Research on Control of Aircraft Trajectory in Simulating Game Scene Based on the Double Euler Method

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Abstract: This study firstly described how to solve the singular problem for aircraft in game scene by using the upside-down relationship between digest and singular zones of Positive Eulerian system and negative Eulerian system. Secondly, it took use of the piecewise cubic spline interpolation to simulate the aircraft trajectory and then through comprehensive two algorithms to control the aircraft in flight, the experimental data has proven that the method proposed in this study can simulate game scene control aircraft flight path. Finally, we made the following conclusions, the dual Euler can make more accurate flight simulation based on the attitude angle but also can improve the accuracy of the Euler equations, which can meet the needs of quick response of different aircraft the great scene in the large-scale three-dimensional scene games.

Key words: Dual euler method, singularity, 3D game scene, aircraft trajectory

INTRODUCTION

In recent years, with great improvement of computer performance and the development of graphics technology, digital signal processing technology, sensor technology and other related technologies, virtual reality technology gradually becomes an important development direction of modern modeling and simulation technology.

Due to the breakthrough of 3D graphics hardware technology, pictures that real-time painted become more and more delicate. Besides 3D game is more diversified and more approximate to the real world, so 3D games have been gradually replaced 2D games and become the mainstream of game market, even online games also tend slowly to 3D. In the aspect of 3D game production, some vendors in Europe, America and Japan have their own mature 3D game engine and the games the made have a large number of fixed players. For example, “Unreal Series”, “Metal Gear Solid II” and so on, they even have celebrated as classics and have a tremendous market potential. There are many high-quality domestic 3D games in recent years, such as long acclaimed "XuanYuan Sword", "XianJian" series began to transit to 3D games (Gunter and Parent, 1990; Zhao, 1987).

As one of virtual reality technology application fields, the aircraft is an outstanding representative of the game field. All flight simulations have links of mathematical simulation, that is when calculating craft attitude, angle $\gamma \phi$ and $\varphi$, Euler equation was used. This is the oldest, simplest, more accurate and more practical. However, from the formula of solving Euler equation by the full amount of motion equation, when $\phi = \pm90^\circ$, the equation has no solution, values of roll Angle and yaw Angle are unable to fixed, causing singularity problem of Euler equation (Sheeke, 1985; Zhang et al., 1999).

Singularity problem is not only a problem of the engineering flight simulation but also a key problem that must be solved in the field of games. During the past, to overcome the singularity, the Quaternion Methods are mainly used and Quaternion Algorithm is theoretically more perfect but a large calculation error exists in the practical application, which affects the calculation accuracy, while Dual Euler method separates the essential area from the overall calculating area and alternate calculation on the inverse relationship of essential area between the ordinary and reversed Euler equation. The essential area is expanded to the whole area and the singularity is overcomes completely and it has no error basically avoid singularity and to simulate aircraft in the game more accurately and efficiently.

DESCRIPTION OF DUAL EULER MODEL

During the definition of the Euler angles system, it Mainly related to two coordinate systems (geodetic...
The Fig. 1. $\phi$ is the angle of the projection of the body axis $x_b$ on the horizontal plane $o_{\text{hka}}$ with the axis $x_c$.

In a horizontal plane, starting from the axis $X$ and counterclockwise is positive. $\phi$ is the angle of the body axis $x_b$ with the horizontal $o_{\text{hka}}$. The direction of axis $x_v$, which pointing to the top, is the positive. $\phi$ is the angle of the aircraft longitudinal symmetry plane $o_{\text{vmn}}$ with the vertical plane containing the axis $x_v$ and look forward from the bottom, the direction of right is positive (Alfred C Robinson, 1958).

According to the concept of flight mechanics, when the ground coordinate system transforms to the body coordinate system, it needs to change three times in turn. If the transformation sequence is the yaw, pitch and roll, it's corresponding to the yaw Angle, roll Angle and pitching Angle of ordinary Euler Angle, respectively write as $\psi$, $\phi$, $\gamma$.

