Back Analysis Method of Foundation Pit Soil Mechanical Parameters
Based on GA-BP Neural Network

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Abstract: In order to ensure the foundation pit construction safety, displacement monitoring was carried on the foundation pit soil. At the same time, according to the measured displacement, the displacement parameters inversion algorithm was taken to calculate the foundation pit soil mechanical parameters, then calculate the stability of foundation pit soil. According to the basic calculation method of genetic algorithm optimizing neural network, in this paper, use uniform design method to design the samples of the foundation pit soil displacement and mechanical parameters. The finite element analysis was used to calculate soil displacement with different parameters. The GA-BP neural network was trained to describe the sophisticated nonlinear relationship between displacement and mechanical parameters of the foundation pit soil. Finally, the actual displacement was input into the trained GA-BP neural network to obtain the soil mechanical parameters. As an example, the soil cohesion, friction angle, elastic modulus and Poisson ratio of Changchun West Railway Station foundation pit were back-analysis by the above method. The calculation displacement obtained by finite element analysis with inversion results and compared with measured data. The comparison result shows that GA-BP neural network has high precision in excavation soil parameters inversion which can meet the needs of engineering.

Key words: Foundation engineering, foundation pit displacement, genetic algorithm, BP neural network, parameters inversion

INTRODUCTION

In the city subway and housing construction, there has been a lot of excavation, the risk of instability in the excavation process, due to the destruction of the structure and properties of the undisturbed soil. In order to guarantee the safety of foundation excavation, it is necessary to evaluate the stability of the foundation soil in real-time. Soil mechanics parameters accurately obtained is a necessary condition for foundation soil stability analysis. And a small amount of laboratory and in situ tests can't be behalf of the entire mechanical properties of the soil, because the test results are discrete with poor representation. In the construction process, there is a certain amount of damage due to the impact of rainfall and free-thaw cycles (Cheng et al., 2010). So, it is very difficult to carry foundation analysis based on soil mechanics parameters obtained by laboratory and in situ tests.

Foundation soil displacement is the parameter which can be accurately obtained most possibly. In 1970s, Kavanagh, Gioda and Maier proposed the idea of the parameters inversion, rock and soil mechanics parameters based on the inversion of rock and soil monitoring results (Li and Yin, 1996). With the development of computing technology, more and more calculation method is applied to the inversion of the rock and soil mechanics parameters, just like Artificial Neural Network (ANN) (Li et al., 1997; Liang et al., 2003), Gradient Algorithm (Levasseur et al., 2008; Rechea et al., 2008), Genetic Algorithm (GA) (Gao and Zheng, 2001, 2003) and Particle Swarm Optimization (PSO) (Wang, 2007). These algorithms have their own advantages and disadvantages, such as ANN with powerful nonlinear mapping ability. But it is easy to fall into local optimal solution in ANN training, because its initial weight and bias values are random, the inversion accuracy of ANN is not high. Gradient Algorithm, GA and PSO often require a combination of numerical algorithms by repeatedly calling the numerical computation as Finite Element Method (FEM). In order to meet the precision of mechanics parameters, the calculation amount is quite large. GA optimization of Back Propagation neural network algorithm (GA-BP) uses genetic algorithm to optimize the neural network initial weights and bias values which can improve the neural network inversion calculation accuracy while avoiding repeatedly numerical calculation so as to save computing time. In this paper, the GA-BP was used to inversion calculated soil Mohr-Coulomb parameters in accordance with monitoring results of Changchun West Railway Station deep foundation soil displacements and inverse calculation results were analyzed.
GA-BP NEURAL NETWORK

GA-BP neural network uses GA to optimize the initial weights and bias values of BP neural network and improves the defects into a local optimal solution in the training process. BP neural network was shown in Fig. 1. The transfer function of the hidden layer was sigmoid function \( f(v) = 1/(1+\exp(-v)) \), the output layer transfer function was purelin function \( f(v) = v \).

For the sample \( p \), the output of the hidden layer nodes is:

\[
y_j = f\left( \sum_{i=1}^{n} w_{ij}u_i + b_j \right)
\]

where, \( n \) is the input layer node number, \( u_i \) is the input for the input layer nodes, \( b_j \) is bias values of the hidden layer nodes, \( w_{ij} \) is the weight between input layer node \( i \) and hidden layer node \( j \) and \( f_i \) is transfer function of hidden layer.

Calculation output of the output layer nodes:

\[
y_k = f\left( \sum_{j=1}^{M} w_{kj}y_j + b_k \right)
\]

where, \( M \) is the number of hidden layer nodes, \( b_k \) is the bias values of the output layer nodes, \( w_{kj} \) is the weight between hidden layer node \( j \) and output layer node \( k \) and \( f_j \) is transfer function of output layer.

Error calculation formula is below:

\[
E = \frac{1}{2} \sum_{i=1}^{n} (Y_{di} - y_i)^2
\]

where, \( Z_{di} \) and \( Y_{di} \) is the desired and calculated output of BP neural network, respectively. And \( m \) is the number of the output layer node.

