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## Influencing Factors Analysis for the Flutter of Wind Turbine Blades

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**Abstract:** Considering the non-stationary Theodorsen aerodynamic force, which was often used for the prediction of the classical flutter of the fixed wing aircraft, the flutter analysis of wind turbine blade airfoil was carried out through the use of the V-g method, based on the equation of motion of wind turbine blade airfoil. So the problem of the flutter critical velocity of blade airfoil should be solved using the eigenproblem and the V-g curve and V- $\omega$  curve will be given. Then, the influencing factors of the flutter critical speed for the wind turbine blade airfoil, that is, the chordwise location of the center of mass, the ratio of flapwise natural frequency to the torsion natural frequency, the ration of mass and the location of three centers (aerodynamic center, stiffness center and the center of mass), were researched in great details. As the conclusions, the guidelines for the blade design, which can improve the flutter critical speed, can be obtained as: (1) Move forward the center of mass, (2) Keep the ratio of natural frequency square not equaling and far away from 1, (3) Choose the area of lower density of air as wind farm and (4) Reduce the distance between the aerodynamic center, stiffness center and the center of mass. These guidelines can be applied for the anti-flutter designs of the wind turbine blade.

**Key words:** Aeroelasticity, blade, flutter, influencing factor, wind turbine

### INTRODUCTION

Wind electricity has become one of the most important representatives as the new energy and clean energy, confronting the current global energy crisis. With the developing the wind electricity industry, wind turbines are focusing on the upsizing. And as one of the most critical parts of the wind turbine, the wind turbine blade is moving for the upsizing and flexibility (Manwell *et al.*, 2009). Because the occurring of wind turbine flutter will give rise to the severe accident, the aeroelatic problem of the wind turbine, especially the dynamic aeroelaticity, that is, flutter, has been the key subject of relative fields (Zhang and Huang, 2011; Hansen *et al.*, 2006). Many researchers have made the studies of wind turbine flutter, totally in two aspects, frequency domain analysis and the time domain analysis. So the critical flutter speed should be given when using the frequency domain analysis (Hansen, 2004, 2007). And the time domain responses of some the wind turbine characteristics will be gotten.

Lobitz (2004) pointed out that two parameters, the chordwise location of the center of mass and the ratio of the flapwise natural frequency to the torsional natural frequency are significant for the flutter per rev flutter speed. And he also researched the classical flutter of the

wind turbine blade and the flutter critical speed would be improved when using the aeroelastic tailoring techniques (Lobitz, 2005).

Up to now, there are little studies on the influencing factors of flutter critical speed of wind turbine blade and there are little studies on the guidelines of the design of the wind turbine blade. In the paper, V-g method should be utilized to carry on the frequency domain analysis of wind turbine blade flutter and predict the flutter critical speed and systematically research the influencing factors of the flutter critical speed. So, the guidelines for the anti-flutter should be expected as conclusions. The outlines are as follows: equation of motion of the wind turbine blade airfoil considering the Theodorsen aerodynamic force, V-g method theory, solution of the flutter critical speed of wind turbine blade airfoil, the influencing factors studies of the flutter critical speed and the conclusions in the end.

### EQUATION OF MOTION OF THE WIND TURBINE BLADE AIRFOIL

The blade airfoil model is given as Fig. 1. The system consists of a tension spring and a torsion spring. Therefore, this 2-D airfoil has two freedoms of degree, that is, the flap ( $h$ , the vertical displacement of the center of

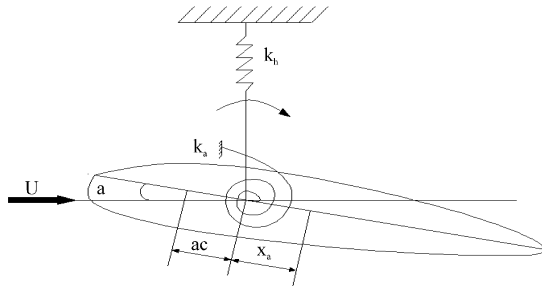


Fig. 1: Blade airfoil model

stiffness, is positive when the airfoil is downward) and the torsion ( $\alpha$ , the pitch angel of airfoil around the center of stiffness, is positive when the airfoil is up windward). Therefore, the aerodynamic force,  $F$ , is positive when the airfoil is downward, the aerodynamic moment,  $T_{\alpha}$ , is positive when the airfoil is up windward.

