Fast Correction Algorithm Research of Image Geometric Distortion in the Image Tracking

Tan Lian, Dang Pei, Luo Qiang and Mutaseem Alsami
1North China Institute of Water Conservancy and Hydroelectric Power, Zhengzhou, 450011, China
2Henan University of Technology, Zhengzhou, 450001, China
3Xi'an Institute of Modern Control Technology, Xi'an, 710001, China
4Department of MIS, College of Applied Studies and Community Service, Danunam University, Danunam, Kingdom of Saudi Arabia

Abstract: In high-performance image tracking system, the image geometric distortion is an important factor restricting accuracy of the algorithm. In order to ensure accuracy and to minimize the computation time, this study proposes a method through research. Radial distortion is the main factor of the image distortion. By the coordinate transformation, this paper got the camera distortion model. This study briefly introduces the image of the principles of geometric distortions. And in the basis of analyzing polynomial algorithm on the coordinate conversion, it approaches a distortion algorithm non-uniform by region. The fixed focus image is established to the corresponding model and non-uniform divided to rectangular areas which are within a polynomial in a high order polynomial. By comparing the correction effect and devotion of time of the correction method between one order region and third order polynomial, the validity of the proposed method by the article can be verified.

Key words: Image distortion, coordinate transformation, distortion model, piecewise algorithm, correction

INTRODUCTION

Image of the geometric distortions acts a notable impact on the quality of image tracking. Algorithm is related to the image on a distortion, the model is greater, and the influence is more serious (Ren et al., 2010). Additionally, background equal algorithm is also influenced by the image geometric distortion. So, it’s important to eliminate distortion in the high quality image tracking system.

Among many factors of distorted images, radial distorted is a major factor of non-linear distortion (Ai et al., 2008; Liao et al., 2000). For this, they have done a lot of research, such as using the polynomial coordinate changing to describe the address mapping between the ideal and distortion, or setting out factors of geometric distortion (Lee et al., 2010). Using the method of polynomial coordinates change to realize the address mapped has been more widely used in the works. However, in order to describe the radial distortion of the from the principle of camera imaged to discover the nonlinear distortion, you must use the third or more of the dual polynomial. So, it’s time consuming and hard to achieve in the real-time image processing system (Shah and Aggarwal, 1996).

On the basis of analyzing polynomial algorithm of coordinate conversion, the article proposes a polynomial algorithm with non-uniform by region. It means that first the image should be divided non-uniformly to rectangle areas and in every area with a polynomial in a high order. Based on the analysis of the image distortion factors namely the radial distortion, this algorithms rules can guarantee the premise of the accuracy to save space of the least conservation model. The algorithm can significantly reduce the operating time. Simultaneously, it can guarantee the accuracy and the space cost can be in a very small scope (Nomura et al., 1992).

IMAGE DISTORTION MODEL

Concept of distortion: For a variety of distortion factors, distinctions would be made between ideal value and a two-dimensional image pixel made by an object point in three-dimensional space of position deviation projected (Asari et al., 1999). Figure 1 shows us the relationship between real image point and ideal image, and the mathematical description can be represented by the following Eq:

\[
\begin{align*}
X &= x + d_x, \\
y &= y + d_y
\end{align*}
\] (1)

Corresponding Author: Mutaseem Alsami, Department of MIS, College of Applied Studies and Community Service, Danunam University, Danunam, Kingdom of Saudi Arabia
Fig. 1: Relationship between real image point and ideal image

Fig. 2: Radial distortion

Fig. 3: Pinhole imaging model

In this Eq., is the actual image coordinates; is the ideal image coordinates; is the nonlinear distortion value. Distortion can be divided into three models: Radial distortion, decentering distortion and thin prism distortion (Hartley and Zisserman, 2000). For rapid image correction and for rapid image correction, we only consider the impact of radial distortion.

Fig. 4: Camera coordinate system and World coordinate system

Figure 2 shows us the two kings of radial distortion. If we takes the distortion center is the origin of coordinates, the mathematical model of radial distortion can be described as:

$$\begin{bmatrix}
\delta_x
\delta_y
\end{bmatrix} = (x, y, r^2 + k_2r^4 + k_3r^6 + L)$$

In this Eq., is the distance of the image dot to the center, is the coordinates of the distortion center, and is the coefficients of radial distortion.