In geometry, when the turn angle is more than a around angle 3600, the cycle of the geometric position is continue. So, we assume that the magnitude Angle is not limited, there are a number of numerical representation for any angular position. To avoid a repeat value, the need to establish clear single value domain. All angle values should be in the scope of $-180^\circ$ to $+180^\circ$, which is in the week angle range.

However, this provision for the space geometric cross-linked system Euler angles is incomplete. If the three corner angle domain is the around angle, there exists double-valued. The following diagrams show the inverted state of aircraft. Figure 2 depicts the express of $|\phi| > 90^\circ$, because of this method is not coincide with the definition of the correct attitude angle, except that somersault flight is inconsistent with people's habit of feeling, it was known as the wonders value. Figure 3 for the representation of $|\phi| \leq 90^\circ$, its attitude angle is strictly conformity with the orthodox definition and conventional concept, so, named intuitive value. Two sets of values are different but there is a pattern: The absolute value of the two same performers is complementary.

Thereby, wonders value domain absolutely produces a double value, only be intuitive definition of a domain the single value. However, For the wonders is the singularity performance of the Euler angles in the definition of domain, it causes the definition of state angle producing qualitative change so that also it makes some illusion and confusion. it is usually should not be used. For example, The Earth's latitude and longitude belong to the visual domain of Euler angles, if it is out of range $\pm 90^\circ$ to describe the latitude that will be superfluous. Therefore, in practical usage, it is to use the
intuitive domain to be sufficient (unless there are particular needs). In fact, that the flight attitude angle passes the singular point \( \pm 90^\circ \) absolutely and accurately to achieve quadrant transform is not easy. Except that the low accuracy or artificially the assumed cross-border conditions, or quadrant is difficult to flip and will always keep their own intuitive value.

In order to simulate the attitude angle of Physical operation, the two required domain range of the mechanical angle are not same, that is the middle corner of the intuitive definition domain (such as positive Euler angles \( \phi \) and passive Euler angles \( \gamma \)) is just \( \pm 90^\circ \) enough, while other angles and three angles of the wonders domain are necessary to be the unrestricted and continuous corner.

Similarly, all other Euler angles (including mathematics and physics simulation) should be intuitive value. That is the second corner can not exceed the scope of \(-90^\circ \) to \(+90^\circ \) but the rest can be identical.

The relationship between angular speed and the plane rotation angular velocity \( \omega_x \), \( \omega_y \), \( \omega_z \) as follows in Eq. 1. Equation 1 is ordinary Euler equations. \( \phi = \pm 90^\circ \) is the singular point of the equation set; \( \phi = 00 \) or \( \phi \) is around \( \pm 1800 \). Because of the high precision, so this is the essence of the equation. If the transformation sequence is yaw, roll and pitch, the corresponding yaw Angle, roll Angle and pitching Angle, etc are anti- Euler Angles, respectively write as \( \varphi \), \( \gamma \), \( \phi \). The relationship between angular speed and the plane rotation angular velocity \( \omega_x \), \( \omega_y \), \( \omega_z \), as follows in anti-Euler Angle (2):

\[
\begin{align*}
\varphi &= (\omega_x \cos \gamma - \omega_z \sin \gamma) / \cos \theta \\
\theta &= \omega_x \sin \gamma + \omega_z \cos \gamma \\
\gamma &= \omega_x - \omega_y \sin \theta
\end{align*}
\]

\[
\begin{align*}
\varphi_i &= (\omega_i \sin \theta_i + \omega_i \cos \theta_i) / \cos \gamma_i \\
\gamma_i &= \omega_i \cos \theta_i - \omega_i \sin \theta_i \\
\theta_i &= \omega_i + \varphi_i \sin \gamma_i
\end{align*}
\]

It can be found that the reversed Euler equations also have a singularity. \( \gamma_i = \pm 90^\circ \) is the singular point of the equation set, it will make \( \phi \) and \( \varphi_i \) toward infinity and be unable to set values; \( \phi \) is 00 or \( \phi \) is around \( \pm 1800 \). Because of the high precision, so this is the essence of the equation.

