Water-table Control Using Ordinary Kriging in the Southern Part of Cameroon

Jorelle Larissa Meli’i, Dieudonné Bisso, Philippe Nouck Njadjock, Théophile Ndougsa Mbarga, Alain François Banga and Eliézer Manguelle-Dicoum

Department of Physics, Department of Earth Sciences, Faculty of Science, University of Yaounde I, Cameroon
Department of Physics, Advanced Teachers Training College, University of Yaounde I, Cameroon
Institute for Geology and Mining Research, Cameroon
Cameroon Academy of Science, Cameroon

Abstract: In this study, application of the kriging technique for the spatial analysis of groundwater levels is shown. The data set consists of measured elevations of the water table at about 70 points during the last five years (2009-2012) in Sangmelima region (South-Cameroon). From these measurements, an experimental variogram was constructed to characterize the spatial variability of water level. Pentaspherical, spherical, exponential, power, linear and gaussian variogram models were fitted to the experimental variogram. The model having the least error 1.77 and 0.001 according to the cross validation criteria was selected by comparing the observed water-table values with the values predicted by empirical variograms. It was determined that, linear model is the best fitted model for the studied area and can be used as tool to conduct desertification control process in this equatorial zone.

Key words: Water-table, geostatistics, groundwater levels, semivariogram, kriging, Cameroon

INTRODUCTION

Many researches have worked on development of water table (Dingman, 2002; Maurer et al., 2002; Wolock, 2003; Fan et al., 2007). Management of this concept is very important to meet the increasing demand of water for domestic, agricultural and industrial use. Various management measures as discharge and industrial implementation zones, require the spatial behaviour of water-table. According to Issaks and Srivastava (1989) and Kunar and Ramadevi (2006), Kriging is an optimal and unbiased spatial method used to evaluate regionalized variables at unsampled locations using an initial data set values and various variogram models such as: pentaspherical, spherical, exponential, gaussian, power, linear, etc. As the selected model influences the prediction of the unknown values, this study is an attempt to determine which theoretical variogram, can give acceptable results to predict the water-table values based on the water-table observations for the last three years in Sangmelima (South-Cameroon) region.

MATERIALS AND METHODS

Description of the study area: The Sangmelima city area is located from longitudes 11°57' to 12°01' and latitudes 2°54' to 2°58' in the southern region of Cameroon, on the northern part of the Congo craton (Fig. 1-3).

The study area has equatorial humid climate characteristics. Annual average minimum and maximum temperatures are 28 and 40°C, respectively while annual average temperature is 34°C. Annual average relative humidity is 81.6%. Rainfall is observed in all seasons. Maximum rainfall is often observed in the time-period from June to October, while minimum rainfall is from December to March. Average annual rainfall is about 150.2 cm. Drainages are intermittent and structurally controlled by foliation and faulting. The main water source of the region is the Afamba River belonging to the Nyong basin system (Fig. 1). The electrical conductivity of the river water ranges from 0.56 to 0.73 dS m⁻¹ and the corresponding range in sodium absorption ratio is from 0.65 to 2.0. There are more than 60 observation wells in this region, geologically covered by various rocks types such as gneiss, shales, dolerite, gabbro and peridotites (Klitgord and Schouten, 1986; Fairhead, 1988; Guinard and Maurin, 1992; Manguelle-Dicoum et al., 1992; Meli’i et al., 2011, 2012). According to the same authors, laterrite, gravel, granite, clay and fractured granite structures are also observed in this region. According to the population encountered in this area, rainfall behaviour, rivers regime, wells and boreholes recharge process have been
Fig. 1: Watersheds map of Sangmelima region

Fig. 2: The simplified topographical map of Sangmelima region

Fig. 3: Stations location map of the studied area
considerably modified during this recent years and are probably due to the climate change and human activities.

**Origin of the data:** The mean data used in this study (70 samples) were collected between 2009, 2010, 2011 and 2012, with drilled piezometers from rivers, swamps and water wells, generally constructed by the municipality city council (Fig. 3-5).

**Geostatistical method:** The variogram measures the squared difference between values as a function of distance. It’s defined according to Isaaks and Srivastava (1989) and Kumar and Ramadevi (2006) by the following equation:

\[
\gamma(h)^* = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [Z(x_i) - Z(x_{i+h})]^2
\]

where, \( \gamma(h)^* \) represents the estimated value of the semivariance for lag \( h \); \( N(h) \) is the number of experimental pairs separated by vector \( h \); \( Z(x_i) \) and \( Z(x_{i+h}) \) are values of variable \( z \) at \( x_i \) and \( x_{i+h} \), respectively. \( x_i \) and \( x_{i+h} \) represent the position in two dimensions.