Where:  $k_h$  is the stiffness of the tension spring,  $k_{\alpha}$  is the stiffness of the torsion spring,  $x_{\alpha}$  is the distance between the center of mass and the center of stiffness,  $ac$  is the distance between the center of stiffness and the aerodynamic center.

Equation 1 is the equation of motion for the wind turbine blade airfoil, which can also has another form non-dimension as (2):

$$\begin{bmatrix} m & mx_{\alpha} \\ mx_{\alpha} & I_{\alpha} \end{bmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \end{Bmatrix} + \begin{bmatrix} k_h & 0 \\ 0 & k_{\alpha} \end{bmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix} = \begin{Bmatrix} F \\ T_{\alpha} \end{Bmatrix} \quad (1)$$

$$\begin{bmatrix} 1 & \bar{x}_{\alpha} \\ \bar{x}_{\alpha} & \bar{r}_{\alpha}^2 \end{bmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \end{Bmatrix} + \begin{bmatrix} \omega_h^2 & 0 \\ 0 & \omega_{\alpha}^2 \bar{r}_{\alpha}^2 \end{bmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix} = \begin{Bmatrix} \frac{F}{mc} \\ \frac{T_{\alpha}}{mc^2} \end{Bmatrix} \quad (2)$$

where,  $m$  means the mass of blade element,  $I_{\alpha}$  means moment of inertia around the center of stiffness,  $r_{\alpha}$  is the rotating radius of airfoil around the center of stiffness,  $c$  means the semi-chord length,  $\omega_{\alpha} = \sqrt{k_{\alpha}/I_{\alpha}}$  is the natural frequency of torsion,  $\omega_h = \sqrt{k_h/m}$  denotes the natural frequency of flap:

$$\bar{x}_{\alpha} = \frac{x_{\alpha}}{c}$$

is the percentage of the distance between the center of mass and the center of stiffness to the semi-chord length,  $\bar{r}_{\alpha} = r_{\alpha}/c$  means the percentage of the rotating radius of airfoil around the center of stiffness to the semi-chord length.

When the wind speed is at the flutter critical speed, the blade airfoil shows simple harmonic oscillation. So,  $h$ ,  $\alpha$  and  $F$ ,  $T_{\alpha}$  and can be presented as following:

$$h = \bar{h}e^{i\omega t}, \alpha = \bar{\alpha}e^{i\omega t}, F = \bar{F}e^{i\omega t}, T_{\alpha} = \bar{T}_{\alpha}e^{i\omega t} \quad (3)$$

where,  $\bar{h}$  means the flap amplitude,  $\omega$  is the flutter frequency,  $\bar{\alpha}$  means the torsioin amplitude,  $\bar{F}$  means the amplitude of the aerodynamic force,  $\bar{T}_{\alpha}$  means the amplitude of the aerodynamic torque.