In this article, we assume that perspective is the pinhole imaging.

Projection model: From the pinhole imaging model (Fig. 3), we can see that the following relationship exists between the camera coordinate system and the imaging plane coordinate system.

$$\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
f & 0 & 0 & X_c \\
f & 0 & 0 & Y_c \\
0 & 1 & 0 & Z_c
\end{bmatrix} \begin{bmatrix}
\mu x \\
\mu y \\
\mu z
\end{bmatrix}$$

(, ) is the pixel coordinates in the image plane coordinate system, is the coordinates in the camera coordinate system. is a constant factor.

Firstly, we make the world coordinate system into the camera coordinate system. As shown in Fig. 4.

Then, do the normalization processing, we can get the projection model:

$$\begin{bmatrix}
x_2 \\
y_2 \\
z_2
\end{bmatrix} = \begin{bmatrix}
s_x & \mu & x_c \\
s_y & 0 & y_c \\
0 & 1 & 1
\end{bmatrix} \begin{bmatrix}
x \\
y \\
z
\end{bmatrix}$$
\((x_c, y_c)\) is the coordinates of the distortion center; \(s_x, s_y\) means the number of pixel on the unit distance in x or y direction; \(\mu\) is called twisting coefficient. It used to describe the distortion caused by optical axis is not perpendicular to the image plane or discretization pixels isn’t rectangular. To simplify the calculation, the value of \(\mu\) is 0 in this study.

**PRINCIPLE OF CALIBRATION DISTORTION**

The essence of calibration image is the change of the space relationship between pixels. So, it should get the original space relationship from geometric transformation.

According to the ideal needle model, the object can be imaged a perfect figure, marked \(f(x, y)\). Because of not all the same between the Actual optical lens and ideal needle model, the distorted image is called aberrations, marked \(g(x, y)\). Assuming that the coordinate of the point of the ideal figure \((x_i, y_i)\) in the distorted image is marked \((x_b, y_b)\), the mapping relationship between \((x, y)\) and \((x_b, y_b)\) is:

\[
\begin{align*}
    x_b &= g(x, y) \\
    y_b &= t(x, y)
\end{align*}
\]  

(5)

In order to get the pixel (gray) value of un-integer, you need to calculate round some pixel integer values on the coordinate, that’s interpolation. The gray interpolation has many types, such as zero order, linear, spline and so on. This article takes method of double linear interpolation to ensure the accuracy and reduce the complicated requirement and it uses four most close value of point \((x_b, y_b)\) to calculate the gray of point \((x_b, y_b)\), to see Fig. 5, the four values nearby are A, B, C, D and each coordinate is \((i, j), (i+1, j), (i, j+1), (i+1, j+1)\), each gray value is \(g(A), g(B), g(C), g(D)\).

First calculating the gray value in points E and F:

\[
\begin{align*}
    g(E) &= (x_b - i)[g(D) - g(C)] + g(C) \\
    g(F) &= (x_b - i)[g(B) - g(A)] + g(A)
\end{align*}
\]  

(6)

Then, the gray value of \((x_b, y_b)\) is:

\[
\begin{align*}
    g(x_b, y_b) &= (y_b - j)[g(A) - g(C)] + (x_b - i)[g(B) - g(D)] \\
    &+ (y_b - j)[g(A) - g(C)] + (x_b - i)[g(B) - g(D)] + g(C)
\end{align*}
\]

So, using the Eq. 5 to get the mapping value \((x, y)\) in the ideal image, then we can get the corresponding place \((x_b, y_b)\) in the distortion image. Owing to \(f(x, y) = g(x_b, y_b)\), so using Eq. 6 can get the gray value. If the steps are taken in every mapping of the ideal image, it means recovery the space between the mappings and then the distortions can be corrected naturally.

