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Free Vibration Analysis of Truncated Conical Composite Shells using the Galerkin Method

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Abstract: Truncated conical shells are extensively used in space crafts, robots, shelters, domes, tanks and in machinery or devices. Thus, the study of their vibrational characteristics has long been of interest for the designers. Moreover, because of the need for light weight designs, composite shell materials have become more and more common. One of the advantages of composite materials is that one can design directional properties into them almost on demand. The purpose of this study was to analytically investigate the vibrational behavior of composite conical shells. Based upon the thin shell theory, the governing equations of motion were derived. The Galerkin method along with beam mode shapes such as weighting functions was employed. The boundary conditions were expected to significantly affect the mechanical behavior of shell-type structures and making use of the beam modal functions made it possible to examine their role in the mechanical behavior of conical shells. The results of the present study, which were in excellent agreement with the existing data from the literature, indicated the considerable effect of boundary conditions on the natural frequencies of shells.

Key words: Conical shells, natural frequency, Hamilton's principle, beam function, Galerkin method

INTRODUCTION

Vibration of shells is important for different fields of engineering applications. Accordingly, many efforts have been made in studying the vibrations of plates and shells with different scales. Sharma and Mittal (2010) presented a review on stress and vibration analysis of composite plates. Jayakumar *et al.* (2006a) studied multi-layer cylindrical shells under electro-thermo-mechanical loads. Jayakumar *et al.* (2006b) also investigated on piezoelectric cylindrical shells under thermal and pressure loads. They presented a closed-form solution, utilizing a classical stress formulation approach to carry out elasto-electro-thermo analysis of generalized plane-strain of a right circular cylindrical shell. Some researchers have considered the free vibrations of conical shells due to their use in nozzles. Garnet and Kemper (1964) analyzed the free vibration of isotropic conical shells using the Rayleigh Ritz method. Recently, Zhao and Liew (2011) considered the vibrations of Functional Graded Materials (FGMs) conical panels and suggested that the effect of thickness on the vibration modes of these structures was an important factor. Investigation of free vibrations of cylindrical shells rotating at high speed was performed by Chen *et al.* (1993). Irie *et al.* (1984) also calculated the natural frequencies of truncated conical shells. Li *et al.* (2009) conducted a field study on free and forced vibrations of truncated conical shells using the Rayleigh Ritz method. Wu and Lee (2001) applied the Differential Quadrature (DQ) method for studying free vibrations of conical shells with variable thickness. Generalized Differential Quadrature (GDQ) method was performed for the first time by Shu (1996) with square differential correction method for the vibration analysis of layered isotropic conical shells. Civalek (2007) used the Discrete Singular Convolution (DSC) to investigate the frequency response of conical shells. Hu *et al.* (2002) studied the vibrations of composite twisted conical shells with respect to the strain tensor. Sofiyev *et al.* (2009) studied the vibrations of orthotropic non-homogeneous conical shells with free boundary conditions. Tripathi *et al.* (2007) studied the free vibration of composite conical shells with random material properties of the finite element method. However, most of the previous works have been conducted in order to simply support boundary conditions. Since boundary conditions may have a significant effect on the response of structural vibrations, in this study, the free vibration of composite conical shells was investigated under various boundary conditions using the solution of beam function and Galerkin method.

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GOVERNING EQUATIONS

According to Fig. 1 and 2, the governing equations of the conical shell with the length of L, thickness of h and radii of R_1 and R_2 on the two ends of the cone and half angle of the head α, based on the approximate extension of Chen *et al.* (1993) were:

