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# RVM Based Compressed Sensing for Optical Fiber Pipeline Data Compression

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**Abstract:** Due to long time continuous operation and the high sampling rate, the distributed optical fiber pre-warning system will generate vast amounts of data. Compressed sensing can sample and compress the signal at the same time thus reduce the data amount to be stored or transferred. Therefore, this article uses compressed sensing approach to compress the optical fiber pre-warning data. In compressed sampling phase, the signal is classified using sparse detection method, then more measurements will be taken for the segment containing threatening event than the segment of normal operation, through this the amount of data can be further reduced. In the signal reconstruction phase, the signal compressed sampling process is modeled by the relevant vector machine and the signal recovery is implemented by probabilistic parameter estimation methods. Using Bayesian framework, the sparsity of the signal and the noise each is modeled by a prior. The sparsity and the noise can be estimated in parameter estimation process and the sparsity needn't be given in advance. The optical fiber pre-warning system is long-running system and the sparsity of the signal will change with time, so the automatic sparsity determination ability is superior to other existing recovery methods. Experimental results show that, under the same measurement, the proposed method can reconstruct the signal with high quality and the reconstructed signal will not affect the positioning result.

**Key words:** Compressed sensing, data compression, optical fiber pipeline, orthogonal matching pursuit (OMP), relevant vector machine (RVM)

## INTRODUCTION

Transporting oil and gas using pipeline is a very important transportation means. However, in the transport process, the safety of the pipeline will be affected by stealing or mining not intentional by the people, natural disaster, natural aging and other accidents. Therefore, monitoring the running status of the pipeline becomes very important. The optical fiber pre-warning system monitors the pipeline status using distributed optical fiber and gives early warning for incidents which will threat the pipeline safety. As a result, very high Nyquist sampling rate need to be taken and this will generate huge amount of online data which is not conducive to data transmission and storage (Yan, 2006). Conve-ntional signal compression method is to firstly acquire the signal using the sampling frequency based on the Nyquist sampling theorem and then the signal is compressed. It can be seen that this type of method can't avoid large amounts of data to be collected. The compressed sensing framework is a technique developed in recent years that can sample and compress the signal simultaneously. If adopted for optical fiber pre-warning system, the sampling rate and the amount of data can be greatly reduced and the difficulties for transmission and storage brought by data overload can be overcome.

The coefficient of the signal sampled based on the Nyquist sampling theory under some transform matrix is sparse. Compressed sensing framework is mainly based on this. The original signal is projected by random matrix when compressed sampling and the signal is recovered when needed using the sparse characteristics described above. At present, there have been a lot of signal recovery algorithms to reconstruct the compressed sampled signal. However, these methods do not take advantage of the probability of signal characteristics. In this paper, the Relevant Vector Machine (RVM) probability model (Tipping, 2001) is used to model the compressed sampling process and recover the compressed sampled signal. The advantage of this method is that the Bayesian framework for modeling the process of compressed sensing. Through the priori for each component of the sparse signal, the most likely value of each component can be gradually estimated in the

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iteration process. The most important point is that the sparsity degree of the signal is not required in advance by this method but can be obtained in the signal recovery process (Ji *et al.*, 2008; Babacan *et al.*, 2010). Meanwhile, the algorithm can get an esti-mate of additive noise due to the signal itself or brought by measurement process. For optical fiber pre-warning signal, because the system's long-running characteristics, the signal sparsity is diff-erent for different time periods, so automatically obtaining the sparsity of each segment is important. The traditional methods, such as MP (Mallat and Zhang, 1993) algorithm and OMP (Tropp and Gilbert, 2007) algorithm, they are controlled by a threshold value for sparsity decision but these two methods require many measurements while the SP (Dai and Milenkovic, 2009) and CoSamp (Needell and Tropp, 2009) algorithm require the sparse degree of the signal provided in advance. In summary, the use of RVM for compressed sensing of optical fiber pre-warning signal has a great advantage.

