Longitudinal and Lateral Coupling Control Trajectory Tracking Algorithm for Intelligent Vehicle

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Abstract: Taking into account the strong dynamic coupling effects when conducting longitudinal and lateral coupling control this study presents a trajectory tracking algorithm for intelligent vehicle. The vehicle’s kinematic model and pose error model are established. The expected yaw velocity can be acquired by planning a virtual path between the vehicle mass center and the look-ahead target point in real time. Then, a sliding mode variable structure trajectory tracking controller is designed based on the longitudinal and lateral coupling nonlinear dynamic model of vehicle. The stability of the control system is analyzed using Lyapunov function method. Finally, the interactive combination control dynamic simulation is realized using Matlab and ADAMS. The multi-body dynamics model of the intelligent vehicle is built in ADAMS and the data interface between Matlab and ADAMS is designed. The co-simulation results show that the proposed algorithm can improve the control performance of the system considering vehicle longitudinal and lateral coupling influence.

Key words: Intelligent vehicle, trajectory tracking, longitudinal and lateral coupling, sliding mode control

INTRODUCTION

As an important part of intelligent transportation system, intelligent vehicle is aimed to assist or replace the driver to drive the car in order to reduce traffic accidents and improve the efficiency of road traffic system. For the reason that the research of intelligent vehicle is helpful to enhance the safety and bring convenience to the road users, it has gained much concern worldwide. (Kim and Son, 2011). Intelligent vehicle trajectory tracking control system refers to the automatic steering control and deceleration control. Its main purpose is to keep the vehicle driving along the expected path and ensure the vehicle’s running safety and ride comfort at the same time (Li and Wang, 2007).

At present, the intelligent vehicle control technology have been carried out related research and obtained a lot of valuable research achievements worldwide. Vehicle longitudinal control concentrates on actions such as joining a fleet, splitting from it and regulating the speed to maintain proper spacing between vehicles, maintaining a relatively constant speed. For example, Baek and Song (2009) proposed a set of longitudinal velocity and distance controllers with switching logic and verified them via Hardware-in-the-loop Simulation (HILS). Khan (2013) utilized Monte Carlo simulation and Bayesian decision model to identify cognitive connected vehicle location and distance information to assist collision avoidance advice and adaptive longitudinal control. While vehicle lateral control is mainly concerned with the road tracking ability, that is, how to control the vehicle along lane mark. For example, Liaw and Chung (2008) applied a second-order nonlinear lateral dynamical model to realize the control of vehicle’s lateral dynamics. Shin et al. (2011) designed a modified LMI-based H lateral controller combining the look-ahead and look-down information.

Currently for intelligent vehicle trajectory tracking control, most researchers mainly decouple the vehicle’s longitudinal and lateral movement and design longitudinal controller and lateral controller respectively. However, the longitudinal movement and the lateral movement of vehicle are actually coupled together and it will increase the controller’s deviation which affects the control precision, if the longitudinal control and lateral control are researched separately. Lee et al. (2009) presented a sliding mode control method for wheeled mobile robots considering the vehicle nonlinear and nonholonomic properties. Liu et al. (2011) designed the vehicle coupling variable structure control law based on Backstepping control. Hu et al. (2011) designed a longitudinal and lateral coupling controller based on the position deviation. However, the vehicle’s lateral error which based

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on the position error feedback control can’t tend to be zero. This study solves the problem effectively by means of the method of vehicle yaw rate feedback control. Previous research work presented a trajectory planning method suitable for curved road considering the longitudinal and lateral coupling function on trajectory (Guo et al., 2013).

This study presents an algorithm for intelligent vehicle trajectory tracking task by considering the nonholonomic constraint features and the longitudinal and lateral strong coupling features of the vehicle.

**MATERIALS AND METHODS**

**Kinematics model:** The control object in this study is an unmanned intelligent vehicle with rear wheel drive and front wheel steer. Due to the movement of vehicle has directivity, the position and orientation of vehicle at some point are described using the world coordinate system oxy and the local coordinate system $o_x y_o$ as shown in Fig. 1.

