Introduction Timing Model under Uncertain Technology Evolution Amplitude

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Abstract: Technology introduction timing is the key to the strategic decision of technology, and it makes an important contribution for the enterprises to gain competitive advantage through technology adoption. Thus, the study on optimization model and influencing factors of enterprise technology introduction timing has important practical significance. This study aims at insufficiencies of existing theories and providing a thorough approach to deeply depict technology evolution process. According to different evolution amplitude, technical progress is divided into technical breakthrough and technical improvement to deeply depict technology evolution process. On this basis, this study first proposes a conceptual model and an optimization model using dynamic programming method of technology introduction timing decision; then relations between optimal introduction time (technical efficiency) and discount rate, technology emergence velocity, technology improvement, current technical efficiency, fixed introduction cost as well as output elastic coefficient and the validity of the model is finally verified by numerical simulation.

Key words: Technology evolution, technology introduction, optimal timing, dynamic programming

INTRODUCTION

The study on optimization model and influencing factors of enterprise technology introduction timing has important practical significance for both technology providers to make sales strategy and technology adopter to invest at the right time. Timing is also considered a kind of rent, especially in uncertain environment filled with turbulence, mutability and randomness. Inappropriate introduction timing will lead to value loss of the enterprises. For the most part, as Dierickx points out, strategy timing is often more important than resources and competence, otherwise the enterprises will suffer time diseconomy. Nowadays, research about technology introduction timing at home and abroad has acquired a series of achievements and there are two main conclusions: the enterprises may fail to keep up with new techniques which diffuse over time (Meade and Islam, 2003; Comina and Hobijn, 2004; Golob and Regan, 2002; Anderson and Newell, 2004; Gander, 2003) and introduction delay can be explained from aspects of profitability uncertainty and uncertainty of technology itself (Fisher et al., 2000; Huggett and Osypina, 2001; Dougan and Bronson, 2003). As a result, research on how uncertain factors can affect introduction timing is crucial in explaining introduction behaviors of the enterprises.

In general, existing researches about technology uncertainty is mainly unfolded on that of the arrival of new technology and the improvement of technical efficiency which are regarded as single random variables to establish the model. For example, Balce and Lippman assume technical evolution process to be multi-continuous technological innovation, the time of which is random. Farzin et al. (1998) assumes random technical sequence of uncertain profitability is hidden in technical evolution process. Doraszelski (2001, 2004) studies multiple technology introduction based on the research by Farzin. These studies suggest that an enterprise will import the technology if its efficiency exceeds a specific threshold value, or it will be skipped. However, they are not pretty good descriptions of factors affecting technological uncertainties during technical evolution process and relationship between introduction timing and uncertain factors (Dai and Xu, 2007). To this end, according to technology evolution amplitude, technical progress is subdivided into technical breakthrough and technical improvement in this study, in order to further characterize technical evolution process and study the decision criterion and influencing factors of technology introduction timing of an enterprise.

MODELING

Modeling approach and conceptual model: Nowadays, when technology is developing rapidly, technology introduction exhibits obvious hysteretic characteristic rather seeking the latest technology. The enterprises take a ‘wait-and-see’ approach, largely because the latest
technology still has room for improvement. Therefore, different enterprises use technology of different phases which presents with coexistence of new and old technologies in the market.

Technology of different phases will affect introduction decision. With the previous study, this study attempts to give technical evolution process a more thorough study, classifying technical progress into technical breakthrough and technical improvement in accordance with different amplitude. In particular, in terms of the theory that the next improvement depends on the time since last innovation, this study extends the model about uncertain process of technology by Farzin. The velocity and possibility of technology’s development are indeterminate in both models.

The investment in a new technique will not be paid off, if the technology is improving rapidly. So introducing decision is irreversible. Sustainable technical progress complicates decision problems, because an enterprise has to balance two types of cost, with error cost of early introduction on one aspect and waiting opportunity cost of expectation for a more efficient technology on the other. Introduction decision relies on the enterprise’s expectation for velocity and degree of technical progress. Thus the concept map of expected introduction profit is shown in Fig. 1.