## BASICS OF COMPRESSED SENSING

Compressed sensing (CS) is a theory that samples and compresses the signal simultaneously and can be divided into two stages including compressive sampling stage and recovery stage. Compressed sampling is based on the sparse characteristics of the signal. Assuming the original signal x is a segment of signal with length N and $\alpha$ is its transform coefficient under the orthogonal transform matrix or orthogonal bases dictionary $\Psi$, the relation of x and $\alpha$ can be written as $x = \Psi\alpha$. If most of the elements of $\alpha$ are zero, then signal x is called sparse under the transform and if the signal can be approximated by K (K<<N) nonzero elements of $\alpha$ by the above equation, then K is called the sparse degree of the signal.

In compressed sampling stage the signal x is projected to a $M \times 1$ dimensional signal y by a measurement matrix $\Phi$ with size M×N (M<<N):

$$y = \Phi x = \Phi\Psi'\alpha = A\alpha \quad (1)$$

where, A can be called the CS matrix. The measure-ement matrix usually used is random Gaussian matrix for its universality. Universality refers to the strong un-correlation between Gaussian random matrices and other orthogonal matrix or dictionaries and this enables accurate reconstruction of the original signal from the measurements. To recover the signal from the measurement with high probability using Gaussian measurement matrix, the measurements number M should satisfy M>K log N. Furthermore, Bernoulli matrix is used in practical applications due to its characteristics of easy hardware implementation and the sub-Gaussian matrix is also used as measurement matrix (Eldar and Kutyniok, 2012).

In recovery stage of compressed sensing, the signal is recovered from the compressed sampled signal. There have been a lot of algorithms for signal recovery. Initially, this problem can be expressed as an optimization problem of $l_0$ norm minimization but this problem is a combinatorial optimization problem and is very difficult to solve. To reduce complexity, the $l_0$-norm minimization problem is transformed into the following $l_1$ minimization problem (Donoho, 2006):

$$\min_{\alpha}\|\alpha\|_1 \quad \text{s.t. } y = A\alpha$$

One representative algorithm of this type is the Basis Pursuit (BP) (Tsaig and Donoho, 2006). The BP algorithm requires a small number of measurements but has very high computational complexity. When considering the additive noise, such algorithms include BPDN (Chen *et al.*, 1998), the Dantzig Selector (Candes and Tao, 2007) and etc.

The other type is the greedy iterative algorithm, such as MP, OMP, CoSamp, SP, etc. MP and OMP determine the sparsity of the signal in the iteration but the algorithm requires more measurements?? and the reconstruction reliability is not very stable. CoSamp and SP require pre-set sparsity K which is much difficult in practice.

## RVM BASED CS OF OPTICAL FIBER PRE-WARNING SIGNAL

**Relevant vector machine for CS:** The Relevance Vector Machine (RVM) is developed on the basis of the support vector machine and is a supervised machine learning methods. This method can create a sparse model for a given data set. Assuming that the observed data set is $\{y_i\}_{i=1}^N$ and the one bye one corresponding input data set is $\{x_i\}_{i=1}^N$, then the relationship between them can be described as following:

$$y = \sum_{j=1}^{J}\alpha_j\phi_j(x) + n = \Phi\alpha + n \quad (2)$$

where, $\Phi = [\Phi_1, \cdots, \Phi_J]$ is a 'design' matrix with size N×J and each column is a basis vector composed of N elements. The vector $n = (n_1, \cdots, n_N)^T$ denotes the additive noise and the coefficient $\alpha = (\alpha_1, \cdots, \alpha_J)^T$ describes the sparse relationship between input and output and if most of the elements are zero.

From Eq. 1 and 2 it can be seen that RVM can be taken as compressed sensing with additive noise. The 'design' matrix $\Phi$ corresponds to the CS matrix A and the sparse coefficient $\alpha$ corresponds to the sparse signal transform coefficients. In compressed sensing, the original signal is $x = s+s_n$ where the pure signal is s and $s_n$ is the additive noise. Therefore, the noise vector n corresponds to the combination of the noise in original signal and quantization noise in compressed sampling. Based on the above considerations, the relevant vector machine can be used to model the compressed sensing process. Achieving the sparse coefficient is equivalent to obtain the sparse signal transform coefficients and the noise vector can be obtained simultaneously.

Assuming each item of the noise n are i.i.d. Gaussian variables with zero-mean and variance $\sigma^2$. Then, the distribution of the measurement vector y is a multivariate Gaussian distribution with mean $\Phi\alpha$ and variance $\sigma^2$.

