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Nurses Staffing and Allocation in Multi-stage Queuing Network with I^2 Patients' Routing for Outpatient Department

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Abstract: A general multi-stage queuing network model with I^2 patients' routing including two tandem queues is established to formulate the behavior of patients flow in Outpatient Department (OD) in a hospital starting from registration, diagnosis, chemical examination, referral, payment and medicine-taking. Focusing on the nurse resources, the formula of performance indicators such as patient' waiting time, probability of nurse' idle are derived by using the system parameters. A mathematical programming model is developed to determine how many nurses are needed and how to allocate to each stage/division to minimize the total costs of patients' waiting time and the nurses' idle time. How to allocate the nurse to each stage is essentially a natural number decomposition problem and thus a neighborhood search combined Simulated Annealing (NS-SA) with Heuristic is developed. Optimal nurse numbers are derived from the enumeration method based on NS-SA. Numerical experiments are conducted to analyze the impact of patients' arrival to the allocation of nurses and the ratio of patients' waiting time and nurses' idle time on the number of nurses needed. The research results can facilitate hospital managers to make decisions on human resources in practice.

Key words: Patients' routing, queuing network, nurse allocation, SA

INTRODUCTION

Over the years, hospital managers continually confront many challenges, such as providing better service quality, with less nurse staff and more patients. Especially, those conflicts are more severe in the Outpatient Departments (OD). For example, there are 171, 024 outpatients in a general hospital in Dalian and 2, 264, 733 outpatients in a famous union hospital in Beijing, China in 2010. How to provide better service level with limited resources and increasing outpatients is a key problem in the hospital so that more and more researchers and practitioners begin to address it in recent years. (Creemers and Lambrecht, 2011).

Health care service systems are essentially queuing systems. Queuing theory as an efficient tool has been used frequently to address the behavior of health-care service systems since Jackson network model was proposed in 1957. Nurse staff allocation in multi-stage queuing system with patients' feedback flow for OD is mainly concerned in this paper. Around of the queuing system in hospital Jlassi *et al.* (2010) made a review on multiple customer types in Emergency Department (ED). According to the performance indicators such as patient

waiting times and the probability of nurse idle times in the multi-severs queuing systems (Abadi *et al.*, 2000) and Balsamo made some analysis. A three tandem queuing system was founded by El-Darzi *et al.* (1998) in a hospital geriatric department. Koizumi *et al.* (2005) formulated a model of patient flows using a queuing network with blocking with finite waiting space and the feedback patients flow. Osorio and Bierlaire (2009) presented an analytic queuing network model which preserves the finite capacity of queues and used structural parameters to obtain the correlation between two stages. The variation in patient flows is described by Brethauer *et al.* (2011) with the help of the work. However, he paid no attention to the structure of the queuing networks and the patients flow variability. Some insights of patient flows just as the feedback patients flow in hospital are derived by Hall *et al.* (2006), Helm *et al.* (2011) and Price *et al.* (2011). Focusing on the staffing allocation (Ruger *et al.*, 2007) identified high-risk patients for triage and resource allocation in ED. Ahmed and Alkhamis (2009) made a simulation optimization model for an emergency department healthcare unit in Kuwait. Izady and Worthington (2012) set nurses staffing requirements for time dependent queuing network in ED with the square

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root method. Bretthauer *et al.* (2011) developed a new heuristic for finite resource allocation in healthcare operations. But the feedback patients flow in queuing system isn't considered in the above papers. The research on the analysis of the structure in OD with feedback patients and patients flow by routing probability is rarely known (Jackson, 1957, 1963; Au-Yeung *et al.*, 2006).