In the linear kriging method, the interpolated value of \( z \) at any point \( x_i \) is given as the weighted sum of the measured values:

\[
Z'(x_i) = \sum_{j=1}^{N} \lambda_j Z(x_j)
\]

where, \( \lambda_j \) is the weight for the observation \( z \) at location \( x_j \), calculated by Eq. 3 so that \( Z'(x_i) \) is unbiased and optimal (minimum squared error of estimation):

\[
\begin{align*}
\sum_{j=1}^{N} \lambda_j &= 1 \quad \text{and} \\
\sum_{j=1}^{N} \lambda_j \gamma(x_i, x_j) &= \gamma(x_i, x_j) \quad i = 1, 2, 3, \ldots, N
\end{align*}
\]

where, \( \mu \) is the Lagrange multiplier and \( \gamma(x_i, x_j) \) the semivariogram between two points \( x_i \) and \( x_j \):

\[
\text{MSE} = \frac{1}{N} \sum_{i=1}^{N} [Z(x_i) - Z'(x_i)]^2
\]

where, \( N \) is the number of samples, \( Z(x_i) \) and \( Z'(x_i) \) are, respectively the estimated value and observed value at the point \( x_i \).

**RESULTS AND DISCUSSION**

**Experimental and fitted variograms models:** The experimental variogram and the fitted variograms (Fig. 6) are calculated using Eq. 1 and values of Table 1 with the data collected from 70 samples. The analytical variograms results obtain are follow (Eq. 5-8):

- **Spherical model:**

\[
\gamma(h) = 3\left[0.4k - 5\left(\frac{h^3}{10}\right)\right]
\]

- **Gaussian model:**

\[
\gamma(h) = 3\left[1.0\exp(-0.2h^2)\right]
\]
Table 1: Values obtained with various variogram models

<table>
<thead>
<tr>
<th>Distance (km)</th>
<th>Experimental model</th>
<th>Spherical model</th>
<th>Gaussian model</th>
<th>Exponential model</th>
<th>Logarithmic model</th>
<th>Penta spherical model</th>
<th>Linear model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>10.6</td>
<td>11.8</td>
<td>10.9</td>
<td>8.9</td>
<td>6.3</td>
<td>7.40.6</td>
<td>10.4</td>
</tr>
<tr>
<td>1.5</td>
<td>16.0</td>
<td>17.9</td>
<td>16.3</td>
<td>12.2</td>
<td>12.5</td>
<td>7142.9</td>
<td>15.9</td>
</tr>
<tr>
<td>2.1</td>
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<td>24.4</td>
<td>22.8</td>
<td>15.6</td>
<td>22.8</td>
<td>39859.0</td>
<td>22.4</td>
</tr>
<tr>
<td>2.5</td>
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<td>28.6</td>
<td>27.0</td>
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<td>28.5</td>
<td>102053.6</td>
<td>26.9</td>
</tr>
<tr>
<td>3.0</td>
<td>32.3</td>
<td>32.8</td>
<td>30.8</td>
<td>19.7</td>
<td>34.0</td>
<td>250070.9</td>
<td>32.2</td>
</tr>
<tr>
<td>3.5</td>
<td>36.9</td>
<td>36.1</td>
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<td>21.2</td>
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</tr>
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<td>22.8</td>
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<td>51.5</td>
<td>41.9</td>
<td>36.6</td>
<td>25.1</td>
<td>48.6</td>
<td>3211271.3</td>
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<tr>
<td>5.5</td>
<td>56.5</td>
<td>41.5</td>
<td>36.8</td>
<td>26.0</td>
<td>52.5</td>
<td>5075288.1</td>
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<tr>
<td>6.1</td>
<td>60.2</td>
<td>39.6</td>
<td>36.9</td>
<td>26.8</td>
<td>55.4</td>
<td>8202439.4</td>
<td>64.5</td>
</tr>
</tbody>
</table>

Table 2: RMSE values obtained with various variogram models

<table>
<thead>
<tr>
<th>Variogram model</th>
<th>Linear model</th>
<th>Spherical model</th>
<th>Gaussian model</th>
<th>Exponential model</th>
<th>Logarithmic model</th>
<th>Penta spherical model</th>
<th>Pentaspherical model</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>1.67</td>
<td>8.40</td>
<td>10.90</td>
<td>19.42</td>
<td>30.94</td>
<td>3130512.5</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 6: Experimental and fitted variogram models

Exponential model:

\[ \gamma(h) = 31 \left[ 1 - \exp(-0.3) \right] \]  
(7)

and linear:

\[ \gamma(h) = 10h \]  
(8)

The analysis of experimental values and those obtained with various analytical variograms (Fig. 6, Table 1) shows that, results of linear model present the best approximation with those of experimental model. But in the last line, the value of the linear model increases when the experimental model value remains constant. Comparison with other models shows that, the linear model is followed by the gaussian, spherical and exponential models. This Table 1 also shows that, the penta spherical model values present very bad correlation with those of the experimental model.