In the Bayesian framework, priors should be established for the parameters $\alpha$ and $\sigma^2$ that need to be solved. The Gaussian distribution with zero mean is taken as the prior function for each element of $\alpha$:

$$p(\alpha|\lambda) = \prod_{i=1}^{J} N(\alpha_i | 0, \lambda_i^{-1}) \tag{3}$$

Because the prior function for each element of the parameter $\alpha$ is controlled by a parameter $\lambda_i$, the model parameters are over parameterized for input data. A hyper prior should be established for each $\lambda_i$ to meet the requirements of the hierarchical Bayesian framework. The hyper prior is:

$$p(\lambda | a, b) = \prod_{i=1}^{J} \Gamma(\lambda_i | a, b) \tag{4}$$

where, $\Gamma(\lambda_i|a, b)$ is the Gamma distribution with shape a and scale b which is conjugate to the Gaussian distribution. The other parameter to be estimated is $\sigma^2$. Let $\beta = \sigma^{-2}$, then the prior is:

$$p(\beta|c, d) = \Gamma(\beta|c, d) \tag{5}$$

According to the above description of the model and the hierarchical priors, the CS recovery problem can be formulated as to compute the posterior $p(\alpha|y, \lambda, \beta)$.

**Parameter estimation for signal recovery:** Based on the above models and Bayesian inference rules, the posteriori probability $p(\alpha, \lambda, \beta|y)$ should be achieved to get the distribution of the parameters. However, there are too many parameters to be estimated in the posterior probability and it can not be calculated directly. The posterior can by be factorized according to Bayesian theorem as:

$$p(\alpha, \lambda\beta|y) = p(\alpha|y, \lambda, \beta)\, p(\lambda, \beta|y) \tag{6}$$

where, $p(\alpha|y, \lambda, \beta)$ is the posterior probability of the sparse signal or its coefficient. It can be decomposed by Bayesian theorem:

$$p(\alpha|y, \lambda, \beta) = p(y|\alpha, \beta)\, p(\alpha|\lambda)/p(y|\lambda, \beta) \tag{7}$$

Because $p(y|\alpha, \beta)$ and $p(\alpha|\lambda)$ are both Gaussian distributions, $p(\alpha|y, \lambda, \beta)$ and $p(y, \lambda, \beta)$ are also Gaussian. The mean and variance of the posterior probability $p(\alpha|y, \lambda, \beta)$ are, respectively as:

$$\Sigma = (\beta\Phi T\Phi + A)^{-1} \tag{8}$$

$$\mu = \beta\Sigma\Phi^T y \tag{9}$$

where, $A = \mathrm{diag}(\lambda_1, \cdots \lambda_N)$. From Eq. 8 and 9 it can be seen that as long as the parameter $\lambda$ is obtained, the mean and variance of the sparse signal can be obtained. The parameter $\lambda$ is solved using the type-II maximum likelihood methods. The method is to maximize the marginalized probability $p(y|\lambda, \beta)$ with respect to the parameter $\lambda$. In fact, the uniform prior distribution for $\lambda$ is used for RVM which is proved to give more sparse solution when using $\alpha = 1$ and $b = 0$ as shape and scale values, respectively (Babacan *et al*., 2010). After omitting the constant, the logarithm of the marginalized probability $p(y|\lambda, \beta)$ with respect to the parameter $\lambda$ is:

$$L = -\frac{1}{2}\log|\Sigma_\lambda| - \frac{1}{2} y^T \Sigma_\lambda^{-1} y$$

where, $\Sigma_\lambda = \beta^{-1}I + \Phi A^{-1}\Phi^T$. Through differentiating the log likelihood leads to $\tilde{\lambda}_i = v_i/\mu_i^2$ and $v_i = 1\lambda_i\Sigma_{ii}$, $\tilde{\lambda}_i$ is the estimation of $\lambda$ and $\Sigma_{ii}$ is the i-th diagonal element of the variance of the posterior in the current iteration. If taking the sparse signal $\alpha$ as the hidden variable and using EM algorithm, the estimation of $\lambda$ is equivalent to differentiating the log likelihood (Tipping, 2001). And the estimation of $\sigma^2$ is:

$$\tilde{\sigma}^2 = \|y - F\mu\|^2 \Big/ \left(N - \sum_{i=1}^{N} v_i\right) \tag{10}$$