$$\begin{aligned}
&\frac{\partial Nx}{\partial x}+\frac{1}{R(x)}\frac{\partial N_{x\theta}}{\partial\theta}+\frac{\sin\alpha}{R(x)}(N_x-N\theta)-\rho h\frac{\partial^2 u}{\partial t^2}=0\\
&\frac{\partial N_{x\theta}}{\partial x}+\frac{1}{R(x)}\frac{\partial N_\theta}{\partial\theta}+\frac{\cos\alpha}{R(x)}\frac{\partial M_{x\theta}}{\partial x}+\frac{\cos\alpha}{R^2(x)}\frac{\partial M_\theta}{\partial\theta}+2\frac{\sin\alpha}{R(x)}N_{x\theta}-\rho h\frac{\partial^2 u}{\partial t^2}=0\\
&\frac{\partial^2 M_x}{\partial x^2}+\frac{2}{R(x)}\frac{\partial^2 M_{x\theta}}{\partial\theta\partial x}+\frac{1}{R^2(x)}\frac{\partial^2 M_\theta}{\partial\theta^2}+\frac{2\sin\alpha}{R(x)}\frac{\partial Mx}{\partial x}\\
&-\frac{1}{R(x)}\left(\sin\alpha\frac{\partial M_\theta}{\partial x}+\cos\alpha N_\theta\right)-\rho h\omega^2\frac{\partial^2 w}{\partial t^2}=0
\end{aligned}\tag{1}$$

where, ρ is the average density in the z direction. The resultant forces and moments can be defined as:

$$\begin{bmatrix}N_x\\N_\theta\\N_{x\theta}\\M_x\\M_\theta\\M_{x\theta}\end{bmatrix}=\begin{bmatrix}A_{11}&A_{12}&A_{16}&B_{11}&B_{12}&B_{16}\\A_{12}&A_{22}&A_{26}&B_{12}&B_{22}&B_{26}\\A_{16}&A_{26}&A_{66}&B_{16}&B_{26}&B_{66}\\B_{11}&B_{12}&B_{16}&D_{11}&D_{12}&D_{16}\\B_{12}&B_{22}&B_{26}&D_{12}&D_{22}&D_{26}\\B_{16}&B_{26}&B_{66}&D_{16}&D_{26}&D_{66}\end{bmatrix}\begin{bmatrix}e_1\\e_2\\e_{12}\\k_1\\k_2\\\tau\end{bmatrix}\tag{2}$$

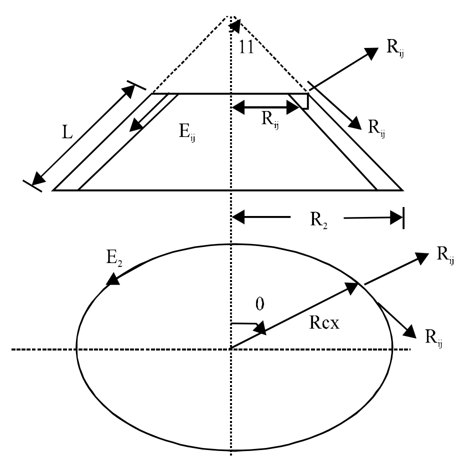

Fig. 1: Top and front view of a truncated conical shell

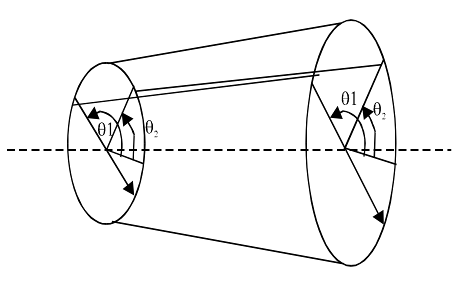

Fig. 2: Side view of a truncated conical shell

In the above equation, A_{ij}, B_{ij} and D_{ij} are stiffness coefficients. Strain and curvature in the middle of the shell were as follows:

$$\begin{aligned}
&e_1=\frac{\partial u}{\partial x},\ e_2=\frac{1}{R(x)}\frac{\partial v}{\partial\theta}+\frac{u\sin\alpha+w\cos\alpha}{R(X)},\\
&e_{12}=\frac{1}{R(x)}\frac{\partial u}{\partial\theta}+\frac{\partial v}{\partial x}-\frac{v\sin\alpha}{R(X)}\\
&k_1=\frac{\partial^2 w}{\partial x^2},k_2=-\frac{1}{R^2(x)}\frac{\partial^2 w}{\partial\theta^2}+\frac{\cos\alpha}{R^2(x)}\frac{\partial v}{\partial x}-\frac{\sin\alpha}{R(X)}\frac{\partial w}{\partial x}\\
&k_{12}=2\left(-\frac{1}{R(x)}\frac{\partial^2 w}{\partial x\partial\theta}+\frac{\sin\alpha}{R^2(X)}\frac{\partial w}{\partial\theta}+\frac{\cos\alpha}{R(X)}\frac{\partial v}{\partial x}-\frac{v\sin\alpha\cos\alpha}{R^2(X)}\right)
\end{aligned}\tag{3}$$

Using Eq. 1 to 3, the governing equations can be obtained based on the movement as follows:

$$\begin{aligned}
&A_{11}=\frac{\partial^2 u}{\partial x^2}+\frac{A_{11}\sin\alpha}{R(x)}\frac{\partial u}{\partial x}-\frac{A_{22}\sin^2\alpha}{R^2(X)}+\frac{A_{66}}{R^2(X)}\frac{\partial^2 u}{\partial\theta^2}+\frac{(A_{12}+A_{66})}{R(x)}\frac{\partial^2 u}{\partial x\partial\theta^2}\\
&-\frac{(A_{12}+A_{66})\sin\alpha}{R^2(x)}\frac{\partial v}{\partial\theta}-\frac{A_{12}\cos^2\alpha}{R(X)}\frac{\partial w}{\partial x}\\
&\frac{A_{22}\sin\alpha\cos\alpha}{R^2(X)}-B_{11}\frac{\partial^3 w}{\partial x^3}-\frac{(B_{12}+2B_{66})}{R^2(x)}\frac{\partial^3 w}{\partial x\partial\theta^2}\\
&-\frac{B_{11}+\sin\alpha}{R(x)}\frac{\partial^2 w}{\partial x^2}+\frac{(B_{12}+B_{22}+2B_{66})}{R^3(x)}\frac{\partial^2 w}{\partial\theta^2}-\frac{B_{22}+\sin^2\alpha)}{R^2(x)}\frac{\partial w}{\partial x}=\rho h\omega^2 u\\
&\frac{(A_{12}+A_{66})}{R(x)}\frac{\partial^2 u}{\partial x\partial\theta}+\frac{(A_{22}+A_{66})\sin\alpha}{R^2(x)}\frac{\partial u}{\partial\theta}+A_{66}\left[\frac{\partial^2 v}{\partial x^2}+\frac{\sin\alpha}{R(x)}\frac{\partial v}{\partial x}-\frac{\sin^2\alpha}{R^2(x)}\right]+\frac{A_{22}}{R^2(x)}\frac{\partial^2 v}{\partial\theta^2}\\
&-\frac{A_{22}\cos\alpha}{R(x)}\frac{\partial w}{\partial\theta}-\frac{(B_{12}+2B_{66})}{R(x)}\frac{\partial^3 w}{\partial x^2\theta}-\frac{B_{22}}{R^3(x)}\frac{\partial^3 w}{\partial\theta^3}-\frac{B_{22}\sin\alpha}{R^2(x)}\frac{\partial^2 w}{\partial x\partial\theta^3}=-\rho h\omega^2 v\\
&-\frac{A_{22}\cos\alpha}{R(x)}\frac{\partial w}{\partial\theta}-\frac{A_{22}\sin\alpha\cos\alpha}{R^2(x)}-B_{11}\frac{\partial^3 u}{\partial x^3}-\frac{(B_{12}+2B_{66})}{R^2(x)}\frac{\partial^3 w}{\partial x\partial\theta^2}\\
&-\frac{2B_{11}\sin\alpha}{R^2(x)}\frac{\partial^2 u}{\partial x^2}+\frac{B_{22}\sin^2\alpha}{R^2(x)}\frac{\partial u}{\partial x}\\
&-\frac{B_{22}\sin^2\alpha}{R^3(x)}\frac{\partial^2 u}{\partial\theta^2}-\frac{B_{22}\sin^3\alpha}{R^3(x)}-\frac{A_{22}\cos\alpha}{R^2(x)}\frac{\partial v}{\partial\theta}\\
&-\frac{(B_{12}+2B_{66})}{R(x)}\frac{\partial^3 v}{\partial x^2\partial\theta}-\frac{B_{22}}{R^3(x)}\frac{\partial^3 v}{\partial\theta^3}\\
&+\frac{B_{22}\sin\alpha}{R^2(x)}\frac{\partial^2 v}{\partial x\partial\theta^2}-\frac{B_{22}\sin^2\alpha}{R^3(x)}-\frac{A_{22}\cos\alpha}{R^2(x)}+\frac{2B_{12}\cos\alpha)}{R(x)}\frac{\partial^2 w}{\partial x^2}-\frac{2B_{22}\cos\alpha}{R^3(x)}\frac{\partial^2 w}{\partial\theta^2}\\
&+\frac{B_{22}\cos\alpha\sin^2\alpha}{R^3(x)}D_{11}\frac{\partial^4 w}{\partial x^4}+\frac{2(D_{12}+2D_{66})}{R^2(x)}\frac{\partial^4 w}{\partial x^2\partial\theta^2}+\frac{D_{22}}{R^4(x)}\frac{\partial^4 w}{\partial\theta^4}\\
&+\frac{2D_{11}\sin\alpha}{R(x)}D_{11}\frac{\partial^3 w}{\partial x^3}+\frac{2(D_{12}+2D_{66})\sin\alpha}{R^3(x)}\frac{\partial^3 w}{\partial x\partial\theta^2}-\frac{D_{22}\sin^2\alpha}{R^2(x)}\frac{\partial^2 w}{\partial x^2}\\
&+\frac{2(D_{12}+D_{22}+2D_{66})\sin^2\alpha}{R^4(x)}\frac{\partial^2 w}{\partial\theta^2}+\frac{D_{22}\sin^3\alpha}{R^3(x)}\frac{\partial w}{\partial x}=\rho h\omega^2 w
\end{aligned}\tag{4}$$

