Pricing by Two-sided Platform under the Condition of Service Differentiation

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Abstract: We study the pricing by platform in two-sided market when it offers differentiated service. In this study, we classify platform services into two categories: Matching service and value-added service. We analyze the pricing, the number of users and the profit of the monopoly platform when it offers differentiated matching service and differentiated value-added service, respectively. The study shows platform can get extra users and profits by offering differentiated service. And a monopoly platform may prefer to charge less for the low quality level and charge more for the high quality level when offer differentiated matching service, while the platform prefer to charge the same for value-added service compared with the condition of offering single value-added service.

Keywords: Two-sided markets, platform, service differentiation, pricing strategy

INTRODUCTION

Platforms in two-sided markets serve two groups of agents in order to facilitate interactions between the two groups and it obtains revenue by charging all agents. There are many examples of two-sided markets, such as e-commerce platform, dating agency, credit cards, etc. In this study, we classify platform services into two categories: Matching service and value-added service. The matching service refers to the services which facilitate the establishment of a buyer-seller transaction, while the value-added service refers to services which cannot affect the probability of a successful match. In the following section, we take the online dating agency—“Shi-Ji-Jia-Yuan” as an example to introduce these services.

“Shi-Ji-Jia-Yuan” is an online heterosexual dating agency and attracts both men and women. First, it offers differentiated matching service. The agency classifies agents into two categories: Free members and paying members. All agents can get to visit all the profiles from the interested ladies and men, while only the paying members have the personalized recommendation service and background check service. Second, it also offers differentiated value-added service, such as wedding photo, wedding planning, etc.

Previous studies of two-sided markets focus on concept definition (Rochet and Tirole, 2004), market characteristic (Evans, 2003), market classification (Evans, 2003), pricing (Cai and Julien, 2003), etc., pricing has been a core issue of the two-sided market theory. As the existence of the cross-group network effects, the pricing in two-sided markets is quite different from the pricing in traditional markets (Weyl, 2006).

Cai and Julien (2003) study the pricing in a game involving two competing platforms. Rochet and Tirole (2004) study the pricing of the monopoly platform and their study focuses on the role of relative price elasticity of demand on the two-sided markets in determining the platform pricing structures. Armstrong (2006) study the pricing of monopoly platform, competing platform and model of “competitive bottlenecks”. The follow-up studies are mostly extended on the basis of those literatures.

However, most scholars assume that platforms only offer one type of service, which is not the case in real situation. So, how should the platform set price to maximize profits when it offers differentiated service and What will happen to the platform’s profit and market share. These problems are worthy of study. Parker and Van Alstyne (2000). Initially study diversified service in information economy. Hanlin and Xi (2006) study the pricing and profits of platform when it provides different quality services, which is related to our model. In this study, we analyze the pricing strategy, the number of agents and the profit of the monopoly platform under the condition of differentiated matching service and differentiated value-added service separately.

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MODELLING SETUP

Monopoly platform A serves two groups of agents, denoted b (buyer) and s (seller). The number of sellers and the number of buyers are normalized to 1 and agents in a group are assumed to be uniformly located along a unit interval with the platform located at x = 0. Sellers and buyers are heterogeneous in the expected utility from buying the platform service and they will decide to buy if their expected earning exceeds the fee charged by the platform. Here, we use 1-x (x represents distance from the customer to the platform) to model agent preferences (i.e., agents with small x have larger gain from buying platform service compared to agents with large x). We model the cross-group network effect with parameter \( a \). The platform has three options: One is only providing high quality service, one is only providing low quality service and the other is providing high quality service and low quality service simultaneously. And platform will charge an agent \( p \) for service of quality \( q \). Every agent will choose to buy the platform service at most one unit.

We assume there is full information and all players (buyers, sellers, the platform) make their decision that maximize their individual utilities at every stage of the game:

- **Stage 1**: Platform decides to choose one strategy from the three options
- **Stage 2**: Platform announces the prices to both buyers and sellers
- **Stage 3**: Agents make their platform adoption decisions according to the utility they can derive. To simplify the model, we don't consider the sunk fixed costs and we set the marginal cost to be zero. We use the backwards induction to solve the problem

In stage three, agents will make their platform adoption decisions. We use \( u(x, q) \) to represent the agent's expected utility, which is located at \( x \) and choose service level \( q \). If agents do not buy platform service, their expected utility is zero. We model the indifferent points as: \( x_0 \) (the point at which agents is indifferent to buying high level service or not), \( x_1 \) (the point at which agents are indifferent to buying low level service or not), \( x_{HL} \) (the point at which agents is indifferent to buying high level service or low level service). Then, we get \( u(x, q_B) = 0, u(x, q_L) = 0, u(x_{HL}, q_B) = u(x_{HL}, q_L) \). When the platform only offers high-quality service, agents of preference \( x \in [0, x_{HL}] \) buy the service, while others not. When the platform offers two kinds of services (\( q_B \) and \( q_L \)), agents of preference \( x \in [0, x_{HL}] \) buy high-quality service, agents of preference \( x \in [x_{HL}, x_1] \) buy low-quality service and agents of preference \( x \in (x_1, 1] \) do not join the platform. In stage two, the platform announces the optimal pricing in the three cases. In first stage, the platform chooses the option which brings maximum profit and announces their service quality.