An enterprise makes introduction decision in light of the maximum expected profits from the technology. Total expected profits consists of four parts: The first part is the profit without adoption; the second part indicates the profit from choosing between introducing and waiting as a technical breakthrough occurs; the third part indicates the profit from choosing between introducing and waiting as a technical improvement occurs; the last part is that of invariant technology. And the latter three parts are discounted at interest rate.

Mathematical model: With regard to production, interest rate and technological investment, basic hypotheses are put forward as follows (Farzin et al., 1998):

- **Hypothesis 1:** In production input, fixed unit cost and product price are $\omega$ and $p$, respectively; moreover, with $p$ unchanged, the enterprise stays competitive after introducing a new technology.
- **Hypothesis 2:** Interest rate of bank loans is $r$ and fixed cost of the technology is $I$.

To construct an optimization model of technology introduction timing, the following hypotheses are brought forth:

- **Hypothesis 3:** Technology introduction happens only once and production will be carried out with the technology all the time.
- **Hypothesis 4:** There are two uncertain factors in technology evolution process, namely the emergence and efficiency improvement of new technology.

The optimization model is built on the basis of above modeling approach and conceptual model. At the very start of technology evolution process, $t = 0$ and technical efficiency $\theta_0$ is engaged into the production. Technical efficiency is denoted as $\theta$ at time $t$, when the enterprise decides to introduce the technology or wait. If introduces, purchase expense will be $I > 0$ and instantaneous profit will be $\pi(\theta)$ with continuously increasing $\pi$. Profit $P$ derived from adoption equals the remainder after purchasing expense has been deducted from discounted value of all revenue. It is referred to:

$$P = \int_0^\infty \pi(\theta)e^{-rs}ds - I = \frac{\pi(\theta)}{r} - I$$

where, $r > 0$ is interest rate.

Suppose that a breakthrough or an improvement or neither occurs in a short interval $\Delta t$. Technical efficiency evolves following the formula below:

$$d\theta = \frac{U\lambda}{\mu}.\theta.\Delta t$$

**Fig. 1:** Concept map of technology introduction decision.
where, U=F(U) and V=G(v, τ) are separately a breakthrough and an improvement. Suppose U>0 and V>0 to avoid technical degradation. τ indicates elapsed time since the last breakthrough and current improvement.

Velocity and degree of an improvement determined by τ rest with elapsed time after the final breakthrough, applying arrival rate μ(τ) and distribution function G(v, τ) of the improvement.

Take V(θ, τ) as total expected profits with technical efficiency θ and improvement time τ. Total expected profits consists of four parts: the first part is the profit which is denoted by π(θ)Δt, of available technology from t to τ+Δt; the second part signifies the profit from choosing between introducing and waiting when a technical breakthrough occurs in [t, τ+Δt], introducing profit is π(θ+u)Δt while waiting profit is V(θ+u, 0) and τ is reset to 0 as long as the technical breakthrough comes with certain probability μ(τ); the third part signifies the profit from choosing between introducing and waiting with a technical improvement in [t, τ+Δt], introducing profit is π(θ+τ)Δt while waiting profit is V(θ+τ, τ+Δt) and technical improvement comes with certain probability μ(τ+Δt) which is associated with τ; the last part is that of invariant technology, namely V(θ, τ+Δt) which comes with probability (1-λΔt-μ(τ+Δt)Δt). And the later three parts have to be discounted. Consequently, the optimization model is represented by a mathematical one, as shown below:

\[
V(\theta, \tau) = \pi(\theta) \Delta t + \frac{1}{1+\lambda \Delta t} \left( \lambda \Delta t \int_{0}^{\tau} \max \left( \frac{\pi(\theta+u)}{r} - 1, V(\theta+u, 0) \right) dF(u) + \mu(\tau) \int_{0}^{\tau} \max \left( \frac{\pi(\sigma+\tau)}{r} - 1, V(\theta+\tau, \tau) \right) dG(v, \tau) \right) + (1-\lambda \Delta t-
\]

