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Study about Paradox of Post Transportation Problem

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Abstract: This is the further study on the basis of previous study, the “transport problem paradox”, aiming to find the discipline when the change is more than the adjustable amount in the optimization process, namely the so-called stage of “post transportation problem paradox”. At first, the view of the minus cost route and the theoretical viewpoint of anti-adjustable route were used. As well as, the theory of adjustment route was taken as a guide. Then it was come up with that the assumption judgment and proven the development trend of the total freight and cargo in the stage of post transportation paradox. At the terminal part, through theoretical analysis, the conclusion is that the total freight continues to increase with the increase of the total cargo and finally the two will have a linear proportional relationship in the stage of post transportation paradox.

Key words: Transportation problem paradox, adjustment route, adjustable amount

INTRODUCTION

In Qi, 1982 published the article named “transportation problem paradox” in the Journal of Operational Research (Qi, 1982) and made a preliminary definition of the transportation problem paradox. That transportation problem paradox refers to the case in which the total freight decreased with the total cargo increasing and other suppliers and demanders uncharged. The condition is that there is a balance of supply and demand between multiple suppliers and multiple demanders. When one supplier’s supply increased and another demander’s demand increased either, the freight of each transportation route without change and tabular method has been applied to obtain the optimal transportation solution. However, Zhou Qi did not further explore the conditions that the transportation problem paradox contains.

In Yun, 1984 first suggested that the reason why transportation problem paradox appears is that there is a minus cost route in his article named “the conditions of transportation problem paradox” (Yun, 1984) which was published in the Journal of Operational Research. Based on tabular method, the necessary and sufficient existing condition of paradox is the presence of minus cost route (i.e., \( u_i+v_j<0 \)) according to the geopotential and dual relationship in the non-degenerate case. What is more, this is also extended to the paradox of general linear programming. After that, some articles also study the existing conditions of the transportation problem paradox from different angles. However, most of them was effected by Lin’s article and only made the deformation. These articles include “the method of looking for general transportation problem paradox” (Yun, 1992), “the general case of transportation - the solution of classic transportation problem paradox” (Xiaohua, 1989), “the paradox of the transportation problem and its appearance conditions” (Ping, 2001), “the transportation problem paradox and its mathematical, economic interpretation” (Miao, 2004), “the existing conditions and solutions of transportation problem paradox” (Yang, 2007), “the discussion about transport problem paradox” (Zhu and Chen, 2009). In recent years, one of the most innovative articles on transportation problem paradox is undoubtedly the “conditions of transportation problem paradox” from (Xia and Ban, 2008) published on Operations Research and Management Science. In this study, the viewpoint of using the theory of anti-tune route to study the transportation problem paradox was put forward. This article abandoned previous studies which were based on geo potential dual. This article raised a new point of view. And it focuses on the discipline of routes’ variation when the best program table of the previous stage was adjusted owing to the both increasing of supply and demand.

First of all, the view of the minus cost route and the theoretical viewpoint of anti-adjustable route were used. As well as, the theory of adjustment route was taken as a guide. Then it was come up with that the assumption judgment and proven the development trend of the total freight and cargo in the stage of post transportation paradox. At the terminal part, through theoretical analysis, the conclusion is that the total freight continues to increase with the increase of the total cargo and finally the two will have a linear proportional relationship in the stage of post transportation paradox.

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OVERVIEW OF POST TRANSPORT PROBLEM PARADOX

Previous studies about the transportation problem paradox are concentrated on the determination of the problem’s existing conditions as well as finding adjustment programs. Research in this area has been very mature and it can quickly and accurately determine whether there exist the conditions of transportation problem paradox on the table of an optimal transportation solutions. In an addition, if there exists, it can also be deduced whether one or more suppliers and demanders increased cargo will cause the total freight reduced. Since the existence of transportation problem paradox needs extremely harsh conditions, or it can also be determined based on common sense that there can not be the unlimited increasing on the cargo with the freight unlimited reducing. This is related to the problem of adjustable amount. Based on previous studies, the adjustable amount is also not difficult to calculate, but the situation at stage after more than that adjustable amount of change is not clear.

