A Research on Compound Base Bidding Price Based on the Game Theory

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Abstract: According to the characteristics of current evaluation method of compound base bidding price, this paper introduces the “Bayesian Nash Equilibrium” in game theory into the bidding quotation system, to provide scientific basis for more valid bidding and bid winning of bidders. Besides, it also probes into problems existing in the game model of bidding quotation in practical application.

Key words: Project bidding, compound base price, game theory, bayesian nash equilibrium

INTRODUCTION

As China’s construction contract market mechanism becomes more and more advanced, project bidding system has also been universally implemented and the competition among construction enterprises becomes increasingly fierce. To ensure that the bidder can win the bid and gain high economic benefits, the contractor should adopt scientific bidding strategy and reasonable bidding quotation. In engineering project bidding competition, each bidder’s fundamental interests collide with each other and their bidding behaviors influence each other. Hence, project bidding is a game process among many interested parties.

GAME THEORY AND BIDDING QUOTATION MODEL

An overview of game theory: As an important branch of operations research, game theory is a kind of mathematical theory and method to research phenomena with adversarial or competitive nature. It is mainly to research how competitors make decision to maximize their utility and keep the equilibrium among different decision-making bodies when they interact with each other. It emphasizes the reciprocal interdependence among decision-making bodies in all aspects, i.e., every decision-making body must determine their optimal strategy based on considering other participants’ possible strategies. (Rasmussen, 2003) The advantage of game theory is particularly reflected in interest conflicting bodies' multiple strategy choices in the case of information asymmetry.

If categorizing the game theory, it can be divided into static game and dynamic game in chronological order of decision-making bodies’ action. Static game means that participants make a choice simultaneously or even if they do not make a choice simultaneously, the latter actor does not know the choice the former actor has made; according to a participant’s understanding of other participants (competitors), can be divided into complete information game and incomplete information game choice according to the former actor’s action.

Dynamic game means that participants’ actions are sequential and the latter actor makes his own. For another, if categorizing the game theory it can be divided into complete information game and incomplete information game. Complete information game means that in the game process, every participant has an accurate knowledge of other participants’ information; otherwise, it is incomplete information game. If combining with two categorization perspectives above, four different types of game can be obtained, namely, static game of complete information, dynamic game of complete information, static game of incomplete information and dynamic game of incomplete information. Four equilibrium concepts corresponding to four types of game above include Nash equilibrium, sub game perfect Nash equilibrium, Bayesian Nash equilibrium and perfect Bayesian Nash equilibrium.

Form and characteristics of bidding: China’s existing project bidding originated from auction theory. After the tenderer publishes the public project information, namely, the unified bill of quantities, the tenderers seal and submit the bidding document according to their own strength and basic information of the project; the tenderee will open

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sealed tenders in unified time and then the reasonable bottom price will win the bid after bidding evaluation.

According to the procedure and characteristics of bidding, it can be seen that in the game process, every tenderer knows their own estimation of the project cost only, namely, private information but does not know others’ estimation of the project cost. Every tenderer’s strategy is made in accordance with their own estimation of the project cost and others’ general estimation, so it is incomplete information; besides, tenderers make the bidding decision independently and despite the different time of bidding document delivery, tenders are opened simultaneously, so it is static game. Obviously, it is a static game of incomplete information, namely, Bayesian Nash equilibrium (Jiao, 2005).

**Current bidding evaluation methods:** At present, there are various bidding evaluation methods in China. From the terms and actual implementation of bid winning, “compound base price” is generally adopted as the bid evaluation standard of bidding quotation to show the competitiveness of bidding. “Compound base price” can effectively eliminate adverse outcomes caused by the leakage of bid project’s base price, reduce the pressure of bidding work and put an end to black box operation.

Instead of providing a base price first, the tenderer takes the average price within the scope of tenderers’ valid quotation as the base price or takes the weighted average through multiplying owners’ base price by some weights and all tenderers’ valid price as the base price which is called compound base price. On occasion, these base prices would be lowered to act as the optimal price. All of these methods determining the base price are called compound base price. In the meantime, all tenderers’ tender quotation deviation is penalized according to fixed penalty rules to reach the quotation mark which can be described by the formula below (Wu, 1995):

\[
B = Y \times w + T \times (1 - w)
\]

where, B is compound base price, Y is base price prepared by the owner, T is average value of all tenderers’ valid quotation, w is weight of y in compound base price.

**GAME MODEL OF COMPOUND BASE BIDDING PRICE**

**Correlated variables setting:** According to bid evaluation method and for the convenience of description, following variables are set here.

The owner’s base price is \( Y = 1 \) and its weight in compound base price is \( w (0 < w < 1) \).

When the tenderers’ quotation is within the owner’s\([a, b]\), it is valid quotation and the proportion of its average value in compound base price is \( 1 - w \).

When the tenderers’ quotation is within\([c, d]\) of the compound base price, it gets a full mark; whenever one percentage exceeds \( d \), p points will be deducted respectively and whenever one percentage is less than \( c \), q points will be deducted;

**Setting:** \( n \) is indicative of valid bid quantity; \( x \) is indicative of our bidding quotation; \( \bar{x} \) is indicative of the average value of other \( n-1 \) valid quotations except for ours; \( B \) is indicative of the compound base price; \( K \) is indicative of the deducted points of our quotation (Hu et al., 2004).