MODEL ANALYSIS

**Profit model:** Add and subtract V(θ, τ+Δt) and divide by Δt to read just the equation, as noted below:

\[
\frac{\Delta V(\theta, \tau)}{\Delta t} + (\tau + \lambda + \mu(\tau)) V(\theta, \tau) = \pi(\theta) + \int_{0}^{\tau} \max \left( \frac{\pi(\theta+u)}{r} - 1, V(\theta+u, 0) \right) dF(u) + \mu(\tau) \int_{0}^{\tau} \max \left( \frac{\pi(\sigma+\tau)}{r} - 1, V(\theta+\tau, \tau) \right) dG(v, \tau)
\]

The introduction timing is optimal with solution θ*.

It is considered the optimal timing of introduction if and only if θ≤θ*(τ) and there exists a unique θ*(τ) for any τ≥0.

This assumption makes intuitive sense, because the enterprises prefer to appropriate techniques rather than advanced ones.

If technical efficiency is θ*(τ), then for any U>0 or V>0, the enterprises will apply the technology. Then the profit equation is:

\[
\frac{\partial V(\theta^*(\tau), \tau)}{\partial \tau} + (\tau + \lambda + \mu(\tau)) V(\theta^*(\tau), \tau) = \pi(\theta^*(\tau)) + \int_{0}^{\tau} \max \left( \frac{\pi(\theta^*(\tau)+u)}{r} - 1, V(\theta^*(\tau)+u, 0) \right) dF(u) + \mu(\tau) \int_{0}^{\tau} \max \left( \frac{\pi(\theta^*(\tau)+\tau)}{r} - 1, V(\theta^*(\tau)+\tau, \tau) \right) dG(v, \tau)
\]

As for θ = θ*(τ), introducing and waiting is thought to be equivalent, so:

\[
V(\theta^*(\tau), \tau) = \frac{\pi(\theta^*(\tau))}{r} - 1
\]

**Boundary conditions:** To get termination condition of the equation, assume arrival rate and improvement distribution act smoothly in the end.

Now that the latest breakthrough varies greatly, assume arrival rate and improvement distribution approach the limit as time goes on that is, limU→∞μ(τ)=μ and limG(v, τ)=G(v), so θ*(τ) is finally stable which is verified by the following proposition.

Since, the latest breakthrough varies greatly, delay level approaches a limit as time goes on, namely limθ→∞θ−θ*, in which θ* is the solution of the Eq. 5:

\[
(\tau + \lambda + \mu(\tau)) \left( \frac{\pi(\theta)}{r} - 1 \right) = \pi(\theta) + \int_{0}^{\tau} \max \left( \frac{\pi(\sigma+\tau)}{r} - 1, V(\theta+\tau, \tau) \right) dG(v)
\]

Suppose improvement distribution and arrival rate are constant over time, a model considering a breakthrough and an improvement is equal to the one only with a breakthrough.
For all \( r > 0 \), if \( G(v; \tau) = 0 \) and \( \mu(\tau) = \mu \), then without loss of generality, let \( \mu = 0 \). If scale is the only difference between a breakthrough and an improvement, a model involving both of those two is therefore equal to the one only considering a breakthrough. The Eq. 5 is simplified to:

\[
(r + \lambda) \left( \pi(\theta) \frac{\pi(\theta) + u}{r} - 1 \right) = \pi(\theta, u) \left( \pi(\theta) \frac{\pi(\theta) + u}{r} - 1 \right) \mathrm{d}F(u) \quad (6)
\]

In case technical efficiency driven by a breakthrough or an improvement exceeds a certain threshold, the enterprises will bring in new technology.

If scale is the only difference between a breakthrough and an improvement, a model considering both of those two is equal to the one involving only a breakthrough. \( \tau \) is not a state variable in the profit equation. It is found the right time for introduction if and only if.