In the following section, we discuss the model of monoprotoly platform offering differentiated matching service and value-added service, respectively.

PRICING STRATEGY ANALYSIS

Matching service: Suppose platform can offer two types of matching service (i.e., \( q_B \) and \( q_L \), \( q_B \) and \( q_L \)) represent the probability of a successful matching between the two groups. The utility of agents is given by the following expressions:

\[
\begin{align*}
    u &= \begin{cases} 
        q_B n (1-x) - p, & \text{agents choose quality level } q_B (i=HL) \\
        0, & \text{agents choose not to join the platform}
    \end{cases}
\end{align*}
\]

\( n \) represent the number of agents in a group.

**Case 1**: Platform only offers quality level \( q_B \).

Setting \( u_B = q_B \alpha n (1-x_B) - p_B = 0 \), we can get the indifferent point \( x_B \). Then, the number of agents \( (n_B = n) \) in a group is equal to \( x_B \). That is to say:

\[
-q_B \alpha \pi_B^* - q_B \alpha n_B - p_B = 0
\]

(1)

There exist several solutions and we discuss the problem in different situations.

**Solution 1**: When \( \Delta = q_B \alpha \left( q_B \alpha - 4 p_B \right) < 0 \), \( \pi_B = q_B \alpha / 4 \), we can get:

\[
\pi_B = \frac{1}{2}, \quad n_B = 2 p_B n_B = \frac{q_B \alpha}{4}
\]

**Solution 2**: When \( \Delta > 0 \), \( \pi_B = q_B \alpha / 4 \), we can get:

\[
\pi_B = n_B = \frac{1}{2} + \frac{\sqrt{q_B \alpha^2 - 4 q_B \alpha p_B}}{2 q_B \alpha}
\]

- Substituting:

\[
\pi_B = n_B = \frac{1}{2} \frac{\sqrt{q_B \alpha^2 - 4 q_B \alpha p_B}}{2 q_B \alpha}
\]

into \( \pi_B = 2 p_B n_B \). We can check that there is no maximum since:

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\[
\frac{\partial^2 \pi_n}{\partial n_H} > 0
\]

So, it is not an equilibrium solution

- Substituting:
  \[
  n_H = \frac{1}{2} \frac{\sqrt{4q_H \alpha^2 - 4q_H \alpha p_H}}{2q_H \alpha}
  \]
  into \( \pi_H = 2p_H n_H \). Similarly, we can get:
  \[
  \frac{\partial^2 \pi_n}{\partial n_H} < 0
  \]
  that is to say maximum exists. Setting:
  \[
  \frac{\partial \pi_H}{\partial n_H} = 0
  \]
  we can get the platform’s optimal pricing, the number of agents in a group and profit:
  \[
  p_H = \frac{2q_H \alpha}{9} < \frac{q_H \alpha}{4}, \quad n_H = \frac{2}{3}, \quad \pi_H = 2p_H n_H = \frac{8q_H \alpha}{27}
  \]
  Since:
  \[
  \frac{8q_H \alpha}{27} > \frac{q_H \alpha}{4}
  \]
  the optimal option for the platform is to set:
  \[
  p_H = \frac{2q_H \alpha}{9}
  \]

**Case 2:** Platform only offers quality level \( q_L \).

Setting \( u_r = q_L, \alpha \) (1-\( x_r \)), \( p_L = 0 \), we can get the indifferent point \( x_L \). Then, the number of agents in a group \( (n_L = n) \) is equal to \( x_L \). That is to say:

\[
- q_L \alpha n^2 + q_L \alpha + p_L = 0
\]  

(2)

The solving processing is the same as case 1.