The position of the vehicle centroid in the world coordinate system is presented as ($X_c$, $Y_c$) and the angle between the vehicle’s heading direction and x coordinate is $\theta_c$. In the word coordinate system oxy, the vehicle’s kinematic model is defined as:

$$
\begin{bmatrix}
X_c \\
Y_c \\
\theta_c
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_c & 0 \\
\sin \theta_c & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
y_c \\
x_c
\end{bmatrix}
$$

(1)

where, $y_c$ and $x_c$ denotes the vehicle centroid velocity and yaw velocity, respectively.

As shown in (1), the position of the vehicle is determined by the vehicle centroid velocity and yaw velocity. In other words, if the change of the vehicle centroid velocity is known, the vehicle trajectory can be determined by controlling the yaw velocity.

The vehicle’s pose error is not the lateral deviation or the orientation deviation of vehicle centroid, but the deviation of a certain point $P$ $(X_p, Y_p)$ in front of the vehicle, which is called the look-ahead point. The distance between the vehicle centroid and the look-ahead point is defined as lookout distance. As shown in Fig. 1, assume that the look-ahead distance of the vehicle is $x_o$, in the local coordinate system of the vehicle, the distance between the look-ahead point and the tangent of road is called lateral deviation $y_c$ and the angle between the center line of the vehicle and the tangent of the road is called orientation deviation $\theta_c$. According to the geometrical relationship of the world coordinate system oxy and the local coordinate system $o_x y_o$, the pose error $p_o$ of intelligent vehicle in the local coordinate system can be defined as:

$$
p_o = \begin{bmatrix}
x_o \\
y_o \\
\theta_o
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_c & \sin \theta_c & 0 \\
-\sin \theta_c & \cos \theta_c & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
X_p - X_c \\
Y_p - Y_c \\
\theta_p - \theta_c
\end{bmatrix}
$$

(2)

where, $(X_p, Y_p, \theta_p)$ denotes the relative position between the vehicle and the look-ahead point in the word coordinate system oxy.

To realize the trajectory tracking of the target path, this study plans virtual path between the vehicle centroid and the desired position of the vehicle. Takes the virtual path $y(\alpha)$ as a cubic polynomial curve:

$$
y = n_1 + n_2 \alpha + n_3 \alpha^2 + n_4 \alpha^3
$$

(3)

and satisfies the following boundary condition:

$$
\begin{align*}
y(0) &= 0 \\
y(\alpha_t) &= y_c \\
\left. \frac{dy}{d\alpha} \right|_{\alpha=\alpha_t} &= \psi
\end{align*}
$$

(4)

where, $\psi$ is the curvature of the vehicle traveling trajectory, $\psi = \omega / v_c$.

Then the virtual path can be represented as follows:
If the intelligent vehicle centroid can track the virtual path $y(x)$ with no deviation, the position of the vehicle centroid on the path equation is $(x, y)$ and the velocity of the vehicle is $v_c$. The orientation of the velocity and the tangent of the curve are consistent, namely, the driving curvature $\psi$ of the vehicle traveling trajectory equals the curvature $\gamma$ of the curve at the position $(x, y)$. Therefore, the ideal yaw velocity $\omega$ can be gotten as follows:

$$\omega = v_c \gamma$$

(6)

Differentiating ideal yaw velocity yields the changing rate of the ideal yaw velocity:

$$\omega = v_c \gamma + v_y \gamma$$

(7)

When the vehicle tracks the target path successfully, the change rate of the ideal yaw velocity represents the change tendency of yaw velocity at this moment. Then the desired yaw rate $\omega_d$ can be represented as:

$$\omega_d = \omega_0 + \alpha \omega |_{\omega_0} = \omega_0 + \alpha \left( \frac{v_y \omega_0}{v_c} + 6 v_y \frac{x_{i}}{x_{i}^y} \right)$$

(8)

where, $\alpha$ denotes the proportionality coefficient.