**POLYNOMIAL COORDINATE CHANGE**

It's the center of geometric distorted correction that Eq 1 describes the change relationship between the image's mapping spaces. However, the reasons caused by distortions are complicated and they're usually acted by many factors. So, it's unrealistic to find the analytic of \(s(x, y), t(x, y)\). In many cases, we can use polynomial to approach the two functions. Assuming the powers of the polynomials is \(k\), so:

\[
\begin{align*}
    x_b &= s(x, y) = \sum_{i=0}^{k} \sum_{j=0}^{k} u_i x^i y^j \\
    y_b &= t(x, y) = \sum_{i=0}^{k} \sum_{j=0}^{k} v_i x^i y^j
\end{align*}
\]  

(7)

The more \(k\) is greater, the more the accuracy of \(s(x, y), t(x, y)\) is better. As to translation, rotation, it can be described by \(k = 1\). But as to nonlinear distortion of radial, \(k\) should be 3 or greater (Nomura et al., 1992).

The coefficient in Eq. 7 is identified by the situation of image distortion. In order to study distortion of image facilities, we can draw a uniform arrow diagram as the ideal figure. And then we can take a photograph of the figure, and that can get the distortion image (Fig. 6). We call this is the aim figure.

![Fig. 6(a-b): Aim figure, (a) Ideal figure (b) Distortion image](image-url)
Firstly getting N points in the ideal figure, define them as \((x_o, y_o), (x_1, y_1), \ldots, (x_N, y_N)\) and then find their coordinate points in the aim figure, define them as \((x_{i_0}, y_{i_0}), (x_{i_1}, y_{i_1}), \ldots, (x_{iN}, y_{iN})\). The N points are named constrained points. They consist of the information of image distortion, and by this can verify the multinomial coefficient of Eq. 4, you can get the change relationship of mapping space place.

For convenience, we defined:

\[
\begin{align*}
U &= [u_{i_0}, u_{i_1}, \ldots, u_{iN}] \\
V &= [v_{i_0}, v_{i_1}, \ldots, v_{iN}] \\
X &= [x_{i_0}, x_{i_1}, \ldots, x_{iN}] \\
Y &= [y_{i_0}, y_{i_1}, \ldots, y_{iN}] \\
H &= \begin{bmatrix}
1 & x_0 & x_0^2 & \cdots & x_0^k & y_0 & x_0^1y_0 & x_0^2y_0 & \cdots & x_0^ky_0 \\
1 & x_1 & x_1^2 & \cdots & x_1^k & y_1 & x_1^1y_1 & x_1^2y_1 & \cdots & x_1^ky_1 \\
1 & x_2 & x_2^2 & \cdots & x_2^k & y_2 & x_2^1y_2 & x_2^2y_2 & \cdots & x_2^ky_2 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
1 & x_N & x_N^2 & \cdots & x_N^k & y_N & x_N^1y_N & x_N^2y_N & \cdots & x_N^ky_N
\end{bmatrix}
\end{align*}
\]

(8)

In Eq. 8, k is the powers of polynomial. Making:

\[
M = \frac{(k+1)(k+2)}{2}
\]

M is the number of polynomial such as:

\[
a(x,y) = \sum_{i=0}^{k} \sum_{j=0}^{k} a_{ij}x^iy^j
\]

so, \(U, V\) are dimensional vector, \(H\) is the matrix of \(M \times N\), Eq. 4 can be rewrites as:

\[
\begin{align*}
X &= HU \\
Y &= HV
\end{align*}
\]

(9)

In Eq. 9, \(X, Y, H\) are constant matrix, \(U, V\) are unknown variable. Because \(N\) (the numbers of tie points) are far more than \(M\), Eq. 9 has not accuracy value. But can get \(\hat{U}, \hat{V}\) the least squares estimation value of \(U, V\) [12]:

\[
\begin{align*}
\hat{U} &= \hat{H}^H H^* X \\
\hat{V} &= \hat{H}^H H^* Y
\end{align*}
\]

(10)

In this study, using \(\hat{U}, \hat{V}\) instead of \(U, V\), taking into Eq. 4, we can get the change relationship said by polynomial image mapping to make geometric distorted correction possible.