**Solution 1:** When \( \Delta = q_L \alpha (q_L \alpha - 4p_L) < 0 \), (i.e. \( p_L < \frac{q_L \alpha}{4} \)), we can get:

\[
\pi_L = \frac{1}{2}, \quad \pi_r = 2p_L n_L = \frac{q_L \alpha}{4}
\]

**Solution 2:** When \( \Delta > 0 \) (i.e. \( p_L > \frac{q_L \alpha}{4} \)), we can get:

\[
n_L = x_L = \frac{1}{2} + \frac{\sqrt{4q_L \alpha^2 - 4q_L \alpha p_L}}{2q_L \alpha}
\]

\[
n_L = \frac{1}{2} \frac{\sqrt{4q_L \alpha^2 - 4q_L \alpha p_L}}{2q_L \alpha}
\]

is not an equilibrium solution

- When:
  \[
  n_L = \frac{1}{2} \frac{\sqrt{4q_L \alpha^2 - 4q_L \alpha p_L}}{2q_L \alpha}
  \]
  platform’s pricing, the number of agents in a group and profit are:
  \[
  p_L = \frac{2q_L \alpha}{9}, \quad \pi_L = \frac{2}{3}, \quad \pi_r = 2p_L n_L = \frac{8q_L \alpha}{27}
  \]
  Since:
  \[
  \frac{8q_L \alpha}{27} > \frac{q_L \alpha}{4}
  \]
  the optimal option for the platform is to set:
  \[
  p_L = \frac{2q_L \alpha}{9}
  \]

**Case 3:** Platform offers quality level \( q_L \) and \( q_U \) \((q_L \geq q_U)\).

We use \( n_{HL} \) to model the total number of agents in a group. Setting \( u_H = 0, \ u_L = 0, \ u_r = u_L \) we can get the indifferent points:

\[
x_L = 1 - \frac{p_L}{\alpha n_L q_H}, \quad x_r = 1 - \frac{p_L}{\alpha n_L q_L}, \quad x_{HL} = 1 - \frac{p_L - p_L}{(q_H - q_L)\alpha n_L}
\]

Assuming:

\[
\frac{p_L}{q_L} \geq \frac{p_L}{q_H}
\]

we can prove \( x_{HL} \leq x_{HL} \leq x_L \). So, the total number of agents in a group is equal to \( x_L \). Then, we have:

\[
n_{HL} = x_L = \frac{1}{2} + \frac{\sqrt{4q_L \alpha^2 - 4q_L \alpha p_L}}{2q_L \alpha}
\]

when \( p_L \) satisfy:

\[
0 \leq p_L \leq \frac{q_L \alpha}{4}
\]

The platform’s profit is:

\[
\pi_{HL} = 2p_L n_{HL} + 2p_L (x_L - x_{HL})
\]

\[
- 2p_L = \frac{4(q_H p_L^2 - 2q_L p_L^2 + q_L p_L^2)}{(q_H - q_L)(q_H \alpha + \sqrt{4q_L \alpha^2 - 4q_L \alpha p_L})}
\]

(3)

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\( \pi_{HL} \) is quadratic in platform’s price \((p_H, p_L)\) and it is concave in these prices. So, platform’s optimal pricing is characterized by the first-order condition. And the equilibrium prices satisfy the following constraints:

\[
\begin{align*}
(5q^2 + q_H q_L p_H^m) + (6q_H q_L + 2q^2)p_H p_L - (3q_H q_L - q_H q_L)(p_H^m)^2 &= 0 \\
4q_H (q_H q_L - q_H q_L p_H^m + 4q_H (p_L^m)^2 + 2q_H (q_H q_L - q_H q_L) + 4q_H q_H q_L q_L - q_H q_L q_L) &= 0
\end{align*}
\]

(4)

We can derive from the Eq. 4. The indifferent point is:

\[
x_{HL} = \frac{1 - p_H - p_L}{(q_H - q_L)\alpha} = \frac{1}{2}
\]

We suggest the following solution procedure to gain the optimal pricing:

- **Step 1**: By solving Eq. 4 simultaneously, we can get optimal pricing \((p_H^*, p_L^*)\) when given a certain \(\alpha\), \(q_H\) and \(q_L\).
- **Step 2**: Substituting \(q_H\) and \(p_L^*\) into \(n_{HL}\), so we can get \(n_{HL}\).
- **Step 3**: Substituting \(p_H^*, p_L^*, n_{HL}\) into \(x_{HL}\), \(x_{L}\), so we can get the indifferent points \(x_{HL}, X_L\).
- **Step 4**: Substituting \(p_H^*, p_L^*, x_{HL}, x_{L}\), \(n_{HL}\) into \(\pi_{HL}\), we can get the platform’s profit.