The data are collected for fitting the model parameters from Dalian Xinhua hospital in Liaoning province, China. A general multi-stage queuing network model with I² patients' routing including two tandem queues is established to formulate the behavior of patients flow in Outpatient Department (OD) in a hospital starting from registration, diagnosis, chemical examination, referral, payment and medicine-taking. Focusing on the nurse resources, the formula of performance indicators such as patient' waiting time, probability of nurse' idle are derived by using the system parameters. A mathematical programming model is developed to determine how many nurses are needed and how to allocate to each stage/division to minimize the total costs of patients' waiting time and the nurses' idle time. How to allocate the nurse to each stage is essentially a natural number decomposition problem and thus a neighborhood search combined Simulated Annealing (NS-SA) with Heuristic is developed. Optimal nurse numbers are derived from the enumeration method based on NS-SA. Numerical experiments are conducted to analyze the impact of patients' arrival to the allocation of nurses and the ratio of patients' waiting time and nurses' idle time on the number of nurses needed. The bottleneck of the system is found. We assume that the nurses are in the broadest sense of the healthcare staff (nurse\medical equipment operator\cashier etc.).

This study is structured as follows: section 2 presents the description of the queuing system model with I² patients' routing in OD. Medical treatment processes flowchart, mathematical formulations by analyzing steady states and the mathematical method to obtain performance indicators are introduced in this section. In section 3 a mathematical programming model is developed to determine how many nurses are allocated to each stage/division to minimize the total costs of patients waiting time and the nurse idle time. A neighborhood search combined Simulated Annealing (NS-SA) algorithm is developed. Section 4 presents numeric analysis of experiment results and section 5 concludes the paper and provides the research direction in future.

QUEUEING SYSTEM IN OD WITH I² PATIENTS' ROUTING

When a patient enters into an outpatient department, he should go to the registration station first and then see doctor. If there is no question in the result after the doctor's diagnosis, he will take the medicine after paying. Otherwise, he needs to make a serial examination process, e.g., CT, blood testing and returns to see doctor after having testing results which defined as referral. The treatment flow chart in OD from patient registration to leaving is described as shown in Fig. 1.

Without loss of generality, a general OD treatment process flowchart is simplified in Fig. 1:

- The patients' external arrival rate is a Poisson distribution with parameter λ . All patients will be served sequentially with the rule FCFS (first come

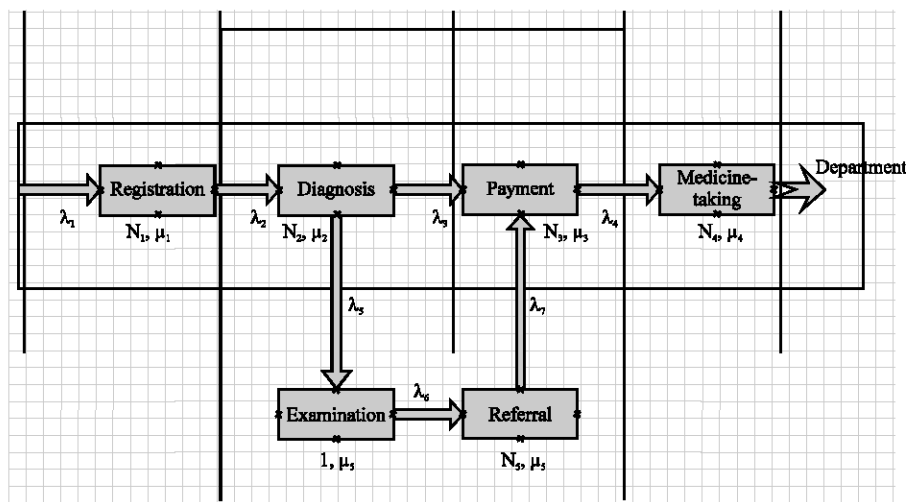


Fig. 1: A multi-stage queuing network with patients' routing structure in OD

first service) before leaving the system. It is assumed there is an infinite buffer between any two stations and the system is steady-stat.

