Dynamic Parameter Schematic Analysis of the Whole-Rocket Vibration Isolation System

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Abstract: The whole-rocket vibration isolation can improve dynamic environment during satellites launching, as well as reliability and security in satellite launching. In this study, analytic expression in multistage launch vehicles vibration isolation coupled dynamic system is firstly established. By using the analytic expression, the physical significance of the Whole-Spacecraft Vibration Isolation (WSVI) technology in whole-rocket vibration isolation system coupled dynamics and other vibration isolation technological approaches are discussed. And the theoretical analysis of the whole-rocket vibration isolation system are given. At last, the whole-rocket system simulation model is established. By using the model, the dynamics theoretical analysis method is verified. Then it is known that the theoretical analysis could provide theoretical directions on the whole-rocket vibration attenuation technique.

Key words: Whole-rocket vibration isolation, whole-spacecraft vibration isolation, vibration transmissibility

INTRODUCTION

During the satellite launching, pneumatic noise which is produced by impulse propulsion of rocket launcher and cowering rubbing air, worsens the mechanics environment where the satellite is located. The satellite is acted on by the dynamics load as motional pattern mostly through carriers; especially the dynamics load becomes worse before separation of the first and second stages in the rockets. Aiming at the phenomenon, the whole-spacecraft vibration isolation technology and launch vehicle component vibration isolation and attenuation technology emerge and enhance the dynamics environment in the satellite launching.

The whole-spacecraft vibration isolation technology transfers effective separation to vibration loading of the satellite through changing the adapter system in hand. Many scholars have achieved a large number of research findings, however, most scholars focus on simple vibration isolation platform manufacturing, not considering the whole-rocket coupled dynamic system which leads to the less effects of vibration isolation lessened effectively (Johnson et al., 1999, 2001; Stewart, 1965; Likun et al., 2006; Zheng, 2003; Denoyer and Johnson, 2001; Xu and Chanan, 2012; Zhang et al., 2010; Edberg et al., 1997; Yang et al., 2005). Zheng simplified the rocket into one subsystem and probed into the physical mechanism in the whole-spacecraft vibration isolation. There are usually two subsystems constituted in the launch vehicle and the processes of the first and second stages’ separation are the worst (Zheng and Tu, 2009). Vibration isolation and attenuation of launch vehicles components enhance the reliability and security in satellite launching in partly vibration attenuation in the rocket instrument capsule and do not reduce the vibration loading effectively that is transferred to the satellite (Du et al., 2004; Lu and Huang, 2008).

In this dissertation, the launch vehicle is divided into two substructures and establishing multistage rocket launcher vibration isolation coupled dynamic system analytic expression. Through analytic expression on vibration transmissibility, it studies the dynamic principle on the whole-rocket vibration isolation technology. By using physical mechanisms, the physical meaning of WSVI is studied which provides theoretical directions for manufacturing the new WSVI platform. And the other approaches for the vibration isolation are explored which open up the new methods in the whole-rocket vibration isolation and attenuation technology. By establishing the whole-rocket simulation model, the dynamic principle analysis and accuracy verification on the whole-rocket vibration isolation coupled system is studied. And the dynamic environment in satellite launching in many ways are improved, thereby ensuring the reliability and security in satellite launching.

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ANALYTICAL THEORY FOR THE WHOLE-ROCKET COUPLED DYNAMIC SYSTEM

The whole-rocket coupled system simplified dynamic model as followed in Fig. 1.

In Fig. 1, the first stage substructure for the rocket launcher mainly includes the first stage fuel tank, the first stage engine, the liquid oxygen tank etc. The second stage for the rocket launcher mainly includes the second stage engine, the second stage engine fuel tank, instrument capsule, fuel tank, the liquid oxygen tank, cowling etc. The first stage substructure and the second stage substructure for the whole-rocket coupled system simplified dynamic model connect through the connecting device and the second stage substructure connects the satellite through the vibration isolation platform.