**Example 1**: Considering a case with the following data: \(\alpha = 0.1\), \(q_H = 3\), \(q_L = 1\), \(p_H > 0\), \(0 < p_L < q_H/4\). Solving the case by Matlab, we obtain \(p_H^* = 0.098\), \(p_L^* = 0.012\). Then, we have \(n_{HL} = 0.86\), \(x_{HL} = 0.5\), \(x_{L} = 0.62\), \(X_L = 0.86\), \(n_{HL} = 0.1067\) and the results satisfy:

\[
p_H > p_L, q_H > q_L
\]

The platform’s profit under different price structures is shown in Fig. 1.

Summarize results of the three cases in Table 1.

**Example 2**: Generate 500 set array \((\alpha, q_H, q_L)\) at random by computer, which belong to \(\alpha \in [0.2, 0.3]\), \(q_H \in [2, 3]\), \(q_L \in [1, 2]\). Get the equilibrium solutions of those conditions:

- Calculate value of \(\pi_{HL}, \pi_H\) under those conditions and plot, as shown in Fig. 2.
- Calculate value of \(p_H^*, p_L^*\) under those conditions and plot, as shown in Fig. 3.

**Proposition 1**: Monopoly platform can increase profit by offering differentiated matching service, since \(\pi_{HL} > \pi_H > \pi_L\).

**Proposition 2**: Monopoly platform should charge less for the low-quality matching service and charge more for the high-quality matching service when provide differentiated matching service, since \(p_H^* > p_L^* > p_L^* > p_L^*\).

**Proposition 3**: Monopoly platform can increase agents by offering differentiated matching service.

**Proof**: The total number of agents in a group is determined by \(q_L\) and \(p_L\). Given \(q_L\) and combined with:

\[
x = \frac{1}{2} \sqrt{a^2 + \frac{4q_L q_L}{2q_L q_L - q_L}}
\]
Table 1: Equilibrium solutions when platform provides differentiated matching service

<table>
<thead>
<tr>
<th>Case</th>
<th>Optimal pricing</th>
<th>No. of agents in a group</th>
<th>Indifferent point</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$p_H = \frac{q_H \alpha}{2}$</td>
<td>$n_H = \frac{q_H V}{2(q_H V - \alpha)}$</td>
<td>$x_H = \frac{q_H V}{2(q_H V - \alpha)}$</td>
<td>$\pi_H = \frac{q_H^2 V^2}{2(q_H V - \alpha)}$</td>
</tr>
<tr>
<td>2</td>
<td>$p_L = \frac{q_L \alpha}{2}$</td>
<td>$n_L = \frac{q_L V}{2(q_L V - \alpha)}$</td>
<td>$x_L = \frac{q_L V}{2(q_L V - \alpha)}$</td>
<td>$\pi_L = \frac{q_L^2 V^2}{2(q_L V - \alpha)}$</td>
</tr>
<tr>
<td>3</td>
<td>$p_H, p_L$</td>
<td>$n_{HL}$</td>
<td>$x_{HL} = \frac{1}{2} x_H, x_L$</td>
<td>—</td>
</tr>
</tbody>
</table>

Table 2: Equilibrium solutions when platform provides differentiated value-added service

<table>
<thead>
<tr>
<th>Case</th>
<th>Optimal pricing</th>
<th>No. of agents in a group</th>
<th>Indifferent point</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$p_H = \frac{q_H V}{2}$</td>
<td>$n_H = \frac{q_H V}{2(q_H V - \alpha)}$</td>
<td>$x_H = \frac{q_H V}{2(q_H V - \alpha)}$</td>
<td>$\pi_H = \frac{q_H^2 V^2}{2(q_H V - \alpha)}$</td>
</tr>
<tr>
<td>2</td>
<td>$p_L = \frac{q_L V}{2}$</td>
<td>$n_L = \frac{q_L V}{2(q_L V - \alpha)}$</td>
<td>$x_L = \frac{q_L V}{2(q_L V - \alpha)}$</td>
<td>$\pi_L = \frac{q_L^2 V^2}{2(q_L V - \alpha)}$</td>
</tr>
<tr>
<td>3</td>
<td>$p_H = \frac{q_H V}{2}$, $p_L = \frac{q_L V}{2}$</td>
<td>$n_{HL} = \frac{q_H V}{2(q_H V - \alpha)}$</td>
<td>$x_{HL} = \frac{1}{2} x_H, x_L$</td>
<td>$\pi_{HL} = \frac{q_H^2 V^2}{2(q_H V - \alpha)} + \frac{q_L^2 V^2}{2(q_L V - \alpha)}$</td>
</tr>
</tbody>
</table>

Value-added service: Most platforms offer value-added service besides matching service.