There exist the condition of multi-supplier and multi-demand, for example, there exist a specific supplier A and a specific demander B. The added supply in A and demand in B will lead to the phenomenon of transportation problem paradox. That is to say the total cargo increased but the total freight reduced and the maximum adjustable amount is \( \theta \). That is to say, when the added cargo in the nodes of A and B is increasing from 0 to \( \theta \), the total freight will continue to reduce, but when the addition of cargo is more than \( \theta \), with the increase of cargo the total freight will increase or not? The question is whether this increase is sustainable? Will it be true or not that when the cargo increase to a certain extent sustainably, it will be come out that freight reduced with cargo increasing namely the transport problem paradox reappears again. This is the so-called “post transportation problem paradox”, which is about the changes in freight and cargo. That is brought about by continuing to add amount to the two determined nodes after the optimization exceeds the maximum adjustable amount.

THEORY OF ADJUST ROUTE

Theory of minus cost route: In Table 1, the letters, from \( A_i \) to \( A_n \), representative supplying places, the letters, from \( B_i \) to \( B_n \), representive demanding places and the letters \( X_{ij} \) representative the optimal volume of transportation from \( A_i \) to \( B_j \).

Suppose that there exists an optimal transportation program table, as shown in Table 1. If there exist this the

<table>
<thead>
<tr>
<th>Table 1: Transportation planning table</th>
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<tr>
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<tr>
<td>( A_1 )</td>
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<td>( \ldots )</td>
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<td>( A_n )</td>
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<tr>
<td>Demand</td>
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</tbody>
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<table>
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<tr>
<th>Table 2: Balanced transportation plan with Cargo and tariffs</th>
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<tr>
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conditions of transportation problem paradox shown in the presence of this table, there must be two nodes, \( A_i \) and \( B_j \), when it add some cargo \( \theta' \), respectively, in the two nodes, the total freight will be reduced. At the same time, there also be some appropriate changes in the optimal solution:

\[
X_{ij} + \theta' \rightarrow X_{i(\theta + \theta')} - \theta' \rightarrow X_{i(\theta + \theta') + \theta' - \theta} \rightarrow \ldots \rightarrow X_{i(\theta + \theta') + \theta'}
\]

or \( X_{ij} + \theta' \rightarrow X_{i(\theta + \theta')} - \theta' \rightarrow X_{i(\theta + \theta') + \theta' - \theta} \rightarrow \ldots \rightarrow X_{i(\theta + \theta') + \theta'} \)

The corresponding adjustment route:

\[
(i, j) \rightarrow (i + \epsilon, j) \rightarrow (i + \epsilon, j + \epsilon) \rightarrow \ldots \rightarrow (i + \epsilon, j + \epsilon, \ldots)
\]

The corresponding adjustment route:

\[
\theta' * (C_{ij} - C_{i(k + \theta')} + C_{i(k + \theta') - \theta} - \ldots + C_{i(n, \theta')}) < 0
\]

or \( \theta' * (C_{ij} - C_{i(k + \theta')} + C_{i(k + \theta') - \theta} - \ldots + C_{i(n, \theta')}) < 0 \)

The corresponding cost of adjustment route:

Therefore, under the conditions of transportation problem paradox, the adjustment route in the optimal program above is the so called the minus cost route (Yun, 1992).

Theory of anti-tune route: As is shown in Table 2, it is the balanced transportation problem.

Take the minimum value for each column in the freight matrix \( (C_{ij})_{min} \) (if there are more than one minimum value, taking any one is ok):

\[
C_{ij} = \min_{\text{finite}} C_{ij}, \quad j = 1, 2, \ldots n
\]

Program as follows: \( X^0 = (C_{ij})_{min} \)

\[
X^0 = b_j, \quad j = 1, 2, \ldots, n, \quad \text{the remaining} \quad X^0 = 0
\]

In Table 2, the letters, from \( A_i \) to \( A_n \), representative supplying places, the letters, from \( B_i \) to \( B_n \), representative
Table 3: Transport solving program with minimum cost in each line

<table>
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<tr>
<th></th>
<th>B₁</th>
<th>B₂</th>
<th>B₃</th>
<th>B₄</th>
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<tr>
<td>A₁</td>
<td>C₁₁</td>
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<tr>
<td>Demand</td>
<td>b₁</td>
<td>b₂</td>
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Table 4: Optimal transportation program with adjust line

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<th>B₁</th>
<th>B₂</th>
<th>B₃</th>
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<td>A₁</td>
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<td>A₄</td>
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<td>Demand</td>
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<td>b₆</td>
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</tbody>
</table>

demanding places and the letters Cᵢⱼ represent the cost of transportation from Aᵢ to Bⱼ. And Xᵢ is the so called minimum program. For the transportation problem such as the following, the non-zero elements in the minimum program Xᵢ has been marked. (Following the same).