Model of analytical theory for the whole-rocket coupled dynamic system: According to Fig. 1, establishing the multistage rocket coupled system dynamics model with the whole-spacecraft vibration isolation platform as follows:

\[ \mathbf{M} \ddot{\mathbf{x}} + (\mathbf{K} + j \mathbf{G}) \mathbf{x} = \mathbf{f} \]  \hspace{1cm} (1)

\( \mathbf{M}, \mathbf{K} \) and \( \mathbf{G} \) represent mass matrix, stiffness matrix and damping matrix, respectively.

\[
\begin{bmatrix}
M_0 & M_1 & M_2 & M_3 \\
M_1 & M_{11} & M_{12} & M_{13} \\
M_2 & M_{21} & M_{22} & M_{23} \\
M_3 & M_{31} & M_{32} & M_{33}
\end{bmatrix}
\]

Element of mass matrix is expressed as follows:

\[
\begin{bmatrix}
K_u & K_x \\
K_x & K_{xx} + k_1 & K_x & K_x \\
K_x & K_{xx} & K_{xx} & K_x \\
K_x & K_{xx} & K_{xx} & K_{xx}
\end{bmatrix}
\]

Element of stiffness matrix is given by:

\[
\begin{bmatrix}
K_u & K_x \\
K_x & K_{xx} + k_1 & -k_1 & K_x \\
-k_1 & K_{xx} & K_{xx} & K_x \\
-k_1 & K_{xx} & K_{xx} & K_{xx}
\end{bmatrix}
\]

Element of damping matrix can be written as:

\[
\begin{bmatrix}
G_u & G_x \\
G_x & G_{xx} & -\eta k_1 & G_x \\
-\eta k_1 & G_{xx} & G_{xx} & G_x \\
-\eta k_1 & G_{xx} & G_{xx} & G_{xx}
\end{bmatrix}
\]

Degree of freedom vector quantity is given by:

\[ \mathbf{x} = \{X, x, x, X_1, x, x, X_2\}^T \]

Incentive vector is expressed as follows:

\[ \mathbf{f} = \{0, 0, 0, 0, 0, f_0\}^T \]

where, \( a \) is the satellite substructure, \( r \) is the satellite substructure boundary. \( s \) is the boundary between the second stage substructure for the launching vehicle and vibration isolator. \( b \) is the second stage substructure for the launching vehicle. \( t \) is the boundary of the connecting device between the second substructure of launching vehicle and the first stage and second stage substructures. \( c \) is the first stage substructure of launching vehicle. \( o \) is the boundary between the launching vehicle second stage substructure and the first and second substructure connecting device. \( f_0 \) is the first stage substructure for launching vehicle by incentive environment.

Fig. 1: Dynamic model of rocket-spacecraft coupling system
Shifting the Eq. 1 to the frequency domain, transforming the equation, it can be got:

$$
\begin{pmatrix}
-\omega^2 M_{sa} + K_a + j G_a \\
K_a + j G_a
\end{pmatrix}
\begin{pmatrix}
X_s \\
X_a
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
(k_a + j n_k k_s) X_s
\end{pmatrix}
$$

(2)

$$
\begin{pmatrix}
-\omega^2 M_{sa} + K_a + j (G_{sa} + n_k k_s) \\
K_a + j G_a
\end{pmatrix}
\begin{pmatrix}
X_s \\
X_a
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
(k_a + j n_k k_s) X_s
\end{pmatrix}
$$

(3)

Now the study adopts the modal variant method and discusses (Tu and Zheng, 2007), conduct the modal into spatial transfer with Eq. 2, 3 and 4:

$$
\begin{pmatrix}
\tilde{X}_s \\
\tilde{X}_a
\end{pmatrix}
= 
\begin{pmatrix}
\psi_s \\
\psi_a
\end{pmatrix}
Q_s
\begin{pmatrix}
X_s \\
X_a
\end{pmatrix}
= 
\begin{pmatrix}
\gamma_s \\
\gamma_a
\end{pmatrix}
Q_s
\begin{pmatrix}
\phi_s \\
\phi_a
\end{pmatrix}
$$