Because of the ordinary and reversed sets of equations are singular, so they cannot be used alone. Combine both of them when used, however, the two sets of equations singular point can be avoided to overcome the singularity of Euler equation. This method is Euler equation.

The Euler equation proposed in the study is based on the inverse relationship of essential area between the ordinary and reversed Euler equation to make partitions replace operation according to the accurate essence area.

When calculating ordinary Euler equation, the advantages of its most accurate and simplest attitude angle in its essence area are expressed; when solving the reversed Euler equation, replace the advantage in its essence area with the disadvantage of the ordinary Euler equation's singularity to avoid the strategic point of singularity. Because it chooses both sections, puts the essence area extension and covers the whole field and the singular area is eliminated; And there is no assumptions ignored the factors of approximation in the model, no principle defect and method errors, the error correction is not needed. Therefore Dual Euler equation can eradicate the singularity method, has a complete domain, calculating accuracy and a calculating accuracy as high as that of ordinary Euler equation. The other methods can be compared and is an ideal full profile equation.

Of course, the moment the reversed Euler equation replace, state initial value needs to be supplied, after replacing, only the parameter of reversed Euler equation can be gotten. But it is not the final parameter of flight attitude angle that required. These will be calculated by coordinate transformation.
Cosine formula of vehicle body coordinate system relative to the ground coordinate system of the positive and negative direction cosine expression between two angle systems as follows:

$$A = [r_1, \theta_1, [\psi], [\varphi], [\gamma], [\psi], [r_1], [\theta_1], [\phi], [\varphi], [\gamma], [\psi], [r_1], [\theta_1]]$$  (3)

Expand the above equation can be obtained direction cosine matrix, then we can obtain:

$$\cos \gamma = \sin \theta / \sin \theta_1$$  (4)

Substitute it into Eq. 2, then the anti-Euler equations consisting of:

$$\begin{align*}
\gamma &= (\omega_1 \sin \theta_1 + \omega_2 \cos \theta_1) \sin \theta_1 / \sin \theta_1 \\
\theta &= \omega_1 + \phi, \sin \gamma
\end{align*}$$  (5)

It shows in the above formula, the singularity of the reversed Euler equation is in $\phi = \pm 90^\circ$, the essence is in $\phi = 0^\circ$ (or $180^\circ$); while the singularity of the ordinary Euler equation is in $\phi = \pm 50^\circ$, the essence is around $\phi = 0^\circ$ (or $180^\circ$). If the essence and its surrounding area are called singular area, there exists a relationship of up and down between the essence area and the singular area of the reversed Euler equation and the ordinary Euler equation.

In practical applications, $\pm 45^\circ$ or $\pm 135^\circ$ can be used as the scope to separate the essence area and the singular area.

Euler equation operates alternately based on the relationship between essence area and the singular area of the reversed Euler equation and the ordinary Euler equation. It gets rid of the singularity of the Euler equation and uses the essence area to improve the solving accuracy.

The flow chart of Euler equation is shown in Fig. 4; including two big branches, operation steps and content are similar in the meantime, the amount of calculation is also basic similar, so the calculating period will be close.

The following figures are data graphs as we use different values, for example: the pitching angle initial value is $0^\circ$, the initial roll angle value is $45^\circ$, yaw angle initial value is $45^\circ$, three axis rate initial value is $0^\circ/s$ and add a $25^\circ/s$ constant pitch rate at $1s$ (Steketee and Badger, 1985) (Fig. 5).

ISSUES ON AIRCRAFT FLIGHT TRACK CONTROL

In the flight simulation, the aircraft's flight track can have two options: One is the aircraft flies along a given line; the other trajectory spline has been confirmed in movement pretreatment. The motion trajectories can be given by using parameter spline curve. First of all, given some discrete points and time parameter as a key frame sequence, these points can be designed, may also be the true data of actual flight and then use these points to produce a interpolation spline curve as the motion trajectory and control the motion by using speed curve method. The other is to control an airplane flight interact in real time, using mouse and keyboard to control the flight of an airplane flight is real time. Control left and right turns, up and down movements, acceleration and deceleration of the aircraft and so on. To achieve the real time effect, the spline interpolation method is no longer used but goes on a distance $(v(t) \cdot \Delta t)$ each time to the speed direction and spins appropriately according to the flight path bending conditions to make it have scrolling effect when turning, till plane in turn when scrolling effect (Li, 2005).