We assume that monopoly platform offers only one type of matching service and two types of value-added service. All agents treat the matching service as the same, while have preference for value-added service. We use $V$ to model value-added service. The utility of agent is given by the following expressions:

$$u = \begin{cases} q_H V (1 - x) + \alpha n - p_H, & \text{agents choose quality level } q_H (i=H,L) \\ 0 & \text{agents choose not to join the platform} \end{cases}$$

$n$ represent the number of agents in a group.

Case 1: Platform only offers quality level $q_H$.

Setting $u_H = q_H V (1-x_H) + \alpha n_H - p_H = 0$, we can get the indifferent point $x_H$. Then, the number of agents in a group ($n_H = n$) is equal to $x_H$, and:

$$n_H = x_H = \frac{p_H - q_H V}{\alpha - q_H V}$$

Substituting $n_H$ into the profit function, we can get:

$$\pi_H = 2p_H n_H - \frac{2p_H^2 - 2p_H q_H V}{\alpha - q_H V}$$

$\pi_H$ is quadratic in platform's price ($p_H$) and it is concave in price $p_H$. When $\alpha$ satisfy $0 < \alpha < q_H V$, the platform's pricing, the number of agents in a group and profit are:

$$p_H = \frac{q_H V}{2}, n_H = \frac{q_H V}{2(q_H V - \alpha)}, \pi_H = \frac{q_H^2 V^2}{2(q_H V - \alpha)}$$

Case 2: Platform only offers quality level $q_L$.

Setting $u_L = q_L V (1-x_L) + \alpha n_L - p_L = 0$, we can get the indifferent point $x_L$. Then, the number of agents in a group ($n_L = n$) is equal to $x_L$.

The solving processing is the same as case 1. So, when $\alpha$ satisfy $0 < \alpha < q_L V$, we can get:

$$p_L = \frac{q_L V}{2}, n_L = \frac{q_L V}{2(q_L V - \alpha)}, \pi_L = \frac{q_L^2 V^2}{2(q_L V - \alpha)}$$

Case 3: Platform offers quality level $q_H$ and $q_L$ ($q_H > q_L$).

We use $n_{HL}$ to model the total number of agents in a group. Setting $u_H = 0$, $u_L = 0$, $u_{HL} = u_L$, we can get the indifferent points:

$$x_H = \frac{q_H V (1-x_H) + \alpha n_{HL} - p_H}{q_H V}, x_L = \frac{q_L V (1-x_L) + \alpha n_{HL} - p_L}{q_L V}, x_{HL} = \frac{q_H V (1-x_H) - (0-H-x_L)}{q_H V}$$

We can proof $x_{HL} < x_H, x_L$. Then, the total number of agents in a group is equal to $x_{HL}$. When $\alpha$ satisfy $0 < \alpha < q_H V$, we can get:

$$n_{HL} = x_{HL} = \frac{q_H V - p_L}{q_H V - \alpha}$$

Substituting $n_{HL}$ into platform's profit Eq.:

$$\pi_{HL} = 2p_H n_{HL} - 2p_H q_L V$$

$\pi_{HL}$ is quadratic in platform's prices ($p_H, p_L$) and it is concave in these prices. So, the equilibrium prices are:

$$p_H = \frac{q_H V}{2}, p_L = \frac{q_L V}{2}$$

And the indifferent points and profit are:

$$x_{HL} = \frac{1}{2}, x_H = \frac{q_H V}{2(q_H V - \alpha)}, \pi_{HL} = \frac{q_H^2 V^2}{2(q_H V - \alpha)} + \frac{q_L^2 V^2}{2(q_L V - \alpha)}$$

Summarize results of the three cases in Table 2:
**Proposition 4:** Diversified value-added service can increase monopoly platform’s profit, while when it offers one type of service, which kind of quality level will bring greater profit is not sure.

**Proof:** We can get from Table 3 that $p_{H} > p_{H}$, $p_{H} = p_{L}$ and the relationship between $p_{H}$ and $p_{L}$ is not for sure.

**Proposition 5:** Monopoly platform should charge the same for value-added services compared with the condition of offering only one type of value-added service when provide differentiated value-added service, since $p'_{H} = p'_{H}$, $p'_{L} = p_{L}$.

**CONCLUSION**

As a method of improving user’s experience and expanding market share, differentiated service strategy becomes more and more important for platform enterprises.

From the analysis above, we can see that monopoly platform can get extra users and profits by offering differentiated service. It is true that differentiation strategy help to increase the platform’s market share, since it offers greater choice for agents. So, more and more agents will be attracted to join the platform. As the existence of the cross-group network, the utility of agent on one side will increase when the number of agents in other group increased. Finally the monopoly platform can get larger agents and greater profit by offering differentiated service.

**REFERENCES**