Validation of the variogram models: The RMSE values obtained with various variograms models (Table 2) shows that, the linear model presents the lowest RMSE 1.67. This result is similar to those obtained previously. According to Marcotte (1995), (Eq. 8) must be used to map the water table level in the studied region.

Predicting the water table map: The variograms (Eq. 5-8) are then used to construct the water table map by kriging with Golden Software, 2002, at the nodes of the 1 km x 1km square grid from the collected depth of some wells and depth of topsoil at some stations where shallow aquifer was detected. These estimated level values are used to draw the iso-depth maps. In Fig. 7-11, the water table appears to be continuous in all the study area with some contrasted observations. The depletion varies with space from 0 to 20 m according to the maps generated by linear, spherical and exponential variograms and from 0 to 160 m according to the map obtained with the logarithmic variogram. If the predicted water table is generally close, the maps are different from one variogram model to another. The figures confirm that, the selected model influences the prediction of the unknown values. In Fig. 7, the isocountours present features of the hills and valleys in relative good correlation with the topography (Fig. 3). The water level is low where the surface of the ground is low and higher where the surface is high. The minimum corresponds to the rivers or swamps whereas the maximum corresponds to the region where soils and weathered rocks are thick.

But, the results of the Fig. 8-11 are not the same. In fact, according to these figures, the isocountours are close but they do not present good correlation with the topography.

Cross validation criteria: The comparison between the measured in field and predicted values of water table by kriging using several analytical model is presented in

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Fig. 7: Water table level (m) map above mean sea level obtained by kriging using linear variogram model

Fig. 8: Water table level (m) map above mean sea level obtained by kriging using spherical variogram model
Fig. 9: Water table level (m) map above mean sea level obtained by kriging using gaussian variogram model

Fig. 10: Water table level (m) map above mean sea level obtained by kriging using exponential variogram model
Table 3: RMS values obtained from various variogram models according to the cross validation criteria

<table>
<thead>
<tr>
<th>Variogram model</th>
<th>Linear</th>
<th>Spherical</th>
<th>Gaussian</th>
<th>Exponential</th>
<th>Logarithmic</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>0.001</td>
<td>2.171</td>
<td>1.406</td>
<td>1.657</td>
<td>5.101</td>
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</tbody>
</table>

Table 4: Comparison between observed data and those obtained by kriging

<table>
<thead>
<tr>
<th>Longitude</th>
<th>Latitude</th>
<th>Experimental</th>
<th>Linear</th>
<th>Spherical</th>
<th>Gaussian</th>
<th>Exponential</th>
<th>Logarithmic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1328.12</td>
<td>327.47</td>
<td>0.61270</td>
<td>0.61870</td>
<td>-7.851360</td>
<td>-4.29632</td>
<td>-5.68949</td>
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<tr>
<td>1328.02</td>
<td>324.55</td>
<td>-0.13690</td>
<td>-0.13728</td>
<td>-7.851360</td>
<td>-3.62224</td>
<td>-4.79407</td>
<td>-9.116576</td>
</tr>
<tr>
<td>1327.52</td>
<td>330.19</td>
<td>-0.083200</td>
<td>-0.083110</td>
<td>-7.851360</td>
<td>-5.98645</td>
<td>-7.06345</td>
<td>-7.818410</td>
</tr>
<tr>
<td>1330.34</td>
<td>326.47</td>
<td>-0.66690</td>
<td>-0.66707</td>
<td>-2.435440</td>
<td>2.82611</td>
<td>-1.76297</td>
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<tr>
<td>1339.44</td>
<td>326.57</td>
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<td>-1.43380</td>
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<td>1339.13</td>
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<td>-0.06828</td>
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<tr>
<td>1326.8</td>
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<tr>
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<tr>
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<td>-0.55086</td>
<td>-7.851361</td>
<td>-6.09103</td>
<td>-7.02012</td>
<td>-8.109330</td>
</tr>
</tbody>
</table>

Table 3 and 4. These results also confirm that, the linear variogram has the low RMSE 0.001 values and appears to be best one. In addition, the map obtained with the linear model Fig. 7, enables to highlight swamps in the area and tests made on wells and boreholes data present a good correlation with kriging results. However, rivers don’t clearly appear, probably due to the narrowing of their courses according to the scale of data.

**CONCLUSION**

The water table influences soil moisture climatology, continental climate dynamics and is very important for peoples in the rural areas, particularly when they need to know at what depth below the surface they can tap far enough into the ground to get a functioning wells or which way pollutants may or may not flow to come to wells or boreholes. The results obtained confirm the usefulness of applying the geostatistic method to investigate water table. This study also shows that, the best fitted variogram in the Sangmelima rural region is the linear model. The map obtained by kriging allows to control the spatial behaviour of water-table in the context of climate dynamics and can guide the urbanization process while preserving the water quality in this region.
REFERENCES