So, substituting (3) into (2), yields:

$$-\omega^2 \begin{bmatrix} 1 & \bar{x}_{\alpha} \\ \bar{x}_{\alpha} & \bar{r}_{\alpha}^2 \end{bmatrix} \begin{Bmatrix} \bar{h} \\ \bar{\alpha} \end{Bmatrix} + \begin{bmatrix} \omega_h^2 & 0 \\ 0 & \omega_{\alpha}^2 \bar{r}_{\alpha}^2 \end{bmatrix} \begin{Bmatrix} \bar{h} \\ \bar{\alpha} \end{Bmatrix} = \begin{Bmatrix} \frac{\bar{F}}{mc} \\ \frac{\bar{T}_{\alpha}}{mc^2} \end{Bmatrix} \quad (4)$$

According to the Theodorsen theory (Theodorsen, 1935), the nonstationary aerodynamic force and moment can be:

$$\bar{F} = \pi \rho_a c^3 \omega^2 \left\{ \frac{\bar{h}}{c} L_h + [L_{\alpha} - (\frac{1}{2} + \bar{a}) L_h] \bar{\alpha} \right\} \quad (5)$$

$$\bar{T}_{\alpha} = \pi \rho_a c^4 \omega^2 \left\{ [M_h - (\frac{1}{2} + \bar{a}) L_h] \frac{\bar{h}}{c} + [M_{\alpha} - (\frac{1}{2} + \bar{a})(L_{\alpha} + M_h) + (\frac{1}{2} + \bar{a})^2 L_h] \bar{\alpha} \right\} \quad (6)$$

where:

$$M_h = \frac{1}{2}, M_{\alpha} = \frac{3}{8} - i \frac{1}{k}, L_h = 1 - i 2C(k) \frac{1}{k}$$

$$L_{\alpha} = \frac{1}{2} - i \frac{1 + 2C(k)}{k} - \frac{2C(k)}{k^2}$$

Substituting (5) and (6) into (4), yields:

$$-\Omega^2 M \begin{Bmatrix} \bar{h} \\ \bar{\alpha} \end{Bmatrix} + K \begin{Bmatrix} \bar{h} \\ \bar{\alpha} \end{Bmatrix} = \frac{\Omega^2}{\mu} A(k) \begin{Bmatrix} \bar{h} \\ \bar{\alpha} \end{Bmatrix} \quad (7)$$

where:

$$M = \begin{bmatrix} 1 & \bar{x}_{\alpha} \\ \bar{x}_{\alpha} & \bar{r}_{\alpha}^2 \end{bmatrix}, K = \begin{bmatrix} R_h^2 & 0 \\ 0 & \bar{r}_{\alpha}^2 \end{bmatrix}$$

$$A(k) = \frac{1}{\mu} \begin{bmatrix} L_h & L_{\alpha} - (\frac{1}{2} + \bar{a}) L_h \\ M_h - (\frac{1}{2} + \bar{a}) L_h & M_{\alpha} - (\frac{1}{2} + \bar{a})(L_{\alpha} + M_h) + (\frac{1}{2} + \bar{a})^2 L_h \end{bmatrix}$$

where  $\rho_a$  is the density of air:

$$\mu = \frac{m}{(\pi \rho_a c^2)}$$

is the ratio of mass:

$$R_\omega = \frac{\omega_h}{\omega_\alpha}$$

is the ratio of the flapwise natural frequency to the torsion natural frequency:

$$\Omega = \frac{\omega}{\omega_\alpha}$$

means the ratio of the flutter frequency of the flapwise natural frequency.

**V-G METHOD THOERY**

V-g method, one of the methods of the frequency domain analysis, mainly based on the harmonically nonstationary aerodynamic force, can be used to solve the flutter equation by solving the complex eigenvalue. This method deals with the real part and imaginary part the complex eigenvalue respectively.

Introducing the artificial structural damping without the structural damping, so the (7) changes as (Zhao, 2007):

$$\begin{bmatrix} m & mx_\alpha \\ mx_\alpha & I_\alpha \end{bmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \end{Bmatrix} + \begin{bmatrix} k_h & 0 \\ 0 & k_\alpha \end{bmatrix} \begin{Bmatrix} h \\ \alpha \end{Bmatrix} = \begin{Bmatrix} F \\ T_\alpha \end{Bmatrix} + \begin{Bmatrix} D_h \\ D_\alpha \end{Bmatrix} \quad (8)$$

Where:

$$D_h = \bar{D}_h e^{i\omega t} = -ig_h m \omega_h^2 \bar{h} e^{i\omega t}, D_\alpha = \bar{D}_\alpha e^{i\omega t} = -ig_\alpha I_\alpha \omega_\alpha^2 \bar{\alpha} e^{i\omega t}$$

where,  $g_h$  means the introduced structural damping coefficient of flap,  $g_\alpha$  means the introduced structural damping coefficients of torsion.