**A POLYNOMIAL ALGORITHM BY REGION**

The simulation experiment and engineering excellence have indicated that it's possible to use polynomial to describe the coordinate conversion between ideal figure and distortion figure. Even if to the complicated nonlinear distortion, you can get a good correction effect if the powers of polynomial you choose are high enough. But according to Eq. 9, with \(k\) greater, its coefficient \(M\) will get greater fast. So, it leads to the big complicated algorithm. Such as \(k = 1\), Eq. 4 is:

\[
\begin{align*}
x_d &= x_o + u_{00}y + u_{10}x \\
y_d &= y_o + v_{00}y + v_{10}x
\end{align*}
\]

(11)

Now, every pixel's coordinate transform need only four times of multiplications and additions. Simultaneity, if \(k = 3\), Eq. 4 equals:

\[
\begin{align*}
x_d &= x_o + u_{00}y + u_{01}y^2 + u_{02}y^3 + u_{10}x + u_{11}xy + u_{12}x^2 + u_{20}x^3 \\
y_d &= y_o + v_{00}y + v_{01}y^2 + v_{02}y^3 + v_{10}x + v_{11}xy + v_{12}x^2 + v_{20}x^3 + v_{30}x^4
\end{align*}
\]

(12)

Completing this pixel's coordinate transform need 32 times of 32 multiplications and 18 times of additions. And the complicated algorithm is more compared with Eq. 11. It's hard to be applied in the real tracking image system.

**Idea of piecewise algorithm:** In order to get high-accuracy of high-order moment and reduce the math operation, this studyproposes a new algorithm which is using the method told above to distinguish satisfied correction effect polynomial:

\[
\begin{align*}
x_p(x,y) &= \sum_{i=0}^{k} \sum_{j=0}^{k} a_{ij}x^iy^j \\
y_p(x,y) &= \sum_{i=0}^{k} \sum_{j=0}^{k} b_{ij}x^iy^j
\end{align*}
\]

(13)

Then, divide the image to several pieces of rectangle areas. And in every small area, the lever of nonlinear-distortion is all controlled in the acceptable range. As shown in Fig. 7.

Fig. 7: Small rectangle area
Because of area is very small, the pixel's coordinate transform can be seen as a linear transformation. Then, we can use a polynomial to describe the coordinate change. In order to identify the coefficient of polynomials, we need investigate four points 1, 2, 3, 4. Assuming their coordinates is \((x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)\). Because we have distinguished the high-order moment such as Eq 13, the corresponding place of four points in the distortion can be calculated precisely, each coordinate is \((x_{1h}, y_{1h}), (x_{2h}, y_{2h}), (x_{3h}, y_{3h}), (x_{4h}, y_{4h})\). Then, the four tie points will be got. And then bringing the four coordinates of the points into Fig. 10, we can distinguish the coefficient of polynomial. Therefore using this method, we can use a first-degree polynomial instead of a high-order moment; it can reduce the math operation markedly.

**Dividing of piecewise area:** According to Eq 11, each polynomial has 6 parameters. If every parameter needs 32 bits, it needs the memory of 24 bytes. So, if the image is divided into so many segmentations, the number of area must be more and the cost of memory for saving model parameters must be considerable. In the other hand, if divided too few, we can't get the satisfied effect. So, it's important to how to divide the image.

From Fig. 6, the parts of fig are non-uniform. So, as to radial distortion, axis distortions are getting little but the round distortion are getting bigger. To get satisfied th acquirement of both accuracy and the space cost, a natural idea is to divide the image with non-uniform. The sub-grids of nonlinear distortion seriously are small and frivolous are larger.

The radial distortion is the main factor of nonlinear distortion (Vasileios). In order to divide the area reasonably, firstly you need study the radial distortion of camera imaging. If the radial distortion only takes place, ignoring the high lever and get following:

\[
\begin{align*}
x &= x_0 (1 + k_1 r^2 + k_2 r^4 + \cdots)
\end{align*}
\]

A \((x, y)\) defined the coordinate gain of \((x, y)\), \(k_1, k_2\) mean the distortion factors of the image in horizontal and vertical. Because of radial distortion is symmetries to original point. We only need to consider the area divide of first quadrant and the others can be completed by principle of symmetry.

Assuming that, divide the first quadrant into \(m\) grids on vertical and \(n\) grids on horizontal. So, the problem come down to how to identifying the series \([x_0, x_1, \ldots, x_m]\) and \([y_0, y_1, \ldots, y_n]\) where, \(0 < x_0 < \ldots, x_m = w, 0 < y_0 < \ldots, y_n = h\). Where \(w, h\) is half of the breadth and the height of the image. Dividing by this, the distortion degree of kinds of rectangle area has the same size. And it ensures the least grids in the premise of accuracy. Figure 8 is the schematic diagram of two series.