**Dynamics model:** Intelligent vehicle is a strong coupling nonlinear system with unmatched uncertainties. In order to research the characteristics of the longitudinal motion, lateral motion and yaw motion easily in the process of trajectory tracking, commonly, the vehicle simplified to three degrees of freedom model when travels on a small curvature and flat road (Hu et al., 2011). At this situation, the role of the suspension and the differences of sideslip property between the left and right tires are ignored. According to the three degrees of freedom vehicle model, the longitudinal and lateral coupling dynamics model can be expressed as:

$$\begin{align*}
    v_x &= -f_x g + \frac{C_L v_y^2}{m} + v_y \omega_0 - 2C_d \frac{v_x}{m} \omega_0 + \frac{F_z}{m} \\
    v_y &= \frac{2(C_L + C_T) v_x}{m} + \frac{2(C_d a - C_d b)}{m} v_y - v_x \omega_0 - \frac{2C_d}{m} \omega_0 + \frac{C_T v_y^2}{m} \\
    \omega_0 &= \frac{C_d a - C_d b}{I_v} v_y \omega_0 - \frac{C_T a^2 + C_T b^2}{I_v} v_y \omega_0 - \frac{C_d a}{I_v} \delta_i
\end{align*}$$

(9)

where, $m$ is the vehicle total mass, $v_c$ is the longitudinal velocity of vehicle, $v_y$ is the lateral velocity of vehicle, $F_z$ is driving/braking force, $I$ is the vehicle rotational inertia, $c_d$ is the coefficient of longitudinal air resistance, $c_d$ is the coefficient of lateral air resistance, $\delta_i$ is the front wheel steering angle, $f_x$ is the coefficient of rolling resistance, $c_s$ is the cornering stiffness of the front tires, $c_s$ is the cornering stiffness of the rear tires, $a$ is the distance from centroid to the front axle, $b$ is the distance from centroid to the rear axle.

The vehicle longitudinal and lateral coupling kinetic model can be simplified as:

$$\begin{align*}
    \dot{v}_x &= f_x + g_i \delta_i + g_i F_z \\
    \dot{v}_y &= f_y + g_i \delta_i \\
    \dot{\omega}_0 &= f_0 + g_i \delta_i
\end{align*}$$

(10)

Where:

$$\begin{align*}
    f_x &= -f_x g + \frac{C_L v_y^2}{m} + v_y \omega_0 \\
    f_y &= \frac{2(C_L + C_T) v_x}{m} + \frac{2(C_d a - C_d b)}{m} v_y - v_x \omega_0 - \frac{2C_d}{m} \omega_0 + \frac{C_T v_y^2}{m} \\
    f_0 &= \frac{C_d a - C_d b}{I_v} v_y \omega_0 - \frac{C_T a^2 + C_T b^2}{I_v} v_y \omega_0 - \frac{C_d a}{I_v} \delta_i
\end{align*}$$

Controller design: Assume that the desired longitudinal acceleration and the desired longitudinal velocity are given information. To reduce the chattering of the control system and improve the robustness of the system, this study designs the sliding mode variable structure controller using the exponential reaching law method. The purposes of designing sliding mode controller are as follows. On the one hand, the vehicle can follow the desired yaw velocity stably by controlling the vehicle's yaw velocity, which enables the vehicle to track the desired trajectory (DeCarlo et al., 1988). On the other hand, the vehicle can track the desired longitudinal velocity stably by controlling the vehicle's longitudinal velocity.

Define the sliding mode switching function as follow (Yang and Kim, 1999):

$$\begin{align*}
    s_i &= v_y - v_y^d \\
    s_{i+1} &= v_x - v_x^d
\end{align*}$$

(11)

where, $v_x$ and $v_y$ denotes the current yaw velocity and the desired yaw velocity of the vehicle, respectively. $v_x$ and $v_y$ denotes the current longitudinal velocity and the desired longitudinal velocity of the vehicle, respectively.

Differentiating (11) and substituting (8) into them, yields the following expression:
\[ s_i = \alpha_i - \alpha_u = f_i + g_i \Delta_i - \alpha_u \]
\[ s_i = v_i - v_p = f_i + g_i \Delta_i + g_i F_i - v_p \quad (12) \]

When designing the sliding mode controller, the control law is normally composed of equivalent control and switching control (Bessa et al., 2008). Equivalent control helps to keep the system state on the sliding surface. While switching control helps to compel the state of the system to slide along the sliding surface, that is to say:

\[ s_i \leq 0 \quad (13) \]

Set \( s_i = 0 \) and the equivalent control of steering angle \( \delta_{el} \) can be represented as:

\[ \delta_{el} = (-f_i + \alpha_i)/g_i \quad (14) \]

Here the switching control of steering angle \( \delta_{sw} \) is designed using the exponential reaching law method, yields:

\[ \delta_{sw} = (-\epsilon_i \text{sgn}(s_i) - k_i s_i)/g_i \quad (15) \]

where, \( \epsilon_i > 0 \) and \( k_i > 0 \).