- The network is a multi-stage queuing system with feedback patients flow where the four stages are M/M/n queuing model with capacity of stage is N_i servers, respectively. The patient arrival rate is a Poisson distribution with parameter λ_i before stage i . The service rate in stage i is independent and an exponential distribution with parameter u_i . The service time in examination stage is a general distribution with parameter u_5 . The input patients flow is λ_5 and the output patients' flow is λ_6
- After seeing a doctor, the patient may have further examination with probability p_1 , enter the 3rd stage with probability p_2 or quit the systems with probability p_3 directly. The patients who enter the examination and see doctor again quit the system with probability p_4 . Then $p_1+p_2+p_3 = 1$
- We note that the patients' total waiting times is W . The expected queue length is and the patients' waiting time is L_i, W_i at stage i , respectively. The mean service intensity at stage i is ρ_i . The waiting time before examination is W_D and the service intensity is ρ . The probability that the system completely lose is π_{i0} and the probability that there are k servers are occupied is π_{ik} at stage i

In the system, the input patient flow of the examination flow is obtained from the routing probability and the external arrival rate λ , then $\lambda_5 = p_1\lambda$. In the equation:

$$\rho = \frac{p_1\lambda}{u_5} < 1$$

is known as the service intensity at examination station. In M/G/1 queuing system, the busy period is:

$$p_c = \rho = \frac{\lambda_5}{u_5} = \frac{p_1\lambda}{u_5}$$

the idle period is $1-\rho$. In other words, the output patients' flow is u_5 when the system is busy and the output patients' flow is u_5 when the system is idle. The equation of output patient can be obtained:

$$\lambda_5 = \frac{p_1\lambda(2-\rho_D)}{D} \tag{1}$$

The patients flow before the referral station:

$$\lambda_5 = \lambda_6 \tag{2}$$

The patients flow before the payment station:

$$\lambda_3 = \frac{p_2\lambda + \lambda_6 = p_2\lambda + p_1\lambda(2-\rho_D)}{D} \tag{3}$$

The patients flow before the medicine-taking station:

$$\lambda_4 = \lambda_3 \tag{4}$$

The expected queue length at stage i is obtained by Eq. 1-4 in steady-state:

$$L(N_i) = \left[\sum_{n_i=1}^{N_i-1} \frac{\omega_i^{n_i}}{n_i!} + \frac{\omega_i^{N_i}}{(1-\rho_i)N_i!} \right]^{-1} \frac{\rho_i \omega_i^{N_i}}{(1-\rho_i)^2 N_i!} \tag{5}$$

Where:

$$\omega_i = \frac{\lambda_i}{u_i \rho_i} = \frac{\omega_i}{N_i} < 1 \tag{6}$$

Equation 6 is essentially the stability condition for Eq. 5 hold in queuing theory. The expected waiting time to enter stage i at steady-state is obtained by equation:

$$W(N_i) = L_i / \lambda_i = \left[\sum_{n_i=1}^{N_i-1} \frac{\omega_i^{n_i}}{n_i!} + \frac{\omega_i^{N_i}}{(1-\rho_i)N_i!} \right]^{-1} \frac{\rho_i \omega_i^{N_i}}{(1-\rho_i)^2 \lambda_i N_i!} \tag{7}$$

The expected patients waiting time before examination station:

$$W_D = \frac{\rho}{2(1-\rho)u_5} \tag{8}$$

The patients total waiting time:

$$W = \sum W(N_i)^T + p_1 W_D \tag{9}$$

The probability that the system completely lose at stage i :

$$\pi_{i0} = \left[\sum_{k=0}^{N_i-1} \frac{(N_i \omega_i)^k}{k!} + \frac{(N_i \omega_i)^{N_i}}{N_i!(1-\omega_i)} \right]^{-1} \tag{10}$$

The probability that there are k servers are occupied at stage i :

$$\pi_{ik} = \begin{cases} \frac{(N_i \omega_i)^k \pi_{i0}}{k!}, & k = 1, 2, \dots, N_i - 1 \\ \frac{N_i^k \omega_i^k \pi_{i0}}{N_i!}, & k = N_i, N_i + 1, \dots \end{cases} \tag{11}$$

Then the expression of the main performance indicators in OD has been obtained.