In Eq. 4, L_{ij} are derivative operators.

SOLVING THE GOVERNING EQUATIONS

To use Galerkin method for solving the governing equations, displacement field functions must be guessed at first. The field should be set in such a way to satisfy the boundary conditions. Displacement field was proposed as follows:

$$\begin{aligned}
&u(x,\theta,t)=A\frac{\partial\phi(x)}{\partial x}\cos(n\theta)\sin(\omega t),\\
&v(x,\theta,t)=B\phi(x)\sin(n\theta)\sin(\omega t),\\
&w(x,\theta,t)=C\phi(x)\cos(n\theta)\sin(\omega t)
\end{aligned}\tag{5}$$

Table 1: Comparison of frequency parameters for an isotropic conical shell with a simply support boundary conditions having different vertex angles

	φ = 30°			φ = 45°			φ = 60°		
n	Irie *et al.* (1984)	Li *et al.* (2009)	Present	Irie *et al.* (1984)	Li *et al.* (2009)	Present	Irie *et al.* (1984)	Li *et al.* (2009)	Present
2	0.7910	0.8431	0.8405	0.6879	0.7642	0.7639	0.5722	0.6342	0.6342
3	0.7284	0.7416	0.7375	0.6973	0.7211	0.7204	0.6001	0.6236	0.6235
4	0.6352	0.6419	0.6368	0.6664	0.6747	0.6737	0.6054	0.6146	0.6144
5	0.5531	0.5590	0.5536	0.6304	0.6336	0.6325	0.6077	0.6113	0.6111
6	0.4949	0.5008	0.4955	0.6032	0.6049	0.6037	0.6159	0.6172	0.6170
7	0.4653	0.4701	0.4661	0.5918	0.5928	0.5919	0.6343	0.6347	0.6346
8	0.4645	0.4687	0.4653	0.5992	0.6005	0.5994	0.6650	0.6653	0.6651