This is a conclusion: making that:

\[
x_i = w \frac{\sqrt{i}}{m}, y_j = h \frac{\sqrt{j}}{n}
\]

Which, \(0 \leq i \leq m, 0 \leq j \leq n\), dividing the area from Fig. 4 and each coordinate of the area keep increasing in a certain range. The difference between the greater coordinate increasing and the smallest is \(k_1 w^2 / m^2 + k_2 h^2 / n^2\) whose evidence is as follow.

Let \(D_0\) represents the rectangle area of \((x_0, y_0)\) on the top right corner acted by \((x_n, y_n)\). The greatest and smallest coordinates gain is:

\[
\begin{align*}
A_{max} &= A(x_0, y_0) = 1 + k_1 x_0^2 + k_2 y_0^2 \\
A_{min} &= A(x_n, y_n) = 1 + k_1 x_n^2 + k_2 y_n^2
\end{align*}
\]

The greatest difference of the coordinate gain is:

\[
\begin{align*}
dA &= A_{max} - A_{min} \\
    &= k_1 (x_0^2 - x_n^2) + k_2 (y_0^2 - y_n^2) \\
    &= k_1 w^2 (\frac{\sqrt{1+i}}{m} + \frac{\sqrt{1+j}}{m})^2 + k_2 h^2 (\frac{\sqrt{1+i}}{n} + \frac{\sqrt{1+j}}{n})^2
\end{align*}
\]

Fig. 8: Dividing of first quadrant image
It evidence that no matter what values $i, j$ are, $d_{ij}$ are all constants. It indicates the distortion degree in each area which divided by Eq. 16 is equal.

We need find the solution of $M$ and $N$ by figure as follows:

$$C (\alpha, \beta) = M \alpha \beta, \alpha > 0, \beta > 0$$

(19)

The constraint condition is follows:

$$E = \frac{1}{\alpha} {\frac{w^2}{\alpha}} + \frac{1}{\beta} {\frac{h^2}{\beta}}$$

(20)

$C (\alpha, \beta)$ is the cost function and it means that the space cost needed to be saved when dividing to $\alpha$ grid on the horizontal and $\beta$ grid on the vertical. $M$ is the memory scale of saving one grid of constraints. $E$ is the limit of area coordinate increasing. It's specified by the detail correction quality. If $E$ is small, the accuracy is high and otherwise is opposite. You need construct a function to get the smallest point:

$$H(\alpha, \beta, \lambda) = M \alpha \beta + k \left( \frac{w^2}{\alpha} + \frac{h^2}{\beta} - E \right)$$

(21)

Through Lagrange multiplier method, get:

$$\frac{\partial}{\partial \alpha} H(\alpha, \beta, \lambda) = M \beta - \frac{k w^2}{\alpha^2} = 0$$

(22)

$$\frac{\partial}{\partial \beta} H(\alpha, \beta, \lambda) = M \alpha - \frac{k h^2}{\beta^2} = 0$$

(23)

$$\frac{\partial}{\partial \lambda} H(\alpha, \beta, \lambda) = k \left( \frac{w^2}{\alpha} + \frac{h^2}{\beta} - E \right) = 0$$

(24)

Find the solution of Eq. 22-24, get:

$$\alpha = \frac{2k w^2}{E}, \beta = \frac{2k h^2}{E}$$

(25)

Owing to the definition of $C (\alpha, \beta)$, according to the formula $m, n$, the grid number is smallest in the premise of error.

$$M = \langle \alpha \rangle, n = \langle \beta \rangle$$

(26)

$\langle \eta \rangle$ means it's value is smallest integer which is equal or greater than $\eta$. When calculate the grid number, Eq. 25 has two parameters $k_1, k_2$ need to be identified. For getting the solution, need construct a matrix as follows:

$$P = \begin{bmatrix} x_1^2 & y_1^2 \\ x_2^2 & y_2^2 \\ \vdots & \vdots \\ x_n^2 & y_n^2 \end{bmatrix}$$

