TV-Seg: Total Variation Segmentation with Imprecise Region

Jinjiang Liu, Jingjing Liang, Yihua Lan, Quanzhou Cheng, Ming Li and Chih-Cheng Hung
School of Computer and Information Technology, Nanyang Normal University, Nanyang 473061, China
 Imaging Processing Business Dept of Beijing E-COM Technology Co., Ltd., Beijing, 100176, China
 Sino-US Intelligent Information Processing Joint Lab, Anyang Normal University, Anyang 455000, China
 Center for Biometrics Research, Southern Polytechnic State University, Atlanta, GA, USA

Abstract: In this study, a new image segmentation method using both region and edge information is proposed to combine the two types of methods or information to achieve accurate segmentation results. The region information is represented by a simple binary mask and the edge information is abstracted from the gradient image. A global optimal total variation model is employed to combine them together. Experimental results demonstrate the efficacy of the proposed method.

Key words: Image segmentation, total variation model, region information

INTRODUCTION

Many segmentation methods have been proposed, including but not limited to as follows: region growing algorithm (Adams and Bischof, 1994; Ojha et al., 2012; Mendoza et al., 2012), Active Contour Models (ACM) (Chan and Vese, 2001; Te Brake and Karssemeijer, 2001) and dynamic programming algorithm (Timp and Karssemeijer, 2004; Cheng and Lin, 2012). Those methods can be mainly classified into two categories: region-based methods and edge-based methods. The two types of methods have their advantages and disadvantages respectively.

One of the most famous edge-based models is the Geodesic Active Contour (GAC) model (Caselles et al., 1997), which is an enhanced version of the snake model (Kass et al., 1988). The edge information is formulated as an edge indicator function by this model. Because only using the edge information, the GAC model is highly sensitive to the initial condition. Another drawback of GAC model is that for images with weak edges or without edges, the edge indicator function will not offer enough information for correct segmentation. Region-based ACM have many advantages over edge-based ones. By using the information inside and outside the contour to control the evolution, the region-based ACM overcome the aforementioned disadvantages. However, for some applications that we need to segment a special target in a complex background, the region-based model fails.

In this study, we propose a new ACM combines the imprecise region information and edge information by a total variation model. The novelty of our model from others is that the region information is formed as a binary mask located around the target region. This mask works as an anchor for contour evolution.

The rest of this paper is organized as follows: Section 2 describes the proposed total variation segmentation with imprecise region information method. Experimental results are shown in section 3. At last a summary and conclusions are given in the section 4.

TOTAL VARIATION SEGMENTATION WITH REGION MASK

The formulation of GAC is in fact a weighted curve length integral that is defined by the following minimization problem:

$$\min_C \left\{ E_{gac}(C) = \int_{L(C)} g(|\nabla I(x)|) |ds| \right\}$$

where $ds$ is the Euclidean length of $L(C)$ is the length of the curve $C$. The function $g$ is an edge indicator function that vanishes at object boundaries, such as:

$$g(|\nabla I|) = \frac{1}{1 + \beta |\nabla I|^\eta}$$

and $g(|\nabla I|) = e^{-\eta |\nabla I|}$, where $I_0$ is the original image and $\beta$, $\eta$, $\kappa$ are arbitrary positive constants. Bresson et al. proved the following equation (Bresson et al., 2007):

Corresponding Author: Yihua Lan, School of Computer and Information Technology, Nanyang Normal University, 473061, Nanyang, China Tel:+86 15225649611
where, \( TV_v(u) = \int \nabla u \cdot \nabla v \, d\Omega \). This means that the GAC energy is equal to the weighted TV-norm when the function \( u \) is a characteristic function of a closed set \( \Omega \subset \Omega \). They also proved that \( u \) is almost allowed to vary continuously between \( [0, 1] \). The advantage of \( TV_v(u) \) over \( E_{GAC}(C) \) is its (non-strict) convexity and easy to add region information.

In many algorithms, such as C-V model (Chan and Vese, 2001) and global geodesic active contour method (GGAC), the region information is only used to ‘help’ the curve move to the strong edge. It reflects the characteristics of the image, but can not express the will of the observer. When we want to segment a special target in a complex background, this kind of region-based model fails. So we need a region term not only contain the image features, but also include target location information. The region term in our proposed model is formulated as follow:

\[
E_{\text{reg}}(u) = \int \nabla (1-2I_{\text{reg}}(x)) \cdot \nabla u \, d\Omega 
\]

(3)

where, \( I_{\text{reg}}(x) \in \{0, 1\} \) for \( \forall x \in \Omega \), is a mask to identify the location information and \( (1-2I_{\text{reg}}(x)) \in \{-1, 1\} \). This term is used to punish the difference between \( u \) and \( I_{\text{reg}} \). Unlike the region term in C-V model, where the mean value of inner region and outer region are updated periodically with the changing of region contour, the binary image \( I_{\text{reg}} \) performs as a fixed anchor to drag the occurring of contour around. Consider \( u \) as a characteristic function, \( E_{\text{reg}}(u) \) takes minimum when \( u \) equals \( I_{\text{reg}} \) and when \( u \) deviates from \( I_{\text{reg}} \, E_{\text{reg}}(u) \) grows. Through a combination of \( E_{\text{reg}}(u) \) and \( TV_v(u) \), a novel energy is obtained to assemble edges around a labeled region to get an accurate segmentation. The proposed model is formulated as follow:

\[
\min_{u,v} \left\{ E_{\text{reg}}(u) = TV_v(u) + \mu E_{\text{reg}}(u) \right\} 
\]

(4)

where, \( \mu \) is an arbitrary positive constant to control the influence of the edge term and the region term.