where A, B and C are fixed parameters and represent the amount of vibrations, n is the number of half waves along the peripheral, ω is angular frequency of vibrations and; ϕ(x) is the meridional function that satisfied boundary conditions of the geometric scaling. ϕ(x) function could be determined from the shell and beam theories using the same boundary conditions. By embedding Eq. 5 in Eq. 4, the residuals R_{ij} could be found. These residuals could be attained by applying operators L_{ij} on the same approximation functions in the following way:

$$\begin{aligned} R' &= L_{11}u + L_{12}v + L_{13}w \\ R'' &= L_{21}u + L_{22}v + L_{23}w \\ R''' &= L_{31}u + L_{32}v + L_{33}w \end{aligned} \quad (6)$$

The Galerkin method was applied as shown below:

$$\begin{aligned} &\int_0^{2\pi}\int_0^L R'\frac{\partial u}{\partial A}x\sin\alpha\,dx d\theta \\ &\int_0^{2\pi}\int_0^L R''\frac{\partial v}{\partial A}x\sin\alpha\,dx d\theta \\ &\int_0^{2\pi}\int_0^L R'''\frac{\partial w}{\partial A}x\sin\alpha\,dx d\theta \end{aligned} \quad (7)$$

By integrating the past three equations, a 3×3 homogeneous system was found. To reach the non-zero solution of this system, the determinant of its coefficients should be equal to zero. By solving the equations, natural frequencies and corresponding modes of vibrations could be found. Numerical results were presented in order to introduce a dimensionless frequency parameter in the following way:

$$\omega_c = \omega R_2\sqrt{\frac{\rho h}{A_{11}}} \quad (8)$$

MODEL VERIFICATION

Table 1 shows the values of frequency parameter for the simply supported boundary conditions of an isotropic conical shell with different vertex angles. For comparison, some results Irie *et al.* (1984) and Li *et al.* (2009) are also comprised in this table. As can be seen, there is good agreement between the two sets of results, which indicates the accuracy and efficiency of the method in studying the vibrations of conical shells.

Table 2: Comparison of frequency parameters for composite conical shells

h/R^*_2	Wu and Lee (2001)	Present
0.01	0.1799	0.1371
0.02	0.2153	0.1788
0.03	0.2397	0.2174
0.04	0.2620	0.2480
0.05	0.2841	0.2709
0.06	0.3061	0.2981
0.07	0.3277	0.3295
0.08	0.3484	0.3647
0.09	0.3680	0.3969
0.10	0.3863	0.4125

Table 2 also shows the possibility of comparing the present results with the ones by Wu and Lee (2001) as far as the vibration response of composite conical shells is concerned. In this table, the frequency parameters of a two-layer conical composite shell with two layers of non-symmetric cross-ply and with the simply supported boundary conditions in both ends are presented for different thicknesses. The results confirmed that the method was suitable for analyzing the vibrations of composite conical shells.

RESULTS AND DISCUSSION

Figure 3 shows that, in the first mode, the frequency parameter of a two-layered cross-ply asymmetric conical shell with a half 30° cone angle under different boundary conditions changed relative to the number of axial half-wave environment. This figure also shows that the frequency of the shell decreased and then increased for all types of the boundary conditions with the increase in the number of half-wave. The results are also confirmed what Wu and Lee (2001), Irie *et al.* (1984) and Li *et al.* (2009) are claimed using different formulations.