Steepest descent method was used to update the BP neural network weights and bias values:

\[
\begin{cases}
   w_{ij}(k+1) = w_{ij}(k) - \alpha \frac{\partial E}{\partial w_{ij}} \\
   b_j(k+1) = b_j(k) - \alpha \frac{\partial E}{\partial b_j}
\end{cases}
\]

where, \( w_{ij}(k+1) \) and \( b_j(k+1) \) is the update weight and bias value, respectively, \( w_{ij}(k) \) and \( b_j(k) \) is weight and bias value before updating, \( \alpha \) is the learning speed, \( t \) is the BP neural network layer.

BP neural network is often easy to fall into local optimal solution when using the steepest descent method so that the neural network fitting accuracy is not high, because the initial weights and bias values of neural network are random values. GA can optimize the initial and bias values of neural network, this algorithm generate solution set range according to the characteristic of problem. Then the problem optimal solution is obtained by genetic manipulation, just like code, select, cross, mutate and decode.

GA was used to optimize the initial weights and bias weights of BP neural network, real coded was used in order to save computing time and the population size was 20~100. The average absolute value of error between the predicted outputs and the desires outputs of the neural network was the individual fitness value \( F \), the calculation formula as follows:

\[
F = \frac{1}{n} \sum_{i=1}^{n} |y_i - o_i|
\]

where, \( n \) is the neural network output nodes number, \( y_i \) is the desired output of the neural network node \( i \) and \( o_i \) is forecast output of node \( i \).

Gamble was used as select operation which determined selection probability \( p_i \) according to the fitness function value, just as follows:

\[
p_i = \frac{f_i}{\sum_{i=1}^{N} f_i}
\]

where, \( F_i \) is the fitness value of individual \( i \), fitness value reciprocal was calculated before individual selected, due to the fitness value was the smaller the better, \( k \) is constant and \( N \) is the number of individuals.

The arithmetic cross algorithm was used as cross operation, calculation method as follow:

\[
\begin{cases}
   x_{i} = \alpha x_{i} + (1-\alpha)x_{i} \\
   x_{i} = \alpha x_{i} + (1-\alpha)x_{i}
\end{cases}
\]

Fig. 1: BP neural network model
where, \(x_i, x_j\) is the original individual real coded, \(x'_i, x'_j\) is the individual after cross operation, \(\alpha\) is cross operation parameter which can be assumed to be constant and can also be set to the variables related to the evolution algebra. Generally \(\alpha\) can be selected according to the characteristic of the problem. The mutation operation can calculate as follow:

\[
a'_i = \begin{cases} 
    a_i + (a_{\min} - a_i) f(g) & r \geq 0.5 \\
    a_i + (a_{\max} - a_i) f(g) & r < 0.5 
\end{cases}
\]  

(9)

where, \(a_{\max}\) is the upper bound of gene \(a\), \(a_{\min}\) is the lower bound of gene \(a\), \(f(g) = r_1(1-g/G_{\max})\), is a random number, \(g\) is the current number of iterations, \(G_{\max}\) is the maximum evolution times and \(r\) is a random number between \([0, 1]\).

GA can obtain the neural network meet the accuracy requirements by these steps. The select operation select individual to make fitness function reach the smallest that is obtained initial weights and bias values which can meet the minimum error value. Then the initial weights and bias values were updated by using the error back propagation.

**FOUNDATION SOIL MECHANICS PARAMETERS INVERSION WITH GA-BP NEURAL NETWORK**

GA-BP neural network used GA to optimize the initial weights and bias value of BP neural network. When inversion calculate foundation soil mechanics parameters with GA-BP neural network, the nonlinear relationship between foundation soil mechanics parameters and displacements was expressed by trained neural network, the steps of this process are as follows:

- Determine the inversion parameters and their range, then construct uniform sheet with uniform design method. Establish the foundation numerical model, in accordance with the uniform sheet, using finite element method to calculate the displacements of the monitoring point of different test.
- Determine the BP neural network hidden layer nodes, with the calculated displacement value as input and the foundation soil mechanics parameters as output. GA was used to optimize the initial weights and bias values of the BP neural network.
- Using the steepest descent method for network training with optimal initial weights and bias, to obtain the neural network can be used for parameter inversion. Then take monitor displacements as input and the output of neural network as the foundation soil mechanics parameters.

**ENGINEERING APPLICATION**

**Engineering overview:** Changchun railway station seep foundation with deep excavation depth and long excavation time, the deepest excavation depth located in the northern of foundation named Y axis, deepest excavation depth was 21.3 m, divided into two excavation stage, the first stage excavation depth was 7–8 m and the second stage excavation depth was 13–14 m. The foundation supporting structure and longitudinal profile were shown as Fig. 2.