(5)

And make the Eq. 2, 3 and 4 into diagonalization, it can be got the modal spatial dynamics Eq. 6, 7 and 8 and among them $\psi, \gamma, \psi, \gamma, \psi, \gamma$ represents the various substructures in its modal coordinates, respectively:

$$
\begin{pmatrix}
D_s + \psi_s^T (k_s + j n_k k_s) \psi_s \\
D_s + \gamma_s^T (k_s + j n_k k_s) \gamma_s
\end{pmatrix}
\begin{pmatrix}
\tilde{X}_s \\
\tilde{X}_a
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
0
\end{pmatrix}
$$

(6)

$$
\begin{pmatrix}
D_s + \psi_a^T (k_a + j n_k k_s) \psi_a \\
D_s + \gamma_a^T (k_a + j n_k k_s) \gamma_a
\end{pmatrix}
\begin{pmatrix}
\tilde{X}_s \\
\tilde{X}_a
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
0
\end{pmatrix}
$$

(7)

$$
\begin{pmatrix}
D_s + \phi_s^T (k_s + j n_k k_s) \phi_s + \psi_s^T (k_s + j n_k k_s) \psi_s \\
D_s + \phi_a^T (k_s + j n_k k_s) \phi_a + \psi_a^T (k_s + j n_k k_s) \psi_a
\end{pmatrix}
\begin{pmatrix}
\tilde{X}_s \\
\tilde{X}_a
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
0
\end{pmatrix}
$$

(8)

In Eq. 6, 7 and 8:

$$
D_s = \text{diag}( -\omega^2 m_s + j n_k k_s + k_s )
$$

$$
D_a = \text{diag}( -\omega^2 m_a + j n_k k_s + k_s )
$$

$$
D_s = \text{diag}( -\omega^2 m_a + j n_k k_s + k_s )
$$

where, $m_s, n_k, k_s, m_a, n_k, k_a, m_s, n_k, k_a$ represent the modal quality, modal damping ratio, modal stiffness, respectively.

Transmissible expression of the vibration analytic of vibration isolation system to the whole-rocket: For discussing the analytic relation of the whole-rocket vibration transmissibility, supposing the vibration isolator only effect in one direction. The joint interfaces between the vibration isolator and the second stage substructure and that between the second stage substructure and the first stage connecting device have the same movement at all degrees of freedom.

Transforming the Eq. 8 can get:

$$
\begin{pmatrix}
D_s + \phi_s^T (k_s + j n_k k_s + k_s + j n_k k_s) \phi_s \\
D_s + \phi_s^T (k_s + j n_k k_s + k_s + j n_k k_s) \phi_s
\end{pmatrix}
\begin{pmatrix}
\tilde{X}_s \\
\tilde{X}_a
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
0
\end{pmatrix}
$$

(9)

According to the method in the bibliography (Zeng and Tu, 2009).

Where:

$$
\phi_s = \{ \phi_s^1, \phi_s^2, ..., \phi_s^N \}
$$

(10)

$$
E_s = \begin{pmatrix}
\phi_s^1 \\
\phi_s^2 \\
\vdots \\
\phi_s^N
\end{pmatrix}
$$

(11)

In the Eq. 12, $\lambda_m - \omega^2 m_{sa} + j n_k k_{sa} = \alpha_m (n - 1, 2, ..., M)$;

$$
\begin{pmatrix}
1 + \lambda \phi_s^1 (k_s + j n_k k_s + k_s + j n_k k_s)^T \\
1 + \lambda \phi_s^2 (k_s + j n_k k_s + k_s + j n_k k_s)^T \\
\vdots \\
1 + \lambda \phi_s^N (k_s + j n_k k_s + k_s + j n_k k_s)^T
\end{pmatrix}
$$