Let  $g_h = g_a = g$ , so yields:

$$-\Omega^2 M \begin{Bmatrix} \bar{h} \\ \bar{\alpha} \end{Bmatrix} + (I + ig) K \begin{Bmatrix} \bar{h} \\ \bar{\alpha} \end{Bmatrix} = \Omega^2 A(k) \begin{Bmatrix} \bar{h} \\ \bar{\alpha} \end{Bmatrix} \quad (9)$$

It can express as the general eigenproblem:

$$(A(k) + M) \begin{Bmatrix} \bar{h} \\ \bar{\alpha} \end{Bmatrix} = \frac{(I + ig)}{\Omega^2} K \begin{Bmatrix} \bar{h} \\ \bar{\alpha} \end{Bmatrix} \quad (10)$$

So, the eigenvalue of the system becomes:

$$\lambda = \frac{(1 + ig)}{\Omega^2} = \lambda_{Re} + i\lambda_{Im} \quad (11)$$

And yields:

$$\omega = \frac{\omega_\alpha}{\sqrt{\lambda_{Re}}}, g = \frac{\lambda_{Im}}{\lambda_{Re}}, V = \frac{\omega_\alpha c}{k\sqrt{\lambda_{Re}}} \quad (12)$$

The calculating procedure of blade airfoil flutter using the V-g method gives:

- Decide the region of reduced frequency of interest:

$$k = \frac{\omega c}{U}$$

$c$  is semi-chord length,  $U$  is air speed)

- Calculate the general aerodynamic matrix  $A(k)$  and solve the complex eigenproblem to yield the eigenvalues  $\lambda$
- Determine the  $g, \omega, V$  at the current reduced frequency using (12)
- Repeat the above procedures at the new reduced frequency according to the step size decided
- Plot the V-g curve consequently

According to the V-g curve, when  $g$  is zero, the flutter condition is satisfied. So, the reduced frequency,  $k$ , at this time should be  $k_F$  and  $V$  determined by (12) should be  $V_F$ , the flutter critical speed. For easy use, let:

$$V_{non} = \frac{V}{\omega_\alpha c} = \frac{1}{k\sqrt{\lambda_{Re}}}$$

which is called reduced speed, can also be treated as the flutter critical speed in the following parts.

**SOLUTION OF THE FLUTTER CRITICAL SPEED OF WIND TURBINE AIRFOIL**

According to the literature (Althaus, 1996), the following parameters table can be introduced and calculated as Table 1.

Based on the parameters above, Fig. 2 gives the V-g graph, which shows the relationship between the reduced

Table 1: Parameters of the wind turbine blade airfoil

Parameters	Values	Parameters	Values
$\bar{x}_a$	0.025	$c$	0.5365 m
$\bar{r}_a$	0.2549	$\bar{\alpha}$	-0.1073 m
$\mu$	167	$k_h$	124-824 N/m
$m$	185 kg	$k_a$	3117 N.m/rad
$R_\omega$	0.0419	$I_a$	12.0245 kg m <sup>2</sup>
$\omega_g$	16.103	$\rho_a$	1.225 kg m <sup>-3</sup>

