substitutes to hedge against similar type of risks. Hedgers can use these contracts as a way to manage their price risk on an expected purchase or sale of the physical metal. They also provide speculators with an opportunity to participate in the markets without any physical backing. Hedgers can diversify risks by opening another futures contract in the opposite position. This hedge allows you to keep your physical gold or silver for the long term while holding a short term position that potentially yields a significant return. Hedging your physical gold and silver positions can be an excellent strategy to diversify risk in your portfolio, take advantage of the volatility and significant corrections in the precious metals markets. Speculators and investors can make profit from fluctuation in gold and silver futures price. Although, these two commodities have different economics uses and they are affected by different economic fundamental.

**RECOMMENDATIONS**

Gold and silver prices are rising over the last decade and they generally provided the high levels of returns. High return attracts many investors to come in this market. Trading gold and silver futures could be considered more of speculative activity than an investment activity comparing from the past. Sometimes the price of the gold and silver are determined by the interaction of supply and demand. In the contest of gold and silver, they are not immediately clear and easy to explain. However, gold and silver have provided excellent returns over the last decade, this does not guarantee the similar performance in the futures. Hedging can greatly reduce the exposure to price risk. It is an important marketing tool for establishing price while retaining considerable marketing flexibility. However, hedging does not guarantee a profit. The hedging decision must still take into account production costs and market outlook. There appears to be considerable risk in trading the commodity futures, not the least of which is the possibility of a change in the process generating the price changes for the gold and silver futures price.

**REFERENCES**
