Comprehensive Assessment Method of Soil and Water Conservation of Forest Ecosystems in China using Correlation Coefficient Between Interval-valued Fuzzy Sets

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Abstract: This study introduces the correlation coefficient between Interval Valued Fuzzy Sets (IVFSs) and proposes a comprehensive assessment method of soil and water conservation of forest ecosystems based on the correlation coefficient, to solve the problem to assess the capability of soil and water conservation for some alternative places and determine the grades of the comprehensive evaluation in different regional conditions. A practical example of soil and water conservation in some alternative places of China is used for fuzzy comprehensive assessment problems as the demonstration of the applications and effectiveness of the proposed assessment method. The method proposed is significant on soil and water conservation in forest ecosystems and provide a new way to assess the capability of soil and water conservation.

Key words: Soil and water conservation, soil erosion, comprehensive assessment, correlation coefficient, interval valued fuzzy set

INTRODUCTION

As one of the world’s major environmental issues, soil and water conservation of forest ecosystems has gained more and more concerns throughout the world. The main purpose of soil and water conservation is to prevent soil erosion and there are two main types of soil erosion: water and wind erosion (Bennett, 1934). Soil erosion is a flattened process which shows dispersed, stripped and transported processes of soil under inner or external forces such as hydro, wind, gravity and human activities (Hudson, 1987). Soil erosion in forest takes away a lot of mud and organic matter leaving only much gravel and stone (Frederick et al., 2003) and has significant adverse impacts on the land and its inhabitants (Toy et al., 2002). Comprehensive treatments of soil erosion need soil and water conservation (Morgan, 2009) which is more and more important than ever before (Elrashidi, 2011).

Soil and water conservation is involved in a matter of physics, chemistry, mathematics, computers, weather, water conservancy, agriculture, forestry, resource, environment, socio-economics and other disciplines. Since the 1920s and 1930s mathematical Einstein (1926) proposed models for understanding the mechanisms of soil erosion and resulting sediment surface runoff and dealt with both gully and sheet erosion. Earliest models were a simple set of linked equations which could be employed by manual calculation. By the 1970s complex computer models with thousands of lines of computer code were expanded to address nonpoint source pollution (Michael Hogan et al., 1973). After that, more and more complex models were improved to handle nuances in micrometeorology, soil particle size distributions and micro-terrain variation. NRCS (Natural Resources Conservation Service) field offices presented the Universal Soil Loss Equation (USLE) which becomes the main model used to measure water erosion by United States government agencies (Renard et al., 1991). Within a Geographic Information System (GIS), new models of topographic potential for the erosion and deposition were used in complex terrains (Mitasova et al., 1996). A comprehensive methodology with integrates of erosion models, GIS techniques and a sediment delivery concept for estimating water erosion and sediment delivery at the watershed scale was presented (Fernandez et al., 2003). Fuzzy methods were described and applied to assessment of forest management with the fuzzy criteria and indicators to solve the sustainable forest development (Menloza and Prabhu, 2003). Three fuzzy methods and linear discriminant analysis were used with the date of forest inventory and analysis to classify six ecological habitats and the classification result was showed to be more accuracy (Zhang et al., 2004). The GIS-based model...
to improve estimation with many characteristics was used in forest science research (Magee-Maeda dog et al., 2006). Fuzzy multiple criteria decision aid was described with the advantage in quantitative and qualitative and an integrated fuzzy multiple criteria methodology was proposed to solve the problem of decision making of forestry management (Tolga and Cengiz, 2011).

In forest ecosystems, vegetation has a significant effort on soil erosion control and is a foremost charismatic of erosion control (Rattan et al., 1998). Increasing vegetation cover ratio on the land helps prevent both wind and water erosion (David et al., 2011). The studies of soil and water conservation function of forest ecosystems in China are mainly based on the vegetation cover ratio (or grass area ratio) to evaluate soil and water conservation. This single evaluation does not fully reflect the differences in soil and water conservation function of forest ecosystems, so the comprehensive assessment of soil and water conservation of forest ecosystems has important theoretical value and practical significance. There are few researches focusing on comprehensive assessment of soil and water conversation of forest ecosystems with fuzzy sets.

In this study, we introduce the correlation coefficient between Interval-valued Fuzzy Sets (IVFSs) and propose an comprehensive assessment method of soil and water conservation of forest ecosystems based on the correlation coefficient, where five criteria (vegetation coverage ratio, depth of forest floor, crown density, forest type and community structure) are represented by IVFSs to assess the capability of soil and water conservation for some alternative places by use of the correlation coefficient between IVFSs and to determine the grades of the comprehensive evaluation in different regional conditions on soil and water conservation in forest ecosystems. This study not only contribute to an objective understanding of the role of forest vegetation on soil erosion control, but also is conducive to the quantitative assessment of forest resources, soil conservation and thus provide a reference for the construction and management of forest resources in China.