However, the above solution involves solving the inverse matrix which increases the computational complexity and may lead to ill-conditioned matrix. To overcome this problem, Tipping proposed an adding and deletion algorithm to calculate the parameter $\lambda$ (Tipping and Faul, 2003). The algorithm rewrites the likelihood function as a function of a certain $\lambda_i$, then obtain the $\lambda_i$ which make the maximum likelihood increment. This method avoids the matrix inversion thus improves the stability of computation and there is not great decline in the final solution.

The two methods described above are both point estimation methods. Bishop proposed a Variational Bayesian (VB) approach to approximate the posterior, other than the point estimation this approach is based on distribution approximation (Bishop and Tipping, 2000). In this method, the posterior probability distribution can be approximated by the product of several probability distributions:

$$p(\alpha, \lambda, \beta|y) \approx q(\alpha)q(\lambda)q(\beta) \tag{11}$$

where, $\approx$ indicates an approximation rather than equal relationship. By applying the variational E-step, each factor of the approximation can be obtained as follows:

$$q(\alpha) = N(\alpha|\mu, \Sigma) \tag{12}$$

$$\mu = \langle\beta\rangle_{q(\beta)} SF^T y \tag{13}$$

$$S = \left(\langle\beta\rangle_{q(\beta)} F^T F + \langle A\rangle_{q(\lambda)}\right)^{-1} \tag{14}$$

$$q(\lambda) = \prod_{i=1}^{J} \Gamma(\lambda_i | \tilde{a}, \tilde{b}_m) \tag{15}$$

$$\tilde{a} = a + 0.5,\ \tilde{b}_m = b + 0.5\left\langle \alpha_i^2 \right\rangle_{q(\alpha)} \tag{16}$$

$$q(\beta) = \Gamma(\beta | \tilde{c}, \tilde{d}) \tag{17}$$

$$\tilde{c} = c + N/2,\ d = d + \left\langle \|y - F\alpha\|^2 \right\rangle_{q(\alpha)} / 2 \tag{18}$$

The above formulas show that all the parameters depend on each other. Iterate these formulas until convergence, the posterior of the sparse signal can be estimated. However, from the above formula can be seen that, the same as the point estimate is that the inverse matrix is still needed to be solved. So, the computation complexity will not fall.

In general, after obtaining the mean of the posterior probability for the sparse signal $\alpha$, the original signal can be estimated as $\hat{x} = \Psi\mu_\alpha$.

**CS of optical fiber pipeline pre-warning data:** In optical fiber pre-warning system, because the system is long-running, the collected signal length is very long. According to the basic principle of compressed sensing, the signal needs to be processed segment by segment on the basis of Nyquist sampling assumption, thus completing the conversion from Nyquist sampling to compressed sampling. From the computational complexity considerations, the segment length should not be too long but too short length can not reflect the compression ability of compressed sensing, so generally the length is taken as 1024.

The noise data accounted for a large proportion and the threatening signal a small proportion in optical fiber pre-warning system monitoring signal. Therefore, to further improve the compression ratio, the OMP-based signal detection method is used to detect whether there is threatening event in the compressed sampling measurement. If there is threatening signal, then more measurement value will be taken otherwise less will be taken.

In maximum likelihood based estimation process, some elements of the parameter $\lambda$ will become very large. If $\lambda_i$ exceeding a threshold, then the corresponding column vector in $\Phi$ will be deleted for parameter estimation. According to Eq. 11, this means the corresponding sparse coefficient will be zero. Then, the final accuracy of the recovered signal will be affected by the value of this threshold. It should be noted here that, although the estimation method proposed by MacKay can accelerate the convergence speed, it will bring ill conditioned matrix $\Sigma^{-1} = \beta\Phi^T\Phi + A$ for too sparse signal. For this reason, using the low convergence speed estimation algorithm is suitable for compressed signal recovery. The adding and deleting algorithm avoids the inverse matrix problem and will not be affected by the ill conditioned matrix. Therefore, this algorithm will be very fast for the recovery of compressed sampled optical fiber pipeline signal. The VB based approach is a distribution estimation algorithm which needs not to set the threshold. Of course, the threshold value is also feasible for this algorithm.