(12)

From Eq. 10, 11 and 12, getting:

$$
D_s + \psi_s^T (k_s + j n_k k_s + k_s + j n_k k_s) \phi_s = (k_s + j n_k k_s + k_s + j n_k k_s) E_s R_s
$$

(13)

Transforming Eq. 11, getting:

$$
\bar{Q}_s = \left( (k_s + j n_k k_s + k_s + j n_k k_s) E_s R_s E_s^T \right)^{-1} \left( k_s + j n_k k_s \right) \bar{X}_s
$$

Define:

$$
\alpha_s = \frac{\lambda_s}{\phi_s^T (k_s + j n_k k_s + k_s + j n_k k_s)}
$$

(15)
According to Eq. 20, it can be got the vibration transmissibility analytic expression 21 from the boundary between the first substructure and the connecting device to the boundary between the satellite and vibration isolator.

Define:

\[
\rho_1 = \frac{\sum_{n=1}^{N} 2\alpha_n (k_n - \alpha m_n) + (\eta_n k_n)}{\sum_{n=1}^{N} (k_n - \alpha m_n)^2 + (\eta_n k_n)^2} \quad \rho_2 = \frac{\sum_{n=1}^{N} \alpha_n k_n}{\sum_{n=1}^{N} (k_n - \alpha m_n)^2 + (\eta_n k_n)^2}
\]

\[
\sigma_1 = \frac{\sum_{n=1}^{N} \phi_n \psi_n (k_n - \alpha m_n)}{\sum_{n=1}^{N} (k_n - \alpha m_n)^2 + (\eta_n k_n)^2} \quad \sigma_2 = \frac{\sum_{n=1}^{N} \phi_n \psi_n k_n}{\sum_{n=1}^{N} (k_n - \alpha m_n)^2 + (\eta_n k_n)^2}
\]

\[
\tau_1 = \frac{\sum_{n=1}^{N} \phi_n \psi_n (k_n - \alpha m_n)}{\sum_{n=1}^{N} (k_n - \alpha m_n)^2 + (\eta_n k_n)^2} \quad \tau_2 = \frac{\sum_{n=1}^{N} \phi_n \psi_n k_n}{\sum_{n=1}^{N} (k_n - \alpha m_n)^2 + (\eta_n k_n)^2}
\]

Then the Eq. 20 can be expressed:

\[
T_m = \frac{\tau_1 + \tau_2}{\frac{1}{k_c^2 + (\eta_n k_c)^2}}
\]

In the Eq. 22:

\[
R_i = k_i + k_j \left( k_i + k_j \right) \rho \psi_1 (k_i + k_j) + \rho \psi_2 \left( k_i + k_j \right)\]

\[
\frac{\sum_{n=1}^{N} \phi_n \psi_n (k_n - \alpha m_n + jn k_n + k_n)}{1 + (k_i + jn k_i)} \sum_{n=1}^{N} \phi_n \psi_n (k_n - \alpha m_n + jn k_n + k_n)
\]

Substituting the Eq. 18 into 19, it can be got the Eq. 20:

\[
\left( \frac{(k_i + jn k_i) \sum_{n=1}^{N} \phi_n \psi_n (-\alpha m_n + jn k_n + k_n)}{1 + (k_i + jn k_i) \sum_{n=1}^{N} \phi_n \psi_n (-\alpha m_n + jn k_n + k_n)} \right) \bar{x}_i = \left( \frac{(k_i + jn k_i) \sum_{n=1}^{N} \phi_n \psi_n (-\alpha m_n + jn k_n + k_n)}{1 + (k_i + jn k_i) \sum_{n=1}^{N} \phi_n \psi_n (-\alpha m_n + jn k_n + k_n)} \right) \bar{x}_i
\]

\[
\left( \frac{(k_i + jn k_i) \sum_{n=1}^{N} \phi_n \psi_n (-\alpha m_n + jn k_n + k_n)}{1 + (k_i + jn k_i) \sum_{n=1}^{N} \phi_n \psi_n (-\alpha m_n + jn k_n + k_n)} \right) \bar{x}_i = \left( \frac{(k_i + jn k_i) \sum_{n=1}^{N} \phi_n \psi_n (-\alpha m_n + jn k_n + k_n)}{1 + (k_i + jn k_i) \sum_{n=1}^{N} \phi_n \psi_n (-\alpha m_n + jn k_n + k_n)} \right) \bar{x}_i
\]