The rest of study is organized as follows. In section 2, we introduce some basic concepts and definitions related to fuzzy sets, IVFSs and Intuitionistic Fuzzy Sets (IFSs). In section 3, the correlation coefficient between IVFSs is introduced by analogy with the definition of the correlation coefficient between IFSs. A comprehensive assessment method of soil and water conversation is proposed based on the correlation coefficient in section 4. A practical example is presented to illustrate the developed approach in section 5. Finally, some final remarks and conclusions of the proposed assessment method are given in section 6.

**PRELIMINARIES**

In this section, we introduce some basic concepts and definitions related to fuzzy sets, IVFSs and IFSs which will be needed in the analysis of the study.

**Fuzzy sets and IVFSs**

**Definition 1:** Zadeh (1965) defined a fuzzy set A in the universe of discourse X as follows:

\[ A = \{ (x, \mu(x)) | x \in X \} \]  

which is characterized by membership function \( \mu_A(x) \): \( X \rightarrow [0, 1] \), where \( \mu_A(x) \) indicates the membership degree of the element x to the set A.

In fuzzy set theory, it is often difficult for an expert to exactly quantify his or her opinion as a number in interval \([0, 1]\). Therefore, it is more suitable to represent this degree of certainty by an interval. From such point of view, Zadeh (1975) first proposed the concept of an Interval-valued Fuzzy Set (IVFS). IVFSs are suitable for capturing imprecise or uncertain information.

**Definition 2:** An IVFS A in the universe of discourse X is given by Zadeh (1975):

\[ A = \{ (x, [\mu^L(x), \mu^U(x)]) | x \in X \} \]  

where, \( \mu^L(x) : X \rightarrow [0,1] \) and \( \mu^U(x) : X \rightarrow [0,1] \) are called a lower limit of membership degree and a upper limit of membership degree of the element x to the set A, respectively, with the condition \( 0 \leq \mu^L(x) \leq \mu^U(x) \leq 1 \). The complementary set \( A^c \) of A is defined as:

\[ A^c = \{ (x, [1 - \mu^L(x), 1 - \mu^U(x)]) | x \in X \} \]

**IVFSs and IFSs**

**Definition 3:** (Atanassov, 1986). Let X be a fixed set. An intuitionistic fuzzy set A in X can be given as

\[ A = \{ (x, \mu_A(x), \nu_A(x)) | x \in X \} \]

where \( \mu_A(x) : X \rightarrow [0,1] \) and \( \nu_A(x) : X \rightarrow [0,1] \) with the condition \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \) for \( x \in X \).

The number \( \mu_A(x) \) denotes the degree of membership of the element x to set A and \( \nu_A(x) \) represents the degree of non-membership of the element \( x \in X \) to set A.

Atanassov and Gargov (1989) transform every IVFS A into an IFS A by \( \mu_A(x) - \mu^L_A(x) \) and \( \nu_A(x) - 1 - \mu^U_A(x) \), that is

\[ A = \{ (x, [\mu^U_A(x), 1 - \mu^L_A(x)]) | x \in X \} \]
CORRELATION IN IVFSS

For each intuitionistic fuzzy set \( A = \{x_i, \mu_A(x_i), v_A(x_i)\}_{i \in X} \) in the universe of discourse \( X = \{x_1, x_2, \ldots, x_n\} \), the correlation coefficient (Bustince and Burillo, 1995) of \( A \) and \( B \) was given by:

\[
K(A,B) = \frac{\sum_{i=1}^{n} (\mu_A(x_i) \mu_B(x_i) + v_A(x_i) v_B(x_i))}{\sqrt{\sum_{i=1}^{n} (\mu_A(x_i))^2 + (v_A(x_i))^2} \sqrt{\sum_{i=1}^{n} (\mu_B(x_i))^2 + (v_B(x_i))^2}}
\]  

(3)

For all \( A, B \in \text{IVFS}(X) \), the correlation coefficient satisfies the following properties:

- If \( A = B \), then \( K(A, B) = 1 \)
- \( K(A, B) = K(B, A) \)
- \( 0 \leq K(A, B) \leq 1 \)