## EXPERIMENT RESULTS

In this study, the simulation is carried out using the signal gathered on scene. The cable is 500m length with depth of 0.5 meter underground. To simulate the threatening event of artificial stolen digging behavior, shovel is used to dig near the optical fiber. The Nyquist sampling rate is 4MHz.To decrease the data to be processed, the sampled signal is down sampled to 44.1 KHz to demonstrate the RVM based algorithm.

For the data compression function of compressed sampling step of CS, there needs to measure the compression ratio. The compression ratio depends on the recovery quality, only the compression ratio meets a certain quality of recovery is feasible. The higher the compression ratio, the worse the signal quality of recovery, whereas the better. The Compression Ratio (CR) is defined as the ratio of the length of the original signal and the measurement number. The recovery quality SNR is defined as the ratio between the energy of the original signal and the sum of error square (Tang *et al*., 2000). SNR is defined as:

$$\mathrm{SNR} = 10\log 10 \sum_{n=1}^{N} x^2(n) \Big/ \sum_{n=1}^{N} [x(n) - \tilde{x}(n)]^2 \tag{19}$$

In the following experiment, the algorithm of the basic relevant vector machine is denoted as RVM, the adding and deleting algorithm for the relevant vector machine is denoted as ADRVM and the variational Bayesian approach is denoted as VBRVM. Firstly, when the segment length of the original signal is 1024 and the measurement number is 90, the signal recovered by ADRVM together with the original signal are shown in Fig. 1. As can be seen from the figure, despite some recovery noise, the signal restored by ADRVM method can be a good approximation of the original signal.

In order to compare the reconstruction quality of the three parameter estimation methods for RVM, in the case of original signal length taken as 1024, the recovery quality is plotted in Fig. 2 when the measurement number varies from 50 to 90 with step 5. As can be seen from Fig. 2 while the adding and deleting algorithm is a sub-optimal method, it has the best quality reconstruction. To further compare the performance of other different algorithms, the recovery quality of OMP and CoSamp algorithm is evaluated under the same condition. For CoSamp, the true sparsity is give in advance for recovery. The SNR of the recovered signal quality by these two algorithms is also plotted in Fig. 2. The results show that ADRVM still gets the best reconstruction quality than OMP and CoSamp. The OMP detection with ADRVM is simulated to further decrease the compression ratio. This is denoted as OADRVM in Fig. 2. The recovery result shows that this algorithm can have almost the same quality as ADRVM and is slightly better or worse than ADRVM for certain measurement number.

Figure 3 shows the CPU time of ADRVM, OMP and CoSamp as the measurement number varies from 50 to 90 with step 5. From the result it can be seen that ADRVM costs the most CPU time and its CPU time will increase with the measurement number as OMP. However, the CPU time of CoSamp will not vary with the measurement number and it costs the least CPU time. Although ADRVM has the best recover quality with the same measurement number, it has the worst CPU time.

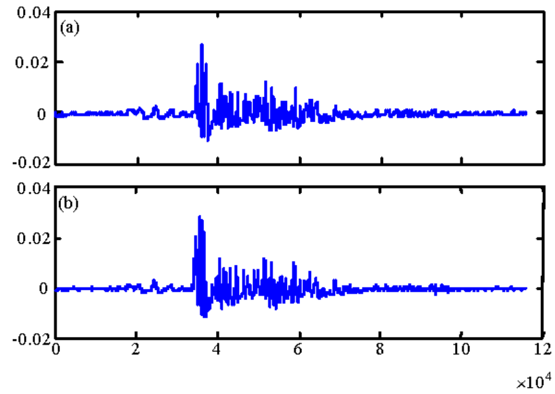


Fig. 1(a-b): Mining signal used in the paper and the recovered signal using ADRVM with measurement number equals to 90