In Table 3, the letters, from Aᵢ to A₄, representative supplying places, the letters, from B₁ to B₆, representative demanding places and the letters Cᵢⱼ, represent the cost of transportation from Aᵢ to Bⱼ. Xᵢ represents the optimal solution of the balanced transportation problem and any value of the total freight of the program can be represented by f(x).

Therefore, it is obvious that:

\[
f(x^0) \leq f(x^*)
\]

In Table 4, the letters, from Aᵢ to A₄, representative supplying places, the letters, from B₁ to B₆, representative demanding places and the letters Cᵢⱼ, represent the cost of transportation from Aᵢ to Bⱼ. It should be noted that the minimum program Xᵢ meet column equilibrium conditions:

\[
\sum_{j=1}^{n} x_{i,j} = b_i, \quad i = 1, 2, \ldots, n
\]  

If it also meet row equilibrium conditions:

\[
\sum_{i=1}^{n} x_{i,j} = a_i, \quad j = 1, 2, \ldots, m
\]

it is easy to know that Xᵢ is fit for the optimal solution conditions at this time, then obviously there will be no paradox. However, if minimum program Xᵢ does not satisfy the column equilibrium conditions, it is an infeasible program of transportation problem, such as the minimum program of the transportation problem shown in Table 3 which is an infeasible solution. But it can be seen that optimal solution Xᵢ as a program which is obtained from a minimum program Xᵢ after appropriate adjustments. If Xᵢ and Xᵢ (non-zero elements circled to show the difference) are simultaneously shown in the table, then draw the adjustment route (indicated by solid lines), it will be more clear. In transportation problem as is shown in Table 3, for example, the minimum program Xᵢ and the optimal solution Xᵢ as well as the adjustment route are marked as the Table 4 shows.

Obviously, the adjustment from the minimum program Xᵢ to optimal solution Xᵢ is in the premise of the condition (3) which satisfies the column balance adjustment. The objective performance of the adjustment is only between the rows of transferred in or transferred out. For minimum program Xᵢ (Xᵢ, Xᵢ), make:

\[
d_i = \sum_{j=1}^{m} x_{i,j} - a_i, \quad i = 1, 2, \ldots, m
\]  

The row in which dᵢ=0 is the so called export line, the row in which dᵢ=0 is the so called balanced line and the row in which dᵢ<0 is the so called import line. In order to satisfy the need for below theoretically proving the adjustment can be seen as a step-wise. For example, when adjust the unit quantity of cargo, the adjustment line having the minimum increased freight (hereinafter referred to as the adjustment difference) is conduct at first. From the final result, balanced line modulcation out should be equal to the total transferred, therefore each step may be seen as simultaneously when it is transferred out of the balanced line and transferred in, so that it is always maintained in the adjustment for the balanced line. For the further step, it can be seen as the process which starting from the export line, via the transit of certain balance line and finally to the import line.

Obviously, starting from any one program X (i.e., not limited to start from the smallest program Xᵢ) and via above planning and adjustment, it can also obtain the optimal solution Xᵢ, at the same time, any one of the programs which meet the column equilibrium conditions (3) can also be got from the adjustment of minimum program Xᵢ. In order to make it facilitate in the narrative, introduce the following definitions:

**Definition 1:** For the program X which meet the column balanced condition (3) and the optimal solution Xᵢ, if \( X_{i,j} > X_{i,j} \) point (i, j) is called as a new value point, if \( X_{i,j} < X_{i,j} \) and row i is the export line of X, the point (i, j) is called as a old value point.