THE NURSES ALLOCATION MODEL

The cost of OD is usually described by the patients waiting time cost and health care staff idle time cost [9]. A mathematical programming model is developed to determine how many nurses are allocated to each stage/division to minimize the total costs of patients' waiting time and the nurse idle time. We assumed the number of nurses N in OD is fixed and the nurses are multi-skilled. Jobs in different stations can be arbitrarily assigned. In other words, all nurses can be allocated to all kinds of job. We note the cost of OD is C , the ratio between nurse's idle time cost and patient's waiting time cost is α . Unit cost expressed in minutes. We also assume that the cycle is T . By Eq. 9-11, a Cost Model Based on Nurses Allocation in Outpatient Departments (CMNAOD) is described:

$$\begin{aligned} & \{T(\sum_{i=1}^5 \lambda_i \alpha W(N_i) + \lambda \alpha p_i W_D) \\ & + \sum_{i=1,3,4} \sum_{k=1}^{N_i-1} (N_i - k) u_i \frac{(N_i \rho_i)^k \pi_{i0}}{k!} + \sum_{i=1,3,4} N_i u_i \pi_{i0} \\ & + \sum_{i=2,5} \beta \sum_{k=1}^{N_i-1} (N_i - k) u_i \frac{(N_i \rho_i)^k \pi_{i0}}{k!} + \sum_{i=2,5} \beta N_i u_i \pi_{i0}\} \\ & \min \text{ s.t. } \sum_{i=1}^5 N_i = N \end{aligned} \tag{12}$$

$$N_i \geq 1, N_i \text{ is integer} \tag{13}$$

$$\omega_i = \frac{\lambda_i}{u_i}, \rho_i = \frac{\omega_i}{N_i} < 1 \tag{14}$$

$$\alpha < 1 \tag{15}$$

The number of nurses is fixed by constraint Eq. 12. Equation 14 is essentially the stability condition in the queuing system. According to the relevant literatures, the nurse's idle time cost is generally bigger than the patient's waiting time cost in Eq. 15.

A neighborhood search combined simulated annealing (NS-SA) algorithm: The optimal solution of CMNAOD is determined by nurse staff allocation which is essentially a natural number decomposition problem with constraints. With the increasing of N , the number of feasible solution becomes very larger. For example, when the $N = 10$, the number of nurses allocation is 84 and $N = 100$, the number of nurses allocation is 156 849. With the increasing of

N , how to find the optimal solution by enumeration becomes very difficult. Thus a neighborhood search combined Simulated Annealing (NS-SA) algorithm is developed.

NS-SA as a meta-heuristic with stochastic neighborhood search has successfully solved many large-scale combinatorial optimization problems [20-22]. CMNAOD model is a nonlinear integer programming model which is essentially a combinatorial optimization problem based on the decomposition of natural numbers. According to the problem character, NS-SA is qualified to solve the problem very well.

To accomplish NS-SA, several core technical issues should be solved such as the initial solution generation, the definition of neighborhood and the cooling schedule. We introduce these critical operations in our NS-SA algorithm in details as followings.

Initial solution generators: The integer coding method is adopted in the algorithm. An example solution can be coded as $X = [N_1, N_2, N_3, N_4, N_5]$. Each variable N_i ($i = 1, 2, 3, 4, 5$) indicates the number of nurses assigned to system node i .

We develop a heuristic approach to generate a good initial solution. The basic idea is described as followings:

- Calculate the minimum number of nurses $\min(N_i)$ assigned to each system node according to constraints Eq. 12-14. Let N_1 is the maximum number of nurses assigned to node 1 by calculating:

$$\max(N_1) = N - \sum_{i=2}^5 \min(N_i)$$

the rest nodes are assigned with the minimum number of nurses. Therefore, a solution can be obtained as X_0 . Let the number of iterations to be $j = 1$

- Let $N_1 = N_1 - 1$ and calculate the corresponding objective value by adding 1 to N_2, N_3, N_4, N_5 , respectively. Save the best solution as X_j , whose objective value is the minimum one
- If $j = \max(N_i) - \min(N_i)$, turn to Eq. 4; else let $j = j + 1$ and turn to Eq. 2
- Select a best solution from the solutions obtained above, whose objective value is the minimum one. Let the best solution is the initial solution X^*

Neighborhood generation strategy: The neighborhood generation is performed on the variable set $X = [N_1, N_2, N_3, N_4, N_5]$. Two random numbers are generated to choose the items which are replaced to generation the

neighborhood of the variable set (i.e., one item plus 1, the other minus 1). Specific operations are described as follows: select one item randomly and judge whether N_i is equal to $\min(N_i)$ or not. If N_i is not equal to $\min(N_i)$, N_i decreases 1; otherwise, N_i adds 1. Then, randomly select another item N_j and j is not equal to i . If N_i adds 1 and N_j is not equal to $\min(N_j)$, N_j decreases 1. If N_j decreases 1, N_j adds 1. Otherwise, reselect j (Kirkpatrick *et al.*, 1983; Vasan and Raju, 2009; Tavares *et al.*, 2011).