Many different approaches can be used to solve this total variation model. Based on Aujol et al. (2006), we modify the model using an approximation variable \( v \):

\[
\min_{u,v} \left\{ E_{\text{reg}}(u) = TV_v(u) + \mu E_{\text{reg}}(v) + \frac{1}{2\theta} \| u - v \|_1^2 \right\} 
\]

(5)

Where the parameter \( \theta > 0 \) is chosen to be small. Since (5) is a strictly convex approximation of (4), its minimizer can be computed by minimizing \( u \) and \( v \) separately. Thus, the following minimization problems are considered:

\[
\min_{v} \left\{ TV_v(u) + \frac{1}{2\theta} \| u - v \|_1^2 \right\} 
\]

(6)

\[
\min_{\nu} \left\{ \mu E_{\text{reg}}(v) + \frac{1}{2\theta} \| u - v \|_1^2 \right\} 
\]

(7)

According to Chambolle et al. (2004), (6) can be solved by using a dual variable \( p \) where:

\[
\begin{align*}
\frac{u^e}{\theta} &= v + \theta \frac{dp}{d\nu} \\
\frac{p^e}{\theta} &= \theta \frac{\nabla u}{1 + \frac{\theta}{2} \| \nabla u \|_1^2}
\end{align*}
\]

(8)

(9)

In all experiments, the time step \( \tau \) is equal to 1/2. The solution of (7) is given by:

\[
v = \min \left\{ \max \{ u - \theta \mu (1 - 2I_{\text{reg}}), 0 \}, 1 \right\} 
\]

(10)

Unlike other region-based methods, \( I_{\text{reg}} \) is fixed during iteration.

**EXPERIMENTAL RESULTS AND DISCUSSION**

To test and evaluate the performance of the proposed segmentation approach, a several experiments have been carried out. In the first example, a polyp image shown in Fig. 1a is used to demonstrate the property of our model. The polyp image is of bad light condition and complex background. A general segment result of our method is shown in Fig. 1.

We also used this image to demonstrate the impact of \( \mu \) on the segmentation result. The parameter \( \mu \) is an arbitrary positive constant to control the influence of the edge term and the region term. A small \( \mu \) means that the result is primarily determined by the edge information, whereas a large \( \mu \) makes the region information to the main contributor. The region image and edge image of the experiment are fixed and shown in Fig. 1b and c. We use:

\[
g(\| \nabla u \|_1) = e^{-\eta \| \nabla u \|_1^2}
\]

to measure the edge information, where \( \eta = 0.2 \), \( \kappa = 0.85 \). Six different values of \( \mu \) range from 0.5 to 0.001 are
selected in our experiments. Image \( u \) and contours corresponding to different \( \mu \) are shown in Fig. 2. The region term measure the difference between image \( u \) and the mask \( I_{\text{ref}} \). A large \( \mu \) allows tiny difference. The edge term has two functions. It smoothes the image by the effect of \( |\Delta u| \) and it forces the counter to the strong edge by \( g(x) \) as well. So we can see from Fig. 2 that the images become smooth as \( \mu \) decreased. The mutation of the last two images illustrates that the region term fails to pull the contour to the correct location.

The robustness of our method with different kinds of mask is shown in Fig 3. The first mask is irregular, scattered and small, while the second mask is partially covered. In both case, the algorithm converges to the same correct segment result. This makes our method quite different from some seed-dependent methods, of which the results heavily depend on the selection of the seeds.

In each group, we present the mask, the segment image and \( u \). In Fig. 4, three different initial \( u \) are employed to show the global property of our model. No matter the initial \( u \) is a random image, a single-value image or a mask, the output of our model has the same structure with different value scales due to the energy of initial images.

In the second example, we compared the proposed method with C-V model and GAC model. Figure 5 demonstrates the proposed method and C-V method in noisy ultrasound image segmentation. It is obviously that the proposed model got more accurate result than the C-V model. Figure 6 compares the proposed method and the
Fig. 4: Global property of our model. In each group, we present 1) initial u and 2) final u. (a) with a random initial u, (b) with a single-value initial u and (c) with a mask initial u.

Fig. 5: Compare the proposed method with C-V. (a) Initial contours, (b) Result of C-V model, (c) Result of our model $\mu = 0.01$ and (d) Result of our model $\mu = 0.015$

GAC model by applying them to an MR image of corpus callosum. For the proposed method, we set $\mu = 0.001$ to

Fig. 6: Compare the proposed method with GAC model. (a) Initial contours, (b) Result of GAC model by force 0.6 (c) Result of GAC model by force 0.8 and (d) Result of our model $\mu = 0.015$

$\mu = 0.01$, while for the traditional GAC model with a small balloon force 0.6, the evolution converges in 5000 iterations and the contour could not pass through the narrow and long part of the object. With a larger balloon force 0.8, the contour could pass over the relatively weaker part of the object. And the initial contour must be inside of the object for GAC model.

CONCLUSION

A new total variation model proposed in this study is designed to combine the edge information and region information. In this paper, we show that the whole region information is not necessary for smoothing region segmentation. Unlike previous region-based methods, the region information is easily represented by a simple binary mask that partially covers the target. We combined GAC model and CV model by TV method. Experimental results show that new model use only gradient and imprecise position information to get better segment results than CV and GAC and it outperforms some popular state-of-art segment algorithms on bad contour condition.

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