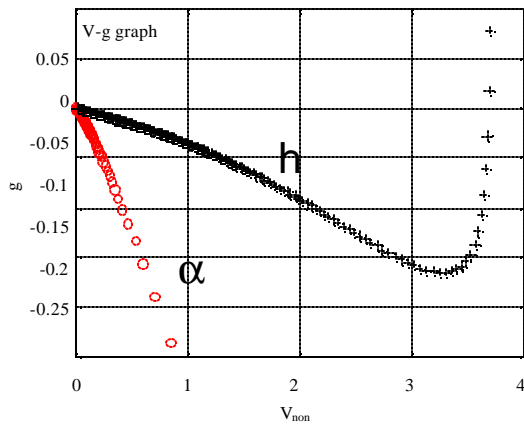


Fig. 2: Relation between the reduced speed and structural damping coefficient

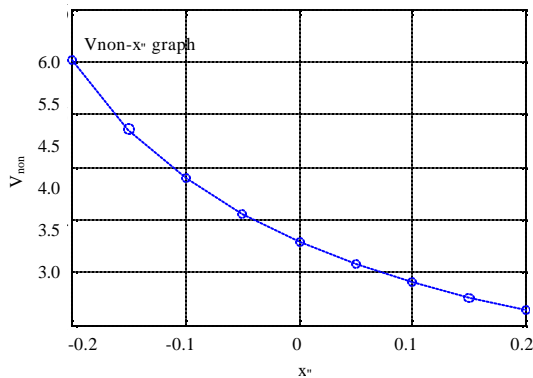


Fig. 3: Relation between the flutter critical speed and the location of the center of mass

speed and the structural damping coefficient, can be obtained based on the theory mentioned above. And the reduced speed equals about 3.6, when the g is zero.

### INFLUENCING FACTOR ANALYSIS OF FLUTTER

The influence of the location of the center of mass on the Flutter critical speed: Letting  $\bar{x}_a = -0.2 \sim 0.2$  with other parameters unchanged, study the influence of the location of the center of mass on the flutter critical speed. When the  $\bar{x}_a$  changes to a new value, the flutter critical

speed will change correspondingly. Thus, the relation between the flutter critical speed and the location of the center of mass will be given as Fig. 3.

According to the Fig. 3, it can be seen that the flutter critical speed will reduce as the  $\bar{x}_a$  changes from -0.2 to +0.2. So, If possible, the center of mass should be allocated ahead of the center of stiffness, in order to improve the flutter critical speed and postpone the flutter.

**Influence of the frequency ratio on the flutter critical speed:** Letting  $R_\omega = 0.2 \sim 1.2$  with other parameters unchanged, study the influence of frequency ratio on the flutter critical speed. When the  $R_\omega$  changes to a new value, the flutter critical speed will change correspondingly. Thus, the relation between the flutter critical speed and the frequency ratio will be given.

Figure 4 demonstrates the results. It is easy to find the flutter critical speed will be minimal when the frequency ratio is near 1, so the design should keep far away the 1 frequency ratio. That is to say, the frequency ratio is the best when it is close to zero or larger than 1, in order to improve the flutter critical speed.

**Influence of the mass ratio on the flutter critical speed:** Letting  $\mu = 1.2 \sim 10$  with other parameters unchanged, study the influence of mass ratio on the flutter critical speed. When the  $\mu$  changes to a new value, the flutter critical speed will change correspondingly. Thus, the relation between the flutter critical speed and the mass ratio will be given.

Figure 5 shows the results. The flutter critical speed is much high when the mass ratio is lower and it became minimal when the mass ration is near 2. Therefore, the mass ratio, 2, should be avoided when designing the wind turbine blade. And the mass ratio should be near 0.5 or the larger the better, in order to improve the flutter critical speed.

**Influence of the three centers on the flutter critical speed:** Reducing the distance of the three centers, that is, aerodynamic center, stiffness center and the center of mass, letting  $\bar{a} = -0.2$ ,  $\bar{x}_a = -0.1$ , Figure 6 shows the result. The flutter critical speed,  $V_{non} = 5.28$ , which is greater than  $V_{non} = 3.6$ , the result according to the Fig. 2. So, it will be seen that reducing the distance between the aerodynamic center, stiffness center and the center of mass will improve the flutter critical speed. And the flutter will be postponed because the new flutter critical speed has been increased than the formal condition.