The metropolis rule is adopted in the algorithm. The neighborhood solution is accepted if the objective value decreases; otherwise, the neighborhood solution is accepted based on the following acceptance probability.

Cooling schedule: The temperature is decreased according to the Equation $t_{k+1} = t_k \cdot p$, where p is the cooling rate and its value is defined between 0.95 and 0.99.

FaTP structure and behavior are introduced. Moreover, the behavior of the distributed shared memory access primitives (read and write operations) is analyzed and the complexities of such primitives have been calculated. Then, the proposed protocol functionality and performance have been measured. Experimental results have shown that FaTP operates properly. It has also shown that its recovery time is reasonable.

NUMERIC ANALYSIS

In order to obtain the value of performance indicators, we must obtain the series of parameters including the external arrival rate of patients λ , the patients routing probability p_i , the per-server service rate u_i , the number of servers N_i . N_i is obtained by the direct observation and p_i is obtained by a general statistical method from data. We use a method of parameter estimation to obtain the values of λ and u_i . The values of the parameters are derived from the maximum likelihood estimation. The Chi-square goodness of fit tests is performed to evaluate the fit of the models. The 171, 024 observations from January 1, 2010 to June 30, 2011 are obtained from Dalian XinHua hospital in China. The value of the external arrival rate is 1.892. The same calculation process of the service rate is done in a similar way. The values of the model parameters are presented as shown in Table 1.

The NS-SA algorithm is implemented by using C # programming language in Visual Studio 2008. The optimal solution of nurse staff allocation is better than the heuristic from Table 2 obviously.

The optimal nurse number is 43 and the optimal solution of nurse staff allocation is: $N_1 = 13, N_2 = 11,$

Table 1: Parameters and description

Parameters and description			
Parameter	Description	Values	i
p_i	Routing probability	0.8149	1
		0.1351	2
λ	External arrival rate	1.892	
u_i	Per-server service rate	0.5137	1
		0.3543	2
		0.5747	3
		0.7748	4
		0.2547	5
T	The cycle	240 min	
D	Examination service rate	1.6	
α	The ratio between nurse's idle time cost and patient's waiting time cost	0.2	

Table 2: The system values of NS-SA and Heuristic

Nurse No.	Optimal solution (NS-SA)	The cost of OD (NS-SA)	Optimal solution (the heuristic)	The cost of OD (the heuristic)
25	[7, 7, 3, 3, 5]	1404.6450	[7, 7, 3, 3, 5]	1404.645
26	[7, 7, 3, 3, 6]	1219.9260	[7, 7, 3, 3, 6]	1219.926
27	[8, 7, 3, 3, 6]	1088.4220	[7, 7, 3, 3, 7]	1176.342
28	[8, 7, 3, 3, 7]	974.3023	[7, 7, 3, 3, 8]	1143.435
29	[8, 7, 4, 3, 7]	933.9087	[7, 7, 3, 3, 9]	1109.887
30	[8, 8, 4, 3, 7]	893.5612	[7, 7, 3, 3, 10]	1076.065
31	[9, 8, 4, 3, 7]	860.7969	[7, 7, 3, 3, 11]	1036.967
32	[9, 9, 4, 3, 7]	848.6831	[7, 7, 3, 3, 12]	991.751
33	[9, 9, 4, 3, 8]	836.9581	[8, 7, 3, 3, 12]	976.861
34	[10, 9, 4, 3, 8]	825.9803	[9, 7, 3, 3, 12]	959.809
35	[10, 9, 5, 3, 8]	815.2530	[9, 8, 3, 3, 12]	945.985
36	[11, 9, 5, 3, 8]	811.4342	[9, 9, 3, 3, 12]	936.891
37	[11, 10, 5, 3, 8]	807.6368	[10, 9, 3, 3, 12]	916.651
38	[11, 10, 5, 3, 9]	804.3238	[11, 9, 3, 3, 12]	900.894
39	[11, 10, 5, 4, 9]	801.3235	[12, 9, 3, 3, 12]	886.880
40	[12, 10, 5, 4, 9]	800.1723	[13, 9, 3, 3, 12]	875.743
41	[12, 11, 5, 4, 9]	799.2524	[13, 10, 3, 3, 12]	865.039
42	[12, 11, 5, 4, 10]	798.6377	[13, 11, 3, 3, 12]	859.196
43	[13, 11, 5, 4, 10]	798.4969	[13, 12, 3, 3, 12]	855.887
44	[13, 12, 5, 4, 10]	798.5866	[13, 13, 3, 3, 12]	846.584
45	[13, 12, 5, 4, 11]	798.8027	[14, 13, 3, 3, 12]	837.469