The Eq. 24 can be transformed into:

\[
T_m = \frac{x_i^{2} + x_2^{2}}{R_i + I_i}
\]

**SCHEMATIC ANALYSIS OF DYNAMIC PARAMETER**

In the Eq. 23, when in the resonance place, vibration transmissibility \(T_m\) can achieve the largest that is \(R_i = 0\).
Through the Eq. 23, it can be seen that when the rocket launcher is in the active launch phase of the first stage, \( \eta_1, k_1, \eta_2, k_2 \) coupled together. Now discussing the Eq. 23 further, the changing tendency of:

\[
\frac{I_1}{(k_i^2 + (\eta_i k_i)^2)^{3/2}}
\]

For the convenience of discussion, firstly supposing the damping in the connecting device is zero and \( \rho_i = \sigma_i = \tau_i = 0 \), then:

\[
\frac{I_1}{(k_i^2 + (\eta_i k_i)^2)^{3/2}} = \left( \frac{\rho_i \eta_i k_i}{k_i^2} - \frac{\sigma_i \eta_i k_i}{k_i} + \frac{2 \rho_i \sigma_i \eta_i k_i^2}{k_i} + \frac{2 \sigma_i \eta_i k_i^2}{k_i} \right)^{3/2}
\]

(24)

If the damping dissipation factor \( \eta_i \) of vibration isolator is increased by adding the damping in structure system (increase the damping Energy-dissipating stiffness), then the vibration transmissibility \( T_n \) can be effectively reduced. As the stiffness term \( k_i \) of the Eq. 23 contains the coupled term, the regular pattern of \( T_n \) by reducing the stiffness of vibration isolator cannot be seen. \( T_n \) relies on the dynamics parameter in the structure of rocket launcher but it can be reduced the vibration transmissibility \( T_n \) by reducing the stiffness of connecting device.

Supposing the damping in vibration device is zero and \( \rho_i = \sigma_i = \tau_i = 0 \), thus:

\[
\frac{I_1}{(k_i^2 + (\eta_i k_i)^2)^{3/2}} = \frac{\eta_i k_i + \rho_i \eta_i k_i - \sigma_i \eta_i k_i + \sigma_i \eta_i k_i - \sigma_i \eta_i k_i}{(k_i^2 + (\eta_i k_i)^2)^{3/2}}
\]

\[
= \frac{1}{k_i} \left( 1 + \frac{\rho_i k_i - \sigma_i k_i + \sigma_i \eta_i k_i}{\eta_i + \eta_i} \right)^{3/2}
\]

(25)

Through the Eq. 25, it can be seen that in the circumstance of making the rocket flying normally, reducing the stiffness of connecting device can effectively reduce the vibration transmissibility \( T_n \).

From the theoretical analysis above, the most direct methods for restraining the vibration to the satellite are found. That is adding the damping of vibration isolator and reducing the stiffness of the first and second stages connecting device during the separation process of violent vibration in the first and second stage. From the bibliography (Zheng and Tu, 2009), to simplify the rocket into one substructure can achieve a conclusion. That is enhancing the damping in vibration isolator and reducing the stiffness of vibration isolator can effectively reduce the vibration transmissibility transferred to the satellite. And in this dissertation, it can be found that in order to reduce the stiffness of vibration isolator, enhancing the whole-rocket coupled system cannot effectively reduce the vibration transferred to the satellite. The vibration isolator could perform a very good effect in the former whole-spacecraft vibration isolation technology in the ground vibrated experiments, however it has a limited effect in flying. However, the new way of vibration isolation at this study is to reduce the stiffness of the first and second stage connecting devices in order to restrain the vibration. And if the whole-rocket vibration isolation platform can support enough damping, it can effectively suppress the vibration of satellites both of the launching and flying.