The controlling variable \( \delta_i \) is:

\[ \delta_i = \delta_{el} + \delta_{sw} = (\alpha_i - f_i - \epsilon_i \text{sgn}(s_i) - k_i s_i)/g_i \quad (16) \]

Similarly, set \( s_i = 0 \) and equivalent control of driving/braking force \( F_{sw} \) can be represented as:

\[ F_{sw} = (-f_i - g_i \Delta_i + v_i)/g_i \quad (17) \]

The switching control of driving/braking force \( F_{sw} \) based on the exponential reaching law is:

\[ F_{sw} = (-\epsilon_i \text{sgn}(s_i) - k_i s_i)/g_i \quad (18) \]

where, \( \epsilon_i > 0 \) and \( k_i > 0 \).

The controlling variable \( F_i \) is:

\[ F_i = F_{sw} + F_{sw} \quad (19) \]

To weaken the tremble of the system on the slide mode, the signum functions \( \text{sgn}(s_i) \) in (16) and (19) are replaced by saturation functions \( \text{sat}(s_i/\Delta_i) \). They are defined as:

\[ \text{sat}(s_i/\Delta_i) = \begin{cases} 
\frac{s_i}{\Delta_i} & |s_i| < \Delta_i \\
\text{sgn}(s_i/\Delta_i) & |s_i| > \Delta_i
\end{cases} \quad (20) \]

where, \( i = 1, 2, \Delta_i \) denotes the thickness of the boundary layer.

To analysis the stability of the control system, the following Lyapunov function is constructed:

\[ J = \frac{1}{2} s_i^2 + \frac{1}{2} e_i^2 \quad (21) \]

Differentiating (21) yields:

\[ J = s_i \dot{e}_i + s_i \dot{e}_i \quad (22) \]

Substituting 11, 12, 16 and 19 into 22, yields:

\[ J - s_i (f_i + g_i \Delta_i - \alpha_i) + s_i (f_i + g_i \Delta_i + g_i F_i - v_p) = s_i (-\epsilon_i \text{sat}(s_i) - k_i s_i) + s_i (-\epsilon_i \text{sat}(s_i) - k_i s_i) \leq -k_i s_i^2 - \epsilon_i |s_i| - k_i s_i^2 - \epsilon_i |s_i| < 0 \quad (23) \]

Analysis from the above expression, the control system has global stability under the designed control law (16) and (19) according to the Lyapunov stability criteria.

**RESULTS**

**Design of the co-simulation system:** To verify the performance of the designed controller, a co-simulation control system is built. The co-simulation process is realized by combining Matlab/Simulink and ADAMS. A desired trajectory module and a sliding mode controller module were established in Matlab/Simulink. ADAMS was used to establish a multi-body dynamic model of intelligent vehicle in Adams/View. In order to simulate the trajectory tracking control system, the ADAMS model should be translated into S-function in Matlab/Simulink (Lee et al., 2012). Co-simulation is the process of simulating a system where two or more separate simulation programs are simultaneously used to model various aspects of the system and these simulation programs communicate during the process (Li and He, 2011).

The vehicle model has to communicate with Simulink for co-simulation. The vehicle model consists of the suspension system, steering system, tire and road surface spectrum. Two input variables and six output variables were selected for the communication. The two input variables are the steering angle \( \delta_i \) and the driving/braking force \( F_i \). The six output variables are \( X_p, Y_p, \theta_p, v_m, v_i \), and
ω. The interactive combination control dynamic simulation was realized through designing the data interface between Matlab and ADAMS, as shown in Fig. 2.

Parts of the intelligent vehicle dynamics model parameters are shown in Table 1.