**Model embodiment**: Embody distributions of technical breakthrough and technical improvement to solve the equation.

**Technical breakthrough**: It is necessary to embody the internal process of technical evolution to solve the equation. To demonstrate that technology evolves in limited steps, suppose a technical breakthrough is a random variable \( U \) distributed on an interval \([u, \bar{u}]\) (\( u \geq 0 \)):

\[
U = \mathcal{U}(\bar{u} - u) + u
\]

where, \( \bar{u} \) complies with \( \beta \)-distribution with degree of freedom \( u_c \) and \( u_s \). If \( u_s > u_c \), the distribution of \( U \) is inclined to the right. As a result, it is probable to make a technical breakthrough with high efficiency.

**Technical improvement**: Suppose a technical improvement is a random variable \( V \) distributed on an interval \([v, \bar{v}]\) (\( v \geq 0 \)):

\[
V = \mathcal{V}(\bar{v} - v) + v
\]

where, \( \mathcal{V} \) is submitted to \( \beta \)-distribution with degree of freedom \( v_s(\tau) \) and \( v_c(\tau) \). If \( v_c(\tau) < v_s(\tau) \), the distribution of \( V \) is inclined to the left which shows it far more likely to achieve a slight improvement while if \( v_c(\tau) > v_s(\tau) \), the distribution is inclined to the right which shows a large improvement.

The arrival time of technical improvement \( \tau \) can be categorized as three cases: time invariant improvement distribution, reduction of technological return and learning effect.

In the case of time-invariance, the arrival time is obtained by setting \( v_s(\tau) = v_1 \) and \( v_c(\tau) = v_2 \) (\( v_1 < v_2 \)); as regards reduction of technological return, owing to the decrease of expected return over time, it is obtained by concave time trajectory, making \( v_s(\tau) \) decrease while \( v_c(\tau) \) increase with \( \tau \) and particularly:

\[
v_s(\tau) = \begin{cases} 
\frac{(v_1 - v_s) \cos(\pi v_1 \tau) + v_1 + v_2}{2}, & \text{if } \tau \leq \bar{\tau} \\
\frac{v_1}{2}, & \text{if } \tau > \bar{\tau}
\end{cases}
\]

\[
v_c(\tau) = \begin{cases} 
\frac{(v_1 - v_s) \cos(\pi v_1 \tau + 1) + v_1 + v_2}{2}, & \text{if } \tau \leq \bar{\tau} \\
\frac{v_1}{2}, & \text{if } \tau > \bar{\tau}
\end{cases}
\]

As regards learning effect, it is derived from hump-shaped time trajectory, so:

\[
v_s(\tau) = \begin{cases} 
\frac{(v_2 - v_s) \cos(2\pi \tau \bar{\tau}) + v_1 + v_2}{2}, & \text{if } \tau \leq \bar{\tau} \\
v_1, & \text{if } \tau > \bar{\tau}
\end{cases}
\]

and:

\[
v_c(\tau) = \begin{cases} 
\frac{(v_2 - v_s) \cos(2\pi \tau \bar{\tau} + 0.5) + v_1 + v_2}{2}, & \text{if } \tau \leq \bar{\tau} \\
v_1, & \text{if } \tau > \bar{\tau}
\end{cases}
\]

which implies expected return increases before decreases, contrary to the case of return reduction:

\[
\mu(\tau) = \begin{cases} 
\frac{\mu \cos(\pi \tau \bar{\tau}) + \bar{\mu} + \mu}{2}, & \text{if } \tau \leq \bar{\tau} \\
\mu, & \text{if } \tau > \bar{\tau}
\end{cases}
\]

With respect to learning effect, hump-shaped time trajectory is applied:

\[
\mu(\tau) = \begin{cases} 
\frac{\mu \cos(2\pi \tau \bar{\tau} + 0.5) + \bar{\mu} + \mu}{2}, & \text{if } \tau \leq \bar{\tau} \\
\mu, & \text{if } \tau > \bar{\tau}
\end{cases}
\]