**Definition 2:** For the adjustment route including the new value point (kᵢ,j) as a starting point and the old value of the point (kᵢ,j) as the end point.
\[(k_{ui}, j) \rightarrow (k_{ri}, j) \rightarrow (k_{ri}, k_{ui}) \rightarrow \cdots \rightarrow (k_{ri}, j) \rightarrow (k_{si}, j)\]  
\[\text{(5)}\]

If in the line above from the starting point \((k_{ui}, j)\) to every point \((k_{ri}, j)\) in the odd position meet the condition of \(X^*_{i\{r+1\}} > 0\), \(0 < s < t\), then (6) is an anti-tune route of \(X^*\). Defined anti-adjustable differential and anti-adjustment amount along the route (6) as follows:

\[\hat{\eta} = (c_{k_{ri}} - c_{k_{ui}}) + (c_{k_{ri}} - c_{k_{ri+1}}) + \cdots + (c_{k_{ri}} - c_{k_{si}})\]  
\[\hat{\eta} = \min_{0 < s < t} \{X^*_{i\{r+1\}}\} > 0\]  
\[\text{(7)}\]

\[\text{Necessary and sufficient conditions of the paradox: If the transport balance problem paradox exist, there are anti-adjustable route (6) in the optimal solution \(X^*\) and in the definition (7) the anti-adjustable difference \(\hat{\eta}\) need to meet the condition as follows:}\]

\[\hat{\eta} > \min_{0 < s < t} c_{k_{ri}} = c_{k_{ri+1}}\]

\[k_{ui}\] is the row number of the new value point \((k_{ui}, j)\), while "\(p\)" is the column number.

Theory of adjust route: The minus cost route described above and the theory of anti-adjustable route are starting from the optimal solution of the transportation problem, to determine whether there is a minus cost route or anti-adjustment route. If being existing, it not only can determine the specific minus cost route and anti-adjustable route, but also can determine to which supplier and demander to add amount may reduce the total freight. Both Minus cost route and adjust amount of the anti-transfer route are limited. The post transportation problem paradox study is about the situation of the change relationship between total freight and total cargo in which the adjustment is more than the adjustable amount.

The fundamental principle of the transportation problem on the operating table method is the basic rule. When transceiver amount of supply and demand increase, the adjustment of the program based on the optimal schedule of prices is always to make sure achieving the lowest freight as the adjustment target. Therefore, the adjustment caused by the increased transceiver amount can be seen the change of the optimal solution along an adjustment route. The increased transceiver cargo in the nodes \(A_i\) and \(B_j\) will be related to one or more adjustment routes. These adjustment routes can be distinguished by \(\Delta y\) which is the increase in value of the freight because of the increase of 1 unit amount transceiver cargo. When add transceiver amount to the points \(A_i\) and \(B_j\), the change of adjustment route always began with the corresponding route of the minimum value \(\Delta y\). \(\Delta y\) can be positive or minus and it can also be 0, at the same time \(\Delta y < C_{ij}\). When \(\Delta y < 0\), increase of the total transceivers amount reduce the freight, namely the above mentioned anti-adjustable route. When \(\Delta y = 0\), the total freight remains unchanged when the transceiver amount increases, when \(\Delta y > 0\), increase of the transceiver amount will lead to the increase of total freight.

In any specific transportation problem, there must be a certain adjustable amount limited in each adjustment route and the adjustable amount is determined by the minimum value of the base variable in the export node of the route. When the increase of transceiver amount is more than the adjustable amount in this route, it will make the route fracture. And additional transceiver amount will lead to the adjustment along the route to be adjusted corresponding to \(\Delta y^0\) (the value of \(\Delta y^0\) is slightly larger than \(\Delta y\)). That is to say, the values of \(\Delta y\) can be racked from small to large and each of the value of \(\Delta y\) corresponds to an adjustment route. When add transceiver amount to the nodes \(A_i\) and \(B_j\), it has always been adjusting along the route which is corresponding to the smallest \(\Delta y\) at first. When the change exceeds the adjustable amount, with the transceiver amount increasing, the adjustment route will break. After that if transceiver amount continue to increase, it will be adjusted along the adjustment route corresponding to the value of a slightly larger \(\Delta y\). So it shows that \(\Delta y\) is always incremented with the increase in the transceiver amount.

\[\Delta y = C_{ij}\] the increasing supplier \(A_i\) will be shipped directly to \(B_j\) and the adjustable amount in this route is infinite.