**Design guidelines for anti-flutter:** From the results above, the design guidelines for the anti-flutter of wind

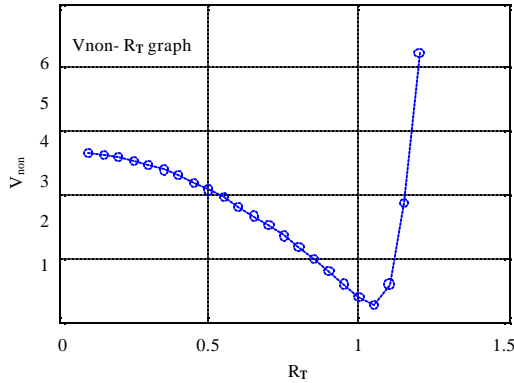


Fig. 4: Relation between the flutter critical speed and the frequency ratio

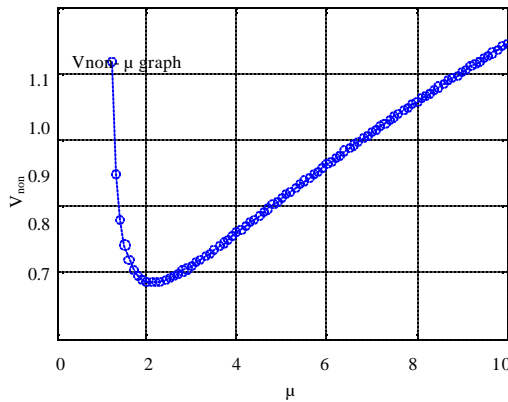


Fig. 5: Relation between the flutter critical speed and the mass ratio

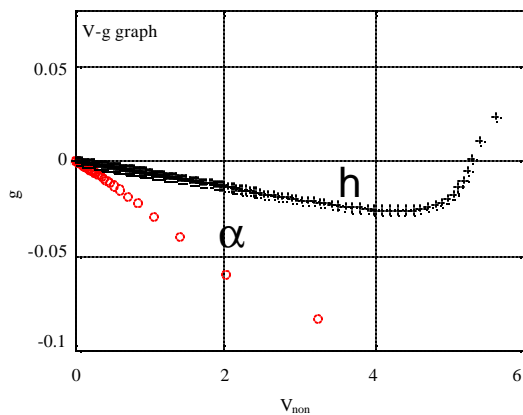


Fig. 6: Relation between the reduced speed and structural damping coefficient

turbine blade, which can improve the flutter critical speed and postpone the occurrence of flutter, can be obtained as following: (1) Move forward the center of mass ahead

of the stiffness center, if possible, before the aerodynamic center (2) Keep the ratio of natural frequency square not equaling and far away from 1, (3) Choose the area of lower density of air as wind farm because of the greater mass ratio, (4) Reduce the distance between the aerodynamic center, stiffness center and the center of mass.

These guidelines will improve the rationality of the design of the wind turbine blades and postpone the occurrence of the wind turbine blade flutter.

### CONCLUSION

The V-g method and nonstationary Theodorsen aerodynamic force were used to predict the classical flutter wind flutter critical speed. The V-g curve will be given, from which the flutter critical speed can yield. Then the influencing factors of the flutter critical speed for the wind turbine blade airfoil, that is, the chordwise location of the center of mass, the ratio of flapwise natural frequency to the torsion natural frequency, the ratio of mass and the location of three center(aerodynamic center, stiffness center and the center of mass), were researched in great details. The guidelines for the blade design, which can improve the flutter critical speed, can be obtained as following: (1) Move forward the center of mass, (2) Keep the ratio of natural frequency square not equaling and far away from 1, (3) Choose the area of lower density of air as wind farm, (4) Reduce the distance between the aerodynamic center, stiffness center and the center of mass.

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