Pile construction was carried on at BC and layout inclinometer tubes at AB before excavating the foundation. When excavating soil 1, use inclinometer to monitor soil 2 slope horizontal displacement in order to ensure the safety of construction. After excavation of soil 1, as the 2 continued to be a construction site, the horizontal of displacement of soil 2 need continuous to monitor. This section of monitoring time was from June 2010 to December 2010, about six months. In this period, the climate of Changchun city was from summer to winter and the slope soil experienced rainfall in the rainy season and freeze-thaw cycles in early winter. The slope soil mechanics parameters changed with slope soil water content increased under the condition of rainfall, these parameters can also damage on condition of freeze-thaw. So, the external environment conditions changes will affect the stability of foundation. In order to ensure stability of the foundation, GA-BP neural network was used to inverse calculation foundation soil mechanics parameters as monitoring foundation soil horizontal displacements and then calculate the foundation soil stability which can provide reliable foundation construction safety.

**The foundation soil displacement monitoring results:**
Ten monitoring points were arranged on the deep

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**Fig. 2:** Schematic diagram of longitudinal profile in excavation
foundation Y axis, the displacement data of monitoring point Y5 was selected as parameter inversion object. The inclinometer monitoring data of these monitoring points was treated into the slope deep soil relative displacement value, Fig. 3 show this data of October 31, November 11 and November 18, 2010. In this monitoring period, the minimum temperature of Changehun city has been below zero at night and above zero during the day, slope soil was under freeze-thaw cycles, the soil at a distance 4-6 m of ground had the larger displacement.

The neural network training samples: According to the Mohr-Coulomb model and associated flow rule, the slope soil mechanics parameters include elastic modulus E, poisson ratio μ, cohesion c and internal friction angle ϕ. At first, set the range of each inversion parameter to construct a neural network training samples. The range of E was 10-155 Mpa, the incremental was 5 Mpa, the range of μ was 0.15-0.44, the incremental was 0.01, the range of c was 10-68 kpa, the incremental was 2 kpa and The range of ϕ was 15-44°, the incremental was 1°. In order to improve the generalization ability of the neural network algorithm, uniform test methods can be used to construct neural network training samples. Put the uniform design parameter values into the finite element model (Fig. 4) and calculate monitoring points (1, 2, 3, 4) horizontal displacement values.

Fig. 3: Monitoring value of relative displacement

Fig. 4: Excavation numerical model

Fig. 5: Curve of average fitness

Took the calculated horizontal displacement value of monitoring points as input value and soil mechanics parameters as output value, then carried on neural network training. The BP neural network input layer nodes were 4, the hidden layer nodes were 12 and the output nodes were 4. The transfer function of hidden layer was the hyperbolic tangent function and the output layer transfer function was purelin function. GA was used to optimize the initial weights and bias values of BP neural network, the size of initial population size was 20, the evolution generation is 100 and the calculated average fitness function value with evolution changes was shown as Fig. 5.

As can be seen from Fig. 5, the average fitness value of the decrease is more noticeable in the evolution generation which was less than 20, evolution generation was 20-80, the average fitness value was the same essentially, evolution generation was 80-100, the average fitness value slightly decreased. The final average fitness was stabled at 0.04 which indicated that the GA found the optimal solution.

Result analysis of parameters inversion: Input the horizontal displacement of nodes which is 0, 2, 4, 6 m from the ground to the trained BP neural network, then cohesion (c), internal friction angle (ϕ), elastic modulus (E) and Poisson ratio (μ) of the slope soil were the output. The calculation results were shown in Table 1.
Table 1: Inversion calculation results of soil mechanical parameters

<table>
<thead>
<tr>
<th>Date</th>
<th>c (kPa)</th>
<th>c (kPa)</th>
<th>E (Mpa)</th>
<th>μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>October 30</td>
<td>55.32</td>
<td>9.98</td>
<td>33.77</td>
<td>0.33</td>
</tr>
<tr>
<td>November 11</td>
<td>41.49</td>
<td>6.46</td>
<td>33.33</td>
<td>0.35</td>
</tr>
<tr>
<td>November 18</td>
<td>36.12</td>
<td>5.62</td>
<td>32.14</td>
<td>0.34</td>
</tr>
</tbody>
</table>

In order to verify the calculation results, input the calculation results of Table 1 to finite element model, calculated the corresponding horizontal displacement value of the inclinometer tube monitoring points and compared with the measured values, the results shown in Fig. 6.

As can be seen from Fig. 6, the slope displacement calculated value in accordance with the parameters inversion that was in good agreement with the measured displacement which indicated the parameter inversion calculation results were correct.

CONCLUSION

The study was based on the basic theory of the GA-BP neural network and carried on researches on the basic steps of this algorithm using in the foundation soil parameters inversion. This algorithm was used to inversion calculate Mohr-Coulomb parameters of the foundation soil, based on Changchun West Railway Station deep foundation safety monitoring results. Finite element method was used to calculation slope displacement with parameter inversion results which is in good agreement with the measure displacement. It is show that the GA-BP neural network is reliable in foundation engineering parameters inversion.

REFERENCES