**Assumption terms:** To make a scientific bidding quotation and achieve a balance between bidders’ profit maximization and mark maximization, following reasonable assumption should be made.

The quota and compiling method adopted by base price is known, so bidders can estimate the absolute number of the owner’s base price by themselves.

In bid documents, the average value of \( w, a, b, c, d, p \) and \( q \) is explicitly stipulated, i.e., they are known constants; besides, it is assumed that all bidders’ quotations are valid.

When determining the bid quotation, influential factors such as cost, technology, quality, credit and task fullness are out of consideration.

**ESTABLISHMENT OF GAME MODEL**

**Equation of compound base price:**

\[
B = w + \frac{(1 - w)}{n} [x + (a - b) \bar{x}]
\]

**Deduction formula and error matrix:** Since \( \bar{x} \) and bidder number \( n \) cannot be known exactly before bidding, error \( E \) exists between our bidding quotation and the compound base price and this error \( E \) can be described by the equation as below:

\[
E = x - B = x - \left[ w + \frac{(1 - w)}{n} [x + (a - b) \bar{x}] \right]
\]

This error \( E \) would cause our bidding quotation to be deducted and the deduction equation is:
\[
\begin{align*}
\frac{p^{x}}{b} - d & = \frac{x}{b} - d & d \geq 0 \\
0 & = c \leq \frac{x}{b} & d = 0 \\
q(c - \frac{x}{b}) & = c - \frac{x}{b} & c \geq 0
\end{align*}
\]

**Game principles:** As for the model above, in fact, it is unnecessary to ensure when our party should provide quotation to get a full mark, because our party can win the bid if only the highest mark of the quotation is obtained. Therefore, in terms of the model above, what should be discussed is under what conditions our party can minimize the deducted points, so that the highest mark of our quotation can be obtained to reach the goal of winning the bid. As a result, the game principle of bidding quotation is to offer a quotation as high as possible under the premise of minimizing deducted points (Liu and Ding, 2002), i.e.:

\[
k = \text{min} \quad x = \text{max}
\]

**Full-mark quotation range:** According to the decision-making principle k-min, if \(k = 0\) (namely, when a full mark is reached) is substituted into the computational formula of compound base price \(B\), following result can be obtained:

\[
\frac{c_{w} + c(1 - w)(n - 1)X}{n - c - c_{w}} \leq x \leq \frac{d_{w} + d(1 - w)(n - 1)X}{n - d - d_{w}}
\]

According to the decision-making principle x-max, the optimal quotation equation is:

\[
x = \frac{d_{w} + d(1 - w)(n - 1)X}{n - d - d_{w}}
\]

**Model analysis:** It can be seen that both full-mark quotation range and the optimal quotation formula are functions of \(X\) and \(n\). Neither \(X\) nor \(n\) can be accurately determined before bidding, so the key to the final quotation decision-making is the accurate estimation of \(X\) and \(n\). As for \(n\), indicative of the number of valid tenders, although it is unknown before bid evaluation, a general scope can be obtained by bidders through information collection and historical empirical estimation. Furthermore, the sensitivity analysis of \(n\) also proves that despite the large deviation of estimation for \(n\), little effect will be caused on the mark result of final quotation.

Hence, the key to the accuracy of bidding quotation model above is the prediction of \(X\). \(X\) indicates the average value of other n-1 bidders' quotations, so the prediction of \(X\) is exactly the prediction of other competitor's quotation.

In bidding process, bidders know their own project cost function only but have no accurate knowledge of their competitors' cost function, so they have to make a prediction according to previous data and their knowledge of general projects' cost level and grasp the probability distribution of competitors' cost function which is the typical static game of incomplete information (Hao et al., 2002). The idea of Bayesian Nash equilibrium in game (Liu and Ding, 2002) theory and assumption above can be applied to finding out each bidder's Nash equilibrium solution with the motivation of maximizing all bidders' expected profit after the game, to reach the average value \(\overline{X}\) of each bidder's quotation. This average value is exactly the bidding quotation for every bidder to make the optimal strategy, i.e.:

\[
\overline{X} = \frac{M + n - 1}{n}v_{1}
\]

where, \(M\) is indicative of the highest value of the bidding quotation, namely, the upper limit value within the valid range of quotation, i.e.:

\[
M = b/Y
\]

\(v_{1}\) is indicative of bidders' estimation of I project's actual cost, a budgetary price can be compiled according to quota of budget and previous engineering experience can be combined for price adjustment, so as to reach the estimation of the project's actual costs.

**CONCLUSION**

From the analysis above, it can be concluded that it is feasible to introduce the Bayesian Nash equilibrium model in game theory into bidding quotation decision-making system based on the bid evaluation system of compound base bidding price and it is completely in conformity with the characteristics of current project bidding in China. However, in actual application, whether the bidding quotation can win the bid or not is not only related to bidders' own determined project cost function but also involves other bidders' estimation of project cost function. When offering bidding quotation, current enterprises in the same region generally determine the project's engineering cost according to local quota of budget which enables stability when each bidder makes the optimal strategy, thus guaranteeing the accuracy of bidding quotation model above.
REFERENCES