ASSUMPTIONS AND PROVING OF POST TRANSPORTATION PROBLEM PARADOX

Assumption: In such two cases:

- The phenomenon of transportation problem paradox does not exist on \((A_i, B_j)\) at the beginning and there is no minus cost route or anti-adjustable in the optimal solution described above which is corresponding to \((A_i, B_j)\)
- There exist transportation problem paradox on \((A_i, B_j)\) at the beginning, that is to say one or more minus cost route or anti-adjustable route in the optimal solution described above is corresponding to \((A_i, B_j)\) and its total adjustable amount is \(\theta^0\). If continues to add transceiver amount to \(A_i\) and \(B_j\), what will
happen. When the increasing transceiver amount is less than \( \theta \), the total freight reduces (including shipping cargo increased but freight constant), due to the presence of the transportation problem paradox conditions. But now the increased transceiver amount exceeds the maximum adjustable amount

If new transceiver \( \Delta x \) would only be added to the points of \( A \) and \( B \), the shipping costs will continue to increase and the phenomenon will not appear that when the total cargo increases, the total freight reduce again.

**Proving:** What going to be proved is that there exist the phenomenon of post transportation problem paradox in the certain supplier \( A \) and demander \( B \). It will not cause the post transportation problem paradox corresponding to these nodes to add transceiver amount to this two, namely, when the transceiver amount of this two nodes is increasing, the total freight will not reduce.

In order to prove what is mentioned above, it must be proved that the broken adjustment route corresponding to smaller \( \Delta y \) will not be repaired, when the adjustment in the adjustment route which is corresponding to larger \( \Delta y \) is done. This will ensure that when the value of \( \Delta x \) is increasing, the value of \( \Delta y \) which is corresponding to \( \Delta x \) is also increasing.

In the adjustment route, odd number node is the export node, while even number node is the import node (base variables are larger than 0). So when the breaking node is at the place of the odd number nodes which is not the starting point on the adjustment route, it has the possibility to make the adjustment route corresponding to the broken node repaired. And the broken node must be the even number node, namely the export node in the original adjustment route:

- When both the current adjustment route and the original adjustment route are 3-node structure, the broken node is on the even number node on the current adjustment route, which is also the even number node on the original adjustment route (Fig. 1). The letters, from \( a \) to \( s \) representative the number of rows and columns and the letters \( C_i \) representative the cost of transportation on row \( i \) and column \( j \) and the letters have the same meaning in all of the figures. In this case, the original adjustment route could not be repaired directly by adding 1 unit transceiver amount, respectively in row \( i \) and column \( j \):
  - The original route: \( (m, j) \rightarrow (m, b) \rightarrow (i, b) \)
  - The current route is the same: \( (m, j) \rightarrow (m, b) \rightarrow (i, b) \)

- If you add 1 unit to the row \( i \) and column \( j \) of the route, respectively, it is obvious that the transceiver amount of node \( (m, b) \) will not increase. On the contrary, this will lead to the decrease of the value of \( (m, b) \) continuously. But as the broken node, the value of node \( (m, j) \) is already 0 and it is impossible to continue to reduce. In one word, it is impossible to be repaired in this case.
- When the current adjustment route has 5-node structure, while the original adjustment route has 3-node structure, the broken node is on the odd number node in the current adjustment route, which is the even number node on the original adjustment route (Fig. 2). In this case, the original adjustment route could not be repaired directly by adding 1 unit transceiver amount respectively in row \( i \) and column \( j \).
Fig. 3: Transportation program with multi-point adjust route

- The current route: \((o, j) \rightarrow (o, b) \rightarrow (m, b) \rightarrow (m, d) \rightarrow (i, d)\)
- The original route: \((m, j) \rightarrow (m, b) \rightarrow (i, b)\)

\((m, b)\) is the broken node and there exist the relationship:

\(C_{m,i} + C_{m,b} + C_{i,b} + C_{q,i} + C_{m,d} + C_{i,d}\)

If \((m, b)\) can be repaired, there exist the relationship:

\(C_{m,i} + C_{i,b} + C_{m,b} + C_{i,b} + C_{q,i} + C_{m,d} + C_{i,d}\)

However, from the relationship, \(C_{m,i} + C_{m,b} + C_{i,b} + C_{q,i} + C_{m,d} + C_{i,d}\) and \(C_{i,b} + C_{m,d} + C_{i,d}\) it can be inferred that \(C_{m,i} + C_{m,b} + C_{i,b} + C_{q,i} + C_{m,d} + C_{i,d}\). So, there exist the contradiction and the broken node can not be repaired.