Taking the transformation between IVFSs and IFSSs (Amatassov and Gargov, 1989) into account, for IVFSs \( A = \{x_i, [\mu_A(x_i), \nu_A(x_i)]\}_{x \in X} \) and \( B = \{x_i, [\mu_B(x_i), \nu_B(x_i)]\}_{x \in X} \), according to the correlation coefficient of IFSSs, the correlation coefficient of IVFSs \( A \) and \( B \) is introduced by:

\[
M(A,B) = \frac{\sum_{i=1}^{n} (\mu_A(x_i) \mu_B(x_i) + \nu_A(x_i) \nu_B(x_i))}{\sqrt{\sum_{i=1}^{n} (\mu_A(x_i))^2 + (\nu_A(x_i))^2} \sqrt{\sum_{i=1}^{n} (\mu_B(x_i))^2 + (\nu_B(x_i))^2}}
\]  

(4)

For all \( A, B \in \text{IVFS}(X) \), the correlation coefficient also satisfies the following properties:

- If \( A = B \), then \( M(A, B) = 1 \)
- \( M(A, B) = M(B, A) \)
- \( 0 \leq M(A, B) \leq 1 \)

COMPREHENSIVE ASSESSMENT METHOD USING THE CORRELATION COEFFICIENT

For a comprehensive assessment problem of soil and water conservation, assessment criteria with respect to alternative places in the evaluations of soil and water conservation are given by using IVFSs.

Suppose that there exists a set of alternative places \( A = \{A_1, A_2, \ldots, A_m\} \). Each alternative place is assessed on \( n \) criteria which are denoted by \( C = \{C_1, C_2, \ldots, C_n\} \). The gained value of a criterion \( C_j \) (\( j = 1, 2, \ldots, n \)) on an alternative place \( A_i \) (\( i = 1, 2, \ldots, m \)) is an IVFS \( \{\mu_i(C_j), \nu_i(C_j)\} \), \( (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n) \) given by the decision maker or expert according to investigating data of the evaluated criteria of soil and water conservation in some places. Thus we can obtain an interval-valued fuzzy assessment matrix \( D = (d_{ij})_{m \times n} \), which is defined as the following form:

\[
D = \begin{bmatrix}
C_1 & C_2 & \cdots & C_n \\
A_1 & [\mu_{11}, \nu_{11}] & [\mu_{12}, \nu_{12}] & \cdots & [\mu_{1n}, \nu_{1n}] \\
A_2 & [\mu_{21}, \nu_{21}] & [\mu_{22}, \nu_{22}] & \cdots & [\mu_{2n}, \nu_{2n}] \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
A_m & [\mu_{m1}, \nu_{m1}] & [\mu_{m2}, \nu_{m2}] & \cdots & [\mu_{mn}, \nu_{mn}]
\end{bmatrix}
\]  

(5)

In comprehensive assessment environments, the concept of ideal point has been used to help the identification of assessment grades for the alternative places. It can provide a useful theoretical construct to evaluate alternative places. Therefore, we define an ideal interval-valued fuzzy set for each criterion in the ideal alternative \( A = [\mu_1^*, \nu_1^*], [\mu_2^*, \nu_2^*], \ldots, [\mu_n^*, \nu_n^*] \) from some ideal situation.

Thus the correlation coefficient between an alternative place \( A \) and the ideal alternative \( A^* \) represented by the IVFSs is given as the follows:

\[
M(A^*, A) = \frac{\sum_{i=1}^{n} (\mu_i^* \mu_i + \nu_i^* \nu_i)}{\sqrt{\sum_{i=1}^{n} (\mu_i^*)^2 + (\nu_i^*)^2} \sqrt{\sum_{i=1}^{n} (\mu_i)^2 + (\nu_i)^2}}
\]  

(6)

The correlation coefficient provides the global evaluation for each alternative place regarding the ideal alternative on all the criteria. From Eq. 6, the larger the value of the correlation coefficient is, the better the alternative place is. Through the correlation coefficient, the assessment grades of all the alternative places can be easily identified according to the relationship between the given assessment grades and the correlation coefficients as shown in Table 1.

**PRACTICAL EXAMPLE**

The following practical example involves a soil and water conservation problem in China. The authorized decision maker or expert need to give the assessment grades of soil and water conservation in some places. The decision maker considers various criteria involving (1) \( C_1 \): Vegetation cover ratio, (2) \( C_2 \): Depth of forest floor, (3) \( C_3 \): Crown density, (4) \( C_4 \): Forest type, (5) \( C_5 \): community structure. The decision maker evaluates six alternative places, \( A = \{A_1, A_2, A_3, A_4, A_5, A_6\} \), based on five criteria, \( C = \{C_1, C_2, C_3, C_4, C_5\} \). The assessment matrix for the alternative place \( A_i \) with respect to the criterion \( C_j \in C \) is given by Table 2.
Table 2: Assessment matrix D