### Simulation results

The desired trajectory was constructed by two straight lines and a five times polynomial curve. The look-ahead distance \( x_0 \) was set to be 3 m and the proportionality coefficient \( \alpha \) in (8) was set to be 0.05. The desired longitudinal initial velocity of the vehicle was 5 m sec\(^{-1}\). The values controller parameters were set as follows: \( \varepsilon_1 = 0.2, \varepsilon_2 = 0.2, k_1 = 1, k_2 = 1, \Delta_1 = 0.2, \Delta_2 = 0.2 \). The initial value of the longitudinal displacement and the longitudinal velocity was set to be 0 m and 6 m sec\(^{-1}\), respectively. The initial value of the lateral displacement and lateral velocity was set to be 2 m and 0.2 m sec\(^{-1}\), respectively. The initial value of the yaw angle and the yaw velocity was set to be 0 rad and 0.2 rad sec\(^{-1}\), respectively. The desired longitudinal acceleration of the vehicle is shown in Fig. 3. The simulation results are shown in Fig. 4-7.

Figure 4 compares the trajectory tracking result using the designed controller. It is observed that the vehicle can track the desired trajectory well throughout the process. As can be seen from the partial enlarged detail, the vehicle realizes the tracking for the desired trajectory after 20 m of the longitudinal displacement.

Figure 5 shows the curves of the real yaw velocity of the vehicle and the desired yaw velocity. When the yaw velocity of the desired path is zero, the real yaw velocity of the vehicle converges to zero quickly. It is observed that the yaw velocity of the vehicle performs tracking well and the vehicle has good control stability and safety.

The tracking error states and deviation are shown in Fig. 6a shows the longitudinal velocity tracking comparison in the process of simulation. Figure 6b presents the longitudinal velocity tracking deviation. It is observed that the longitudinal velocity deviation can quickly converge to zero. Figure 6c presents the lateral
deviation curve. There exists a certain variation during the vehicle crossing from the line path to the curve path. However, the lateral deviation remains in the range of [-0.1 0.1] m. Figure 6d displays the orientation deviation curve. It asymptotically converges to zero after 5 sec and there is no variation during the whole simulation.

Figure 7 shows the control input variables (driving/braking force and front wheel steering angle) of during the process of trajectory tracking. Because of the change of the longitudinal velocity during the time period 15-20 and 35-50 sec, the absolute value of the driving/braking force of the intelligent vehicle increases. During the time period 20-60 sec, the front wheel steering angle changes from positive to negative maximum because of the curvature transformation.

Analysis from Fig. 4-7, the proposed controller has a good performance. It is able to correct the tracking deviation quickly and track the desired trajectory with different curvature accurately.

Fig. 6(a-d): Longitudinal and lateral tracking states and deviation, (a) Comparison of longitudinal velocity. (b) Longitudinal velocity deviation and it can quickly converge to zero. (c) Lateral deviation remains in the range of [-0.1 0.1] m. (d) Orientation deviation asymptotically converges to zero after 5 sec and there is no variation during the whole simulation.
Fig. 7(a-b): Input variables of the controller, (a) The absolute value of the driving/braking force increases because of the change of the longitudinal velocity during the time period 15-20 and 35-50 sec and (b) During the time period 20-60 sec, the front wheel steering angle changes from positive to negative maximum because of the curvature transformation.

CONCLUSION

This study has presented a control method of the intelligent vehicle trajectory tracking considering the nonholonomic constraint features and the longitudinal and lateral coupling features. This study deduced the vehicle’s desired yaw velocity based on the vehicle’s kinematic model and pose error model and then a sliding mode variable structure trajectory tracking controller is designed based on the longitudinal and lateral coupling nonlinear dynamic model of vehicle. The algorithm makes vehicle longitudinal axis tending to the tangent direction of the desired trajectory through controlling the vehicle yaw velocity and makes the vehicle tracking the expected longitudinal velocity through controlling the driving/braking force and the front wheel steering angle. A multi-body dynamics model of the intelligent vehicle is built in ADAMS. The interactive combination control dynamic simulation between Matlab/Simulink and ADAMS is realized through designing the data interface between Matlab and ADAMS. The effectiveness of the proposed algorithm is verified by the simulation.

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