- When the current adjustment route has five-node structure, while the original adjustment route has seven-node structure, the broken node is on the odd number node of the current adjustment route, which is the even number node in the original adjustment route (Fig. 3). In this case, the original adjustment route could not be repaired directly by adding 1 unit transceiver amount respectively in row \(i\) and column \(j\):

- The current route: \((o, j) \rightarrow (o, b) \rightarrow (m, b) \rightarrow (m, d) \rightarrow (i, b)\)
- The original route: \((q, j) \rightarrow (q, h) \rightarrow (m, h) \rightarrow (m, b) \rightarrow (s, b) \rightarrow (s, f) \rightarrow (i, f)\)

Fig. 4: Change of total cargo and freight at different stage, the stage of transportation problem paradox is from “a” to “i” in Fig. 4, the stage of post transportation issue paradox is from “j” to “n”; the stage of linear relationship is from “m” to “n” \((\Delta y = C_{m,b} \Delta x)\)

\[(m, b)\] is the broken point and there exist the relationship:

\[C_{m,i} + C_{m,b} + C_{i,b} + C_{q,i} + C_{m,d} + C_{i,d}\]

From the relationship:

\[C_{m,i} + C_{m,b} + C_{i,b} + C_{q,i} + C_{m,d} + C_{i,d}\]

It can be inferred the fact that:

\[C_{m,i} \rightarrow C_{m,b} \rightarrow C_{i,b} \rightarrow C_{q,i} \rightarrow C_{m,d} \rightarrow C_{i,d}\]

However, from the relationship:

\[C_{m,i} \rightarrow C_{m,b} \rightarrow C_{i,b} \rightarrow C_{q,i} \rightarrow C_{m,d} \rightarrow C_{i,d}\]

It can be inferred the fact that:

\[C_{m,i} \rightarrow C_{m,b} \rightarrow C_{i,b} \rightarrow C_{q,i} \rightarrow C_{m,d} \rightarrow C_{i,d}\]

So, there exist the contradiction and the broken point can not be repaired.

And so on, it can be proved that there exist contradictions when the current adjustment route has the structure with \(2m+1\) nodes and the original adjustment route has the structure with \(2m+1\) nodes, the broken node.
can not be repaired. This process has proved that when the adjustment is carried out in adjustment route corresponding to larger $\Delta y$, the broken adjustment route corresponding to the smaller $\Delta y$ can not be repaired.

Therefore, when it is in the stage of post transportation, the phenomenon that cargo increases but freight reduces is impossible to appear again on the certain two nodes. Further more, when the cargo of the certain supplier $A_i$ and demander $B_i$ is increasing, the increase value of $\Delta y$ is always incremented. When $\Delta y = C_{i_{op}}$, the increase cargo of $A_i$ is shipped directly to $B_i$ and the adjustable amount is infinity. It will become the linear relationship between the increase value of freight $\Delta y$ and the increase amount of cargo $\Delta x$, namely $\Delta y = C_{i_{op}} \Delta x$. The above relationship is shown in Fig. 4.

**CONCLUSION**

Predecessors have done many studies on the stage of transportation problem paradox, while fewer studies on stage after this adjustment of transportation problem paradox. So, this article focuses on and prove how the trends of the total cargo and the total freight change where there is no transport problem paradox or after the stage of paradox adjustment. If there is no conditions of transport problem paradox, the optimum value of the program's entire transport freight is constantly on the rise with the increase of the total cargo. Finally, it presents the situation of linear growth. Of course, a variation of the total freight is different owing to the cargo adjustments at the different stages.

This is actually the problem of static optimal or dynamic optimal program, that is to say, the optimal transport program which usually be used is only optimal in the static case. Once the cargo of transportation nodes can be adjusted to some extent, to seek the optimal solution requires the use of transportation problem paradox and the content of post paradox stage. The revelation of the change in this process is that endlessly increasing cargo in certain nodes would lead to overall non-optimization, so it should try to keep a coordinated proportion of the cargo between each node. The best state is that any combination nodes are at the end of each transportation problem paradox stage. It doesn't enter the stage of post transportation problem paradox. This is the only way to reach the true sense of the optimal.

**REFERENCES**