<table>
<thead>
<tr>
<th>A (Alternative places)</th>
<th>C_1: Vegetation cover ratio</th>
<th>C_2: Depth of forest floor</th>
<th>C_3: Crown density</th>
<th>C_4: Forest type</th>
<th>C_5: Community structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1 (Ganshu_01)</td>
<td>[0.80, 0.85]</td>
<td>[0.30, 0.33]</td>
<td>[0.60, 0.70]</td>
<td>[0.40, 0.60]</td>
<td>[0.40, 0.60]</td>
</tr>
<tr>
<td>A_2 (Ganshu_02)</td>
<td>[0.55, 0.65]</td>
<td>[0.00, 0.01]</td>
<td>[0.25, 0.35]</td>
<td>[0.40, 0.60]</td>
<td>[0.10, 0.30]</td>
</tr>
<tr>
<td>A_3 (Guizhou_01)</td>
<td>[0.75, 0.85]</td>
<td>[0.50, 0.55]</td>
<td>[0.55, 0.60]</td>
<td>[0.80, 1.00]</td>
<td>[0.10, 0.30]</td>
</tr>
<tr>
<td>A_4 (Guizhou_02)</td>
<td>[0.56, 0.80]</td>
<td>[0.45, 0.50]</td>
<td>[0.10, 0.15]</td>
<td>[0.60, 0.80]</td>
<td>[0.10, 0.30]</td>
</tr>
<tr>
<td>A_5 (Guangxi_01)</td>
<td>[0.88, 0.90]</td>
<td>[0.66, 0.84]</td>
<td>[0.70, 0.80]</td>
<td>[0.40, 0.60]</td>
<td>[0.10, 0.30]</td>
</tr>
<tr>
<td>A_6 (Guangxi_02)</td>
<td>[0.80, 0.85]</td>
<td>[0.66, 0.90]</td>
<td>[0.68, 0.70]</td>
<td>[0.40, 0.60]</td>
<td>[0.10, 0.30]</td>
</tr>
</tbody>
</table>

Table 3: Assessment results between assessment grades and correlation coefficients

<table>
<thead>
<tr>
<th>Grade</th>
<th>Very low</th>
<th>Low</th>
<th>Median</th>
<th>High</th>
<th>Very high</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1 (Ganshu_02)</td>
<td>A_2 (Guizhou_02)</td>
<td>A_3 (Guangxi_01)</td>
<td>A_4 (Guizhou_01)</td>
<td>A_5 (Guangxi_01)</td>
<td>A_6 (Guangxi_02)</td>
</tr>
</tbody>
</table>

Then we define ideal IVFSs for criteria in the ideal alternative from some ideal situation (Zhang et al., 2009) as follows:

$$A^* = \{[0.9, 1], [0.9, 1], [0.8, 0.9], [0.8, 1.0], [0.6, 0.9]\}$$

By applying Eq. 6, we can give the correlation coefficient between an alternative place A_i and the ideal alternative A^* as follows:

$$M(A^*, A_i) = 0.7585, M(A^*, A_i) = 0.4058,$$
$$M(A^*, A_i) = 0.8071, M(A^*, A_i) = 0.5954,$$
$$M(A^*, A_i) = 0.8260, M(A^*, A_i) = 0.8213$$

Therefore, according to the relationship between the assessment grades and the correlation coefficient in Table 1, the assessment grades for all the alternative places are given in Table 3.

From Table 3, we can see that these places for A_1 (Guizhou_01), A_2 (Guizhou_01) and A_6 (Guangxi_02) have very high conservation grades, the place of A_1 (Ganshu_01) has high conservation grade and the places of A_2 (Guizhou_02) and A_4 (Guangxi_02) have median conservation grades. Also, we can give the ranking order of optimal conservation grade:

$$A_1 > A_2 > A_3 > A_4 > A_5 > A_6$$

CONCLUSION

In this study, we have provided the correlation coefficient between IVFSs and an assessment method of soil and water conservation of forest ecosystems by use of the correlation coefficient. The comprehensive assessment method was demonstrated by a practical example of the soil and water conservation, where each alternative place with respect to a set of criteria represented by IVFSs. As expected, the proposed method can be used to give the assessment grades of all the alternative places according to the relationship of between the assessment grades and the correlation coefficient. Besides, the techniques proposed in this paper can provide more useful approaches to efficiently provide the appropriate assessment references for the decision maker or expert. The proposed assessment method not only improves the existing assessment method but also finds a new way for the permeation of IVFS theory to the comprehensive assessment of the soil and water conservation of forest ecosystems. This assessment method proposed in the paper is simple and practical in the assessment process and a practical example shows that proposed method is reasonable and effective.

In the future, we shall continue working in the application of the proposed assessment method to more complex assessment problems and other domains.

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