**MODEL SIMULATION AND ANALYSIS**

The simulation is carried out using MATLAB 7.0 in order to intensively study the relationships of introduction timing and the optimal decision criterion with other parameters, verifying effectiveness of the model and influence of all parameters on technology introduction timing and then validate the conclusion about optimal decision criterion.
**Parameter setting:** Based on Fazin study, assume an enterprise planning adopting a new technology manufactures similar products following the production function $y = f(x) = x^\theta$. If $p$ and $w$, respectively the fixed price of output and input, the profit function of the enterprise is:

$$\pi(\theta) = \max_{x} py - wx = (1 - \alpha)(\alpha/w)^{(1-\alpha)} (p\theta)^{(1-\alpha)}$$

The parameters and the ones impacting technology evolution are set as shown in Table 1.

**Simulation results and analysis:** Treat optimum technical efficiency $\theta^*$ as the function of a variable and other parameters fixed, the simulation results of functions involving technical evolution parameters are described below in Fig. 2-8:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Parameters</th>
<th>Value</th>
</tr>
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<td>$\alpha$</td>
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</tr>
<tr>
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<td>$\gamma_3$</td>
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<tr>
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<td>$\theta_0$</td>
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</table>

![Graph 1](image1)

**Fig. 1: Graph 1**

**Fig. 2: $\theta^*$ as function of $\lambda$**

**Fig. 3: $\theta^*$ as function of $\bar{u}$**

**Fig. 4: $\theta^*$ as function of $u$**

**Fig. 5: $\theta^*$ as function of $r$**

**Fig. 6: $\theta^*$ as function of $I$**

**Fig. 7: $\theta^*$ as function of $\theta_0$**
of arrival rate. Figure 2 suggests $\theta^*$ increases with $\lambda$. Now a higher arrival rate is more likely to compensate a more efficient technology that is instantly available, the enterprise is motivated to delay introduction which means the motive grows with amplitude of technical progress

- **$\theta^*$ as function of $u$:** The only possibility of high efficiency is enough to cause some delay. Regard $\theta^*$ as a function of $\bar{u}$ and $u$ and expected breakthrough is variable. As can be seen from Fig. 3 and 4, both a higher breakthrough expectation and a higher standard deviation can raise waiting expectation.

- **$\theta^*$ as function of $r$:** The enterprise imports the technology in advance owing to a high discount rate. Figure 5 indicates that the smaller $r$ is, the less the hysteretic value and the higher the discount rate. As profit from efficient technology reduces with a higher discount rate, waiting expectation is lowered.

- **$\theta^*$ as function of $I$:** The enterprise imports the technology in advance owing to an increase in fixed investment. Figure 6 shows $\theta^*$ increases with $I$. It is apparent that fixed installation cost of a new technology must balance with high technical efficiency.

- **$\theta^*$ as function of $\theta_d$:** Introduction delay of the enterprise on the edge of technical efficiency is probably greater than that of the enterprise using backward technology. As noted in Fig. 7, the more effective the existing technology, the higher the hysteretic value.

- **$\theta^*$ as function of $\alpha$:** In Fig. 8, $\theta^*$ increases with the larger value of $\alpha$, in other words, the higher the productivity is, the slower is the pace of technology introduction. However, for lower $\alpha$, hysteretic value increases with the decrease of output elasticity. This is known as "inefficient technology trap".

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**CONCLUSION**

This study subdivides technical progress into technical breakthrough and technical improvement in accordance with different evolution amplitude and brings forward an optimization model using dynamic programming method so as to acquire the decision criterion and effects of influencing factors of technology introduction timing through model analysis and numerical simulation. Specific conclusions are detailed as follows:

1. In case technical efficiency driven by a breakthrough or an improvement exceeds a certain threshold, the enterprises will bring in the technology.
2. The enterprises will adopt the technology in advance due to the increase of discount rate and fixed investment; introduction delay of the enterprises on the edge of technical efficiency is probably greater than that of the enterprises with backward technology; high efficient enterprises are more apt to slow down technology introduction than inefficient ones.

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