(27)

$$Q = \begin{bmatrix} 1 & \frac{x_1}{x_0} & \frac{x_2}{x_0} & \ldots & \frac{x_n}{x_0} & 1 \\ 1 & \frac{y_1}{y_0} & \frac{y_2}{y_0} & \ldots & \frac{y_n}{y_0} & 1 \end{bmatrix}$$

$$K = [k_1, k_2]^T$$

$$(x_0, y_0), \ldots, (x_n, y_n)$$ and $$(x_0, y_0), \ldots, (x_m, y_m)$$ are the model of distinguish of high time polynomial which is to select the data of constrained point coordinate in group N. $P$ is a constant matrix in dimension $N \times 2$. $Q$ is constant vector in dimension $N$. So, according to Eq. 15, it's:

$$Q = PK$$

(28)

Owing to the analysis of LS, you can get the estimated parameter vector $K$:

$$K = (P^T P)^{-1} P^T Q$$

(29)

Let $K$ instead of $K$ taken into function (25), combining with (26) and calculate the allow error is number $m$ and $n$ which is in the conditions of $E$. Then, taken into (16), you can get the reasonable figure area divide.

**SIMULATION ANALYSIS**

In order to compare the coordinate change of high time polynomial and one time polynomial slide algorithm's correction result and the each operation time, Correcting to the distortion grid of Fig. 6. The scale is $384 \times 288$, that $w = 192, h = 144$.

Figure 9 is the area dividing image. Compared with Fig. 10b, two algorithms all get the clear correction.
Fig. 10(a-b): Result of correction, (a) Result of correction by cubic polynomial, (b) Result of correction by first-degree polynomial in piecewise

Table 1: Comparison of the results of the two methods

<table>
<thead>
<tr>
<th>a</th>
<th>Coordinates</th>
<th>b</th>
<th>Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>a_1</td>
<td>-182.138</td>
<td>b_1</td>
<td>-182.138</td>
</tr>
<tr>
<td>a_2</td>
<td>-127.69</td>
<td>b_2</td>
<td>-127.70</td>
</tr>
<tr>
<td>a_3</td>
<td>0.139</td>
<td>b_3</td>
<td>0.139</td>
</tr>
<tr>
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<td>b_4</td>
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<tr>
<td>a_5</td>
<td>98.86</td>
<td>b_5</td>
<td>98.86</td>
</tr>
<tr>
<td>a_6</td>
<td>180.43</td>
<td>b_6</td>
<td>181.43</td>
</tr>
</tbody>
</table>

Fig. 11(a-b): Result of correction, (a) Distorted image in Intelligent tracking system and (b) Result of correction by the new method

In order to test the accuracy of one time grid model, we need get 6 grids of points of intersection, corresponding with a_1-a_6 in (a), b_1-b_6 in (b), next is the list of their coordinates. As shown in Table 1.

By looking from the average time of 5 degree operation, using the method of first-degree polynomial in piecewise cost less time Table 2.

Table 2: Comparison of the run time of the two methods

<table>
<thead>
<tr>
<th>Cubic polynomial (ms)</th>
<th>First-degree polynomial in piecewise (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>282</td>
<td>101</td>
</tr>
<tr>
<td>308</td>
<td>91</td>
</tr>
<tr>
<td>285</td>
<td>96</td>
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<tr>
<td>286</td>
<td>108</td>
</tr>
<tr>
<td>285</td>
<td>87</td>
</tr>
</tbody>
</table>

In order to verify the practicability of the algorithm, the author designed the intelligent vehicle tracking system controlled by double DSP. Fig. 11 shows the result of correction by the system.

From the figures, the first-degree polynomial in piecewise has achieved good results. This method has good correction effect and little algorithm. It's a good method of the fast distortion correction.
CONCLUSIONS

The principle of correction by first-degree polynomial in piecewise has been analyzed. This study introduces the image of the principles of geometric distortions. And in the basis of analyzing polynomial algorithm on the coordinate change, it approaches a polemical algorithm non-uniform by region. The fixed focus image is established to the corresponding model and non-uniform divided to rectangular areas which are within a polynomial in a high order polynomial. By comparing the correction effect and deviation of time of the correction method between one order region and third order polynomial, the validity of the proposed method by the article can be verified.

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