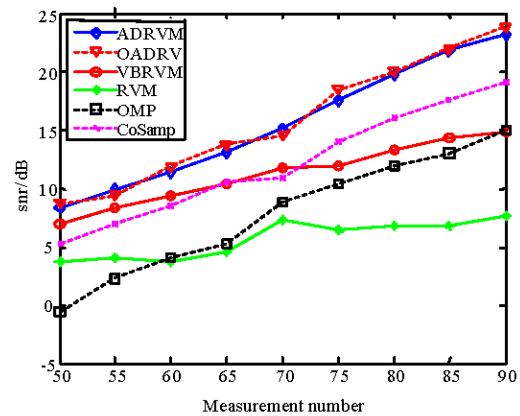


Fig. 2: SNR comparison of RVM methods, CoSamp and OMP as the measurement number varies from 50 to 90

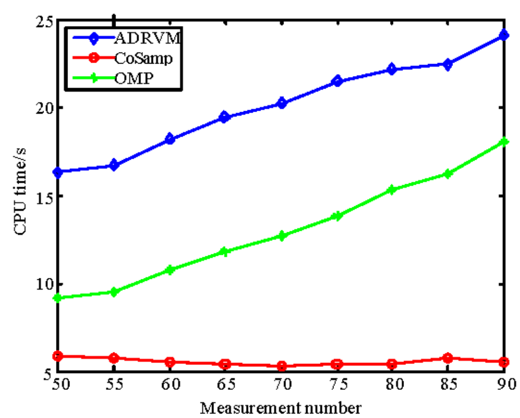


Fig. 3: CPU time of ADRVM, CoSamp and OMP as a function of measurement number

Table 1: Positioning difference in time

| Measurement number | OADRVM ($\Delta$t s) | ADRVM ($\Delta$t s) |
|---|---|---|
| 50 | 2.91e-5 | 2.37e-5 |
| 55 | 5.42e-5 | 5.92e-6 |
| 60 | 4.90e-6 | 5.92e-6 |
| 65 | 3.00e-6 | 1.97e-6 |
| 70 | 3.00e-6 | 0.99e-6 |
| 75 | 1.00e-6 | 0.99e-6 |
| 80 | 0 | 0 |
| 85 | 0 | 0 |
| 90 | 0 | 0 |

Finally, positioning experiment on the reconstructed signal is executed. Assuming the length of the pipeline is L and the time difference of the signal pass by the first end and the end is $\Delta_t$, the position of the threatening events is $x = (L-v\Delta t)/2$ (Yan, 2006). The above equation indicates the key of location is to find the time difference. In this paper, the cross-correlation method is used to get the time difference. Let x(n) and y(n) are the signal achieved in the first end terminal and the end terminal respectively, then the cross-correlation function is:

$$R(m) = \frac{1}{N}\sum_{n=0}^{N-1} x(n)y(n+m)$$

Assuming the maximum is got when $m = m_0$, the time difference is $\Delta t = m_0 T_s$ where $T_s$ is the Nyquist sampling time interval. In this paper, the difference of $\Delta t$ between original and the recovered signal is used to evaluate the positioning result with measurement number varies from 50 to 90 with step 5. Only the algorithms ADRVM and OADRVM are evaluated for positioning. The results are shown in Table 1. As can be seen from the table, when the measurement number exceeds 80 or CR is 12.8 for ADRVM, the reconstructed signal can have the same positioning result as the original signal.

## CONCLUSION

Since the Nyquist sampling rate of optical fiber pre-warning system is too high to produce large amounts of data, this paper uses compressive sampling methods for data compression to reduce the amount of data. To further reduce the amount of data, the OMP detection method is first used to detect whether a threat signal is in the measurement. Then, more measurement values will be taken if there is a threat signal, or less. The Relevant Vector Machine (RVM) is used to model the compressive sampling process for data recovery. Based on the Bayesian framework, the sparse data and noise variance each are given a probability with a prior. Three kinds of parameter estimation methods are used for data recovery. Although the sparsity degree of each segment of the optical pre-warning data varies with time, RVM can identify the sparsity in the data recovery process, so it is very practical for a long time actual signal. Experimental results show that RVM can accurately recovery the optical pre-warning data and the CR can reach 12.8. Finally, the positioning experiments prove that the reconstructed data will not affect the positioning result.

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