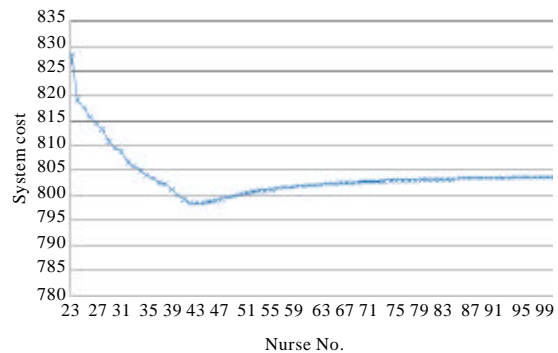


Fig. 2: The optimal number of nurse in CMNAOD

$N_3 = 5, N_4 = 4, N_5 = 10$. The cost of OD is 798.49 min with the nurse idle time costs and the patient waiting time costs. The reason that the optimal solution exist is with the increasing nurse number, the patients expected

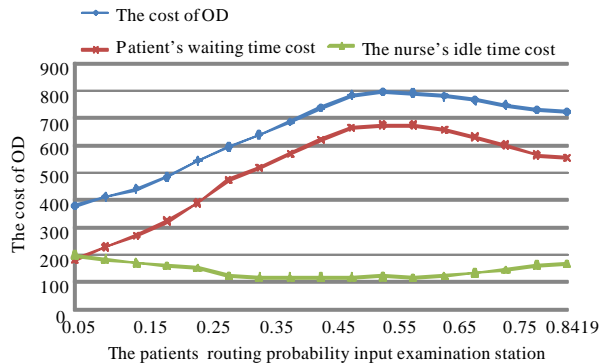


Fig. 3: λ for the impact of system values

waiting time cost in the system is significantly increasing is significantly decreasing and the nurse idle time cost in the system is significantly increasing.

p_1 for the impact of system values: With the patient routing probability input examination station p_1 , varying from (0.1, 1.892) and other parameters are fixed, as $N = 30$, so the changes of the costs of OD, the nurse idle time costs and the patient waiting time costs are shown in Fig. 3. From Fig. 3, some insights are obtained as following:

- With the increasing patient routing probability input examination station, the patient waiting time costs in the system is significantly increasing before $p_1 = 0.45$. But after $p_1 = 0.45$, the values of the patient waiting time costs are stable. The main reason for this phenomenon is that the examination station is a regulator to our queueing systems (Zhu *et al.*, 2013)
- With the increasing patient routing probability input examination station, the nurse idle time costs in the system is significantly decreasing before $p_1 = 0.45$. But after $p_1 = 0.45$, the values of the nurse idle time costs become stable with the same reason as above

CONCLUSION

In this study, a general multi-stage queueing system model with I^2 patients' routing including two tandem queues about OD is addressed. A mathematical programming model is developed with nurse staff allocation and a NS-SA algorithm is developed. Numerical experiments are conducted to analyze the impact of patients' arrival to the allocation of nurses and the ratio of patients' waiting time and nurses' idle time on the number of nurses needed. The research results can facilitate hospital managers to make decisions on human resources in practice.

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