**SIMULATED VERIFICATION IN THE ANALYSIS OF THE WHOLE-ROCKET DYNAMICS**

The principle analysis of the dynamic parameters is conducted with the simplified whole-rocket vibration isolation coupled system. The principle analyzed verification is conducted, making use of calculating figures in detail.

The large rocket launcher coupled system mainly consists of satellites, the first substructure of rocket launcher and the second substructure of rocket launcher, the roll booster, adapter and cowling. The material of cowling is adopted with fiber reinforced polymer skin and the rest shells use LF6 material. The roll booster, liquid oxygen tank and engine are adopted with RBE3 unions to connect the barycenter of the components to the covering union surrounding. And the components of rocket launcher is simplified. Module for the typical components of rocket launcher is showed in Fig. 2.

Making use of the limited meta-module in the whole-rocket coupled system, the accurate verification is conducted by analytic expression in principle analysis and only considering the vertical vibration of satellite. As the first stage substructure in the rocket launcher is changed by time, the simulation in the typical time can be proceeding in the simulated verification of the

Fig. 2: Finite element model of rocket-spacecraft coupling system
Fig. 3: Vibration transmissibility comparison of different damping rocket-spacecraft coupling system

whole-meter coupled system. And the vibration before the separation of the first and second stage substructure is taken as the typical time in the process of principle analysis and verification.

Figure 3 shows that only enhancing the damping \( \eta \) of vibration isolator can get the vibration transmissibility from the boundary between the first stage substructure and the stage connecting device to the connecting boundary of the isolator device.

In Fig. 3, the vibration transmissibility in the first and second stratum can be reduced by enhancing the damp of the vibration isolator. The vibration transmissibility in the first stratum can lower 22.5% at maximum and that in the second stratum can lower 64% at maximum. And the vibration transmissibility in the second stratum lowers more apparently than the one in the first stratum.

Then only reducing the stiffness \( k \) of the first and second stage connecting device, whose simulation results as Fig. 4.

From the Fig. 4, it can be found that both the first and second stratum vibration transmissibility lower if only reducing the stiffness of the first and second stratum connecting device. And the vibration transmissibility in the first stratum lowers 19.5% at maximum; the one in the second stratum lowers 61.5% at maximum.

At last, the simulation is conducted while enhancing the damping of the vibration isolator and lowering the stiffness of the first and second stage connecting device, the simulation results in the Fig. 5 as contrast.

In Fig. 5, it can be found that the vibration transmissibility in the first stratum lowers 24% and the vibration transmissibility in the second stratum lowers 84.6%. And increasing the damping of the vibration isolator and reducing the stiffness of the first and second stage connecting device can have a better suppression effect. Then the accuracy of principle analysis is verified which provides a new vibration isolation approach for the whole-meter vibration isolation.

CONCLUSION

The whole-meter vibration isolation technology can enhance the dynamic environment in the satellite launching. By establishing the vibration transmissibility analytic expression of the multistage rocket coupled dynamics system, the physical significance of WSVI technology from the analysis of the whole-meter vibration isolation principle are revealed. And another
effective approach for the vibration isolation is proposed. By using the analytic expression, it can be concluded that increasing the damping of the vibration isolation platform and reducing the stiffness of the first and second stage connecting device is the most direct method to restrain the vibration transferred to the satellite. At last, it is tested accurately from the principle analysis by the whole-rocket simulation model. The principle analytic method could provide theoretical guidance for the whole-rocket vibration isolation and attenuation technology as well as improve the reliability and security in satellite launching further.

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REFERENCES