Spline interpolation algorithms: To determine the motion of objects is to determine the object position of each time in the world coordinate system $Q_{wp}$, which is to determine the relationship between the object’s local coordinate system and world coordinate system. The object's motion model (1.1) shows that is to determine the amount of rotation matrix and translation in fact, which rotation matrix (1) determines the object's position, translation quantity $M = (x(t), y(t), z(t))$ determines the position of the object (Kochanek and Bartels, 1984).

Aircraft trajectory is generally a smooth curve, for which we used as a piecewise cubic spline interpolation trajectory (Kochanek and Bartels, 1984).

$$p_i(u) = p_i b_0(u) + p_{i+1} b_1(u) + T_{i+1} b_2(u) + T_i b_3(u)$$

$$\mu \in (0, 1), I = 0, 1, 2, ..., n$$  (6)

where, $b_0$, $b_1$, $b_2$, $b_3$ is the interpolation group, $p_i$ is for a given sequence of interpolation points, $T_i$ is tangential at the interpolation point.

$$\begin{align*}
b_0 &= 2u^3 - 3u^2 + 1 \\
b_1 &= -2u^3 + 3u^2 \\
b_2 &= u^3 - 2u^2 + u \\
b_3 &= u^3 - u^2
\end{align*}$$  (7)

Selection of tangent vectors: Using this tangential guarantee spline curve tangential continuity and increases the spline shape control flexibility:
Fig. 4: Process of dual euler method

\[ T_i = \frac{1 - t}{2} [(1 + b)(p_i - p_{i-1}) + (1 - b)(p_{i+1} - p_i)] \quad (8) \]

where, \( t \) is tensor parameters, \( b \) is the offset parameter.

To uniquely determine an interpolation at \( N+1 \) points parametric spline curve to the data points must be known assigns the corresponding parameter values \( \Delta \), namely the implementation of this set of data points parameterized. In this study, the cumulative chord length parameterization:

\[
\begin{align*}
    u_0 &= 0 \\
    u_i &= u_{i-1} + |\Delta p_{i-1}|
\end{align*}
\]

where:

\[ I = 1, 2, ..., n, \quad \Delta p_{i-1} = |p_i - p_{i-1}| \quad (9) \]

Given the parameter of \( U \in (u_i, u_{i+1}) \), the parameter domain transformation:

\[ u = \frac{U - u_i}{u_{i+1} - u_i} \]

the points on the spline can be calculated from the (1.2) formula.

Accumulated chord length parameterization does not mean that the interpolation curve of cumulative chord length as a parameter, nor is the arc length.
Fig. 5: Different gesture angles

Fig. 6: Continue
Fig. 6(a-b): (a) Flight path and (b) Trajectory parameters parameterization. In order to better simulate the object along the interpolated curve movement, when in motion control interpolation curve must be re-arc length parameterization.

ANALYSIS AND DISPLAY OF TEST DATA

This simulated aircraft took off from the longitude of 180°54 and latitude of 34°14, the initial velocity is zero, the initial attitude angle is zero, after sliding, climbing, uniform level flight, constant acceleration, turning, diving, climbing and other processes, through simulation, flight path shown in Fig. 6a shows, track parameters shown in Fig. 6b shows.

CONCLUSION

Application of virtual reality technology as one of the areas of the game field, aircraft is an outstanding representative of the field. In this study, the use of double Euler different algorithm is based on equations between positive and negative essence of Europe and singular zones alternating operation relationship upside down, out of the plight of the singularity of the Euler equation and use Digest improve solution accuracy but also the use of spline interpolation algorithm uses a piecewise cubic spline interpolation as a trajectory, when in motion control arc interpolation curve re-parameterization, confirmed by simulation algorithm thought of this article can better simulate game scene aircraft trajectory control.

REFERENCES