The figure represents the effect of boundary conditions on the vibration behavior of shells so that the wave number corresponding to the smallest frequency (fundamental frequency) is different for different boundary conditions. According to the curves, the number of half-wave of the fundamental frequency was equal to 4, 5, 5, 4, 2 and 4 for boundary conditions of FS

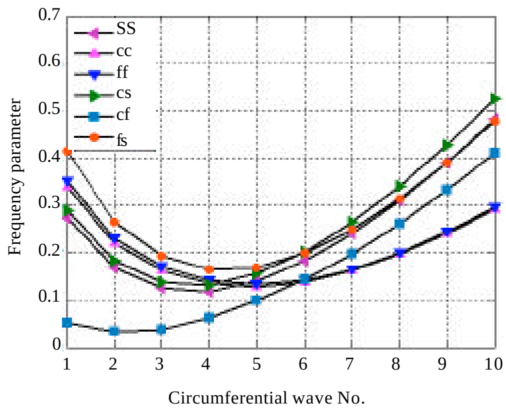

Fig. 3: Changes in the number of half-wave frequency parameter setting for composite conical shells under various boundary conditions

(Free-Simply Support), CF (Clamped-Free), CS (Clamped-Simply Support), FF (Free-Free), CC (Clamped-Clamped) and SS (Simply Support-Simply Support), respectively.

CONCLUSION

This article showed that the Galerkin method with beam functions can be used well in calculating the natural frequency of the truncated conical shells with different boundary conditions. It can be also concluded that the boundary conditions significantly affected the response of structural vibrations. It was also found that no matter what the boundary conditions were, the frequency of the shell decreased and then increased with the increase in the number of half-wave.

REFERENCES

Chen, Y., H.B. Zhao, Z.P. Shen, I. Grieger and B.H. Kroplin, 1993. Vibrations of high speed rotating shells with calculations for cylindrical shells. J. Sound Vibr., 160: 137-160.

Civalek, O., 2007. Numerical analysis of free vibrations of laminated composite conical and cylindrical shells: Discrete Singular Convolution (DSC) approach. J. Comput. Applied Math., 205: 251-271.

Garnet, H. and J. Kemper, 1964. Axisymmetric free vibrations of conical shells. J. Applied Mech., 31: 458-466.

Hu, X.X., T. Sakiyama, H. Matsuda and C. Morita, 2002. Vibration of twisted laminated composite conical shells. Int. J. Mech. Sci., 44: 1521-1541.

Irie, T., G. Yamada and K. Tanaka, 1984. Natural frequencies of truncated conical shells. J. Sound Vibr., 92: 447-453.

Jayakumar, K., D. Yadav and B. Nageswara Rao, 2006a. A piezoelectric cylindrical shell under thermal and pressure loads. Trends Applied Sci. Res., 1: 214-225.

Jayakumar, K., D. Yadav and B.N. Rao, 2006b. A multi-layer cylindrical shell under electro-thermo-mechanical loads. Trends Applied Sci. Res., 1: 386-401.

Li, F.M., K. Kishimoto and W.H. Huang, 2009. The calculations of natural frequencies and forced vibration responses of conical shell using the Rayleigh-Ritz method. Mech. Res. Commun., 36: 595-602.

Sharma, A.K. and N.D. Mittal, 2010. Review on stress and vibration analysis of composite plates. J. Applied Sci., 10: 3156-3166.

Shu, C., 1996. Free vibration analysis of composite laminated conical shells by generalized differential quadrature. J. Sound Vibr., 194: 587-604.

Sofiyev, A.H., K.A. Korkmaz, Z. Mammadov and M. Kamanli, 2009. The vibration and buckling of freely supported non-homogeneous orthotropic conical shells subjected to different uniform pressures. Int. J. Pres. Ves. Pip., 86: 661-668.

Tripathi, V., B.N. Singh and K.K. Shukla, 2007. Free vibration of laminated composite conical shells with random material properties. Compos. Struct., 81: 96-104.

Wu, C.P. and C.Y. Lee, 2001. Differential quadrature solution for the free vibration analysis of laminated conical shells with variable stiffness. Int. J. Mech. Sci., 43: 1853-1869.

Zhao, X. and K.M. Liew, 2011. Free vibration analysis of functionally graded conical shell panels by a meshless method. Compos. Struct., 93: 649-664.