Game Theory Analysis of Renewable Energy Construction Developers and Consumers

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Abstract: Many different interest-related parties are involved in the full life cycle of the project. They make final decisions in a rational or non-rational way via interactions in terms of surveying construction, dismantling stages and show the building as a result of interaction among various aspects. This study focuses on applying game theory to analyzing the decisions of the various stakeholders involved in the construction of renewable buildings, including decision-making of developers and consumers in the various stages of the total life cycle. If the building is renewable energy construction, the developer can obtain earnings either taking strategy with high price or strategy with low price, if the building is traditional construction, the developer will take strategy with high price for profit. For buyers with preference, they would buy renewable energy construction buildings certainly. But for buyers without preference, they may not buy any buildings, or few of them would choose to buy.

Keywords: Game theory, renewable resources, Hotelling model, Bayesian model

INTRODUCTION

With the development of construction market, an industry applying renewable energy to the new construction has appeared, the developers held both the entry and the wait-and-see attitude on emerging markets. Compared with other factors, consumers are most concerned about the use of building performance and the purchase price when consumers buying real estate. In game theory, the Hotelling price competition model and the Bayesian game model is a classic price analysis model (Wang and Xiao, 2008). So far, the game model has been widely used in various industries.

Recently, many efforts have been made to develop using game theory analysis methods to analyze the behavior (strategic interaction and strategy equilibrium) between the developers in the real estate development market and establishing a framework for the analysis of the real estate market. Such as in the progress of transaction between developers and consumers, it is effective to balance the transaction price and the amount by game model analysis. But game theory analysis for participants of the renewable energy construction is rarely.

This article will apply Hotelling price competition model and Bayesian model to the construction of renewable energy market, analyze the impact of the behavior between developers and consumers on prices and the equilibrium condition of the game.

GAME ANALYSIS BETWEEN THE DEVELOPER AND THE DEVELOPER

Model assumption: Building energy efficiency that the real estate development industry advocated has been progressively implementation. It includes renewable measures for the construction of the main structure such as external wall tiles, window materials, roof insulation with renewable technology or renewable materials and the improvement of the way energy consumed during building use. It aims at using renewable energy technologies in the construction to transform consumable energy into renewable energy in running process. When the developers are in the driving position in the develop real estate, they will have to face the choice of the type of construction: construction of renewable energy or traditional architecture.

In the following model that assuming two developers for the game players and each individual is rational, they unilateral seek methods to maximize their own interests. Parties choose the developed building structure in the real estate market autonomously and they have two choices: Development and non-development, so it resulting in the four cases: (Development, development), (non-development, development), (development, non-development), (non-development, non-development). When both are developed or not developed, they belong to the development of similar products and do not have obvious comparison. When one of them is developed that

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is, (development, non-development), (non-development, development), these two categories are treated as one case (Kuhn and Song, 2004). Then apply Hotelling price competition model to convert it.

**Model building:** Assume that a city is a linear city of 1 length, consumers are evenly distributed in the interval 0-1 range and the density of its distribution function is 1. There are two developers here, one developed the real estate of renewable energy and another developed the traditional energy construction. Consumers consider the building structure, life convenience, work convenience, traffic and other factors when they select the real estate. These costs are proportional to distance and the cost of the unit distance is t, the consumers living in x pay the travel costs x for buying real estate of the developer 1 and they pay the travel costs t(1-x) to purchase from the developer 2. The two cases of developer’s position distribution are as follows: one is located at both ends of [0, 1], the other is located at any position in the interval [0, 1].

**Model analysis**

**Methods:** The first case: it’s locating at both ends of [0, 1]: As is shown in the Fig. 1, the developer 1 who is the insider to develop traditional architectural structure is located at x = 0, the developer 2 who is the insider to develop renewable energy building structure is located at x = 1. Taking p_i as the price of the developer i, then the strategy of developer i set for S_i = {0, 1}, i = 1, 2, c_i is the cost of the developer i, D(p_i, p_j) is the demand function: i = 1, 2.

There exists a point x in [0, 1], where either consumers purchase the house property of developer 1 or buy the real estate of developer 2, the travel cost they spend is equivalent. That is, it satisfying the following conditions:

\[ p_i + tx = p_j + t(1-x) \]

Then:

\[ D_i(p_i, p_j) = x = \frac{p_j - p_i + t}{2t} \]

\[ D_j(p_i, p_j) = 1-x = \frac{p_i - p_j + t}{2t} \]

![Fig. 1: Two developers at both ends of [0, 1]](image)

The profit functions of the two developers are as follows:

\[ F_i(p_i, p_j) = (p_i - c_i)D_i(p_i, p_j) = \frac{1}{2t}(p_i - c_i)(p_j - p_i + t) \]

\[ F_j(p_i, p_j) = (p_j - c_j)D_j(p_i, p_j) = \frac{1}{2t}(p_i - c_j)(p_i - p_j + t) \]

Developer 1 and 2 option the maximum price difference between selling price and cost in order to maximum their profits. Figure out the first derivative:

\[ \frac{\delta F_i}{\delta p_i} \]

and let:

\[ \frac{\delta F_i}{\delta p_i} = 0 \]

computing the optimal reaction function to get the following results:

\[ \frac{\delta F_i}{\delta p_i} = p_i + c_i + t - 2p_i = 0 \]

\[ \frac{\delta F_j}{\delta p_j} = p_j + c_j + t - 2p_j = 0 \]

Solving the simultaneous equations:

\[ p_i^* = \frac{c_i + 2c_j}{3} + t \]

\[ p_j^* = \frac{2c_i + c_j}{3} + t \]

\((p_i^*, p_j^*)\) is the Nash equilibrium point who is corresponding to the equilibrium outcome. Each developer's profit is:

\[ F_i = F_j = \frac{1}{2t}\left(\frac{c_i - c_j}{3} + t\right)^2 \]

The second case: it's locating at any position in the interval [0, 1]:

Figure 2 shows that the developer 1 is still in the left of the developer 2 and at a(a>0), the developer 2 is located at 1-b(b>0), moreover, 1-a-b>0. Here let the travel cost be the quadratic function of the distance, the point that
Fig. 2: Two developers at any position of [0, 1] consumers away from the two place consuming the same travel costs should satisfy the following equation:

\[ p_1 + t(x - a)^2 = p_2 + t(1 - b - x)^2 \]

Then:

\[ D_1(p_1, p_2) = x = a + \frac{1 - a - b}{2} + \frac{p_2 - p_1}{2t(1 - a - b)} \]

\[ D_2(p_1, p_2) = 1 - x = b + \frac{1 - a - b}{2} + \frac{p_1 - p_2}{2t(1 - a - b)} \]

The profit functions of the two developers are as follows:

\[ E_1(p_1, p_2) = (p_1 - c_1)D_1(p_1, p_2) - (p_1 - c_1) \left[ a + \frac{1 - a - b}{2} + \frac{p_2 - p_1}{2t(1 - a - b)} \right] \]

\[ E_2(p_1, p_2) = (p_2 - c_2)D_2(p_1, p_2) - (p_2 - c_2) \left[ b + \frac{1 - a - b}{2} + \frac{p_1 - p_2}{2t(1 - a - b)} \right] \]

Figure out the first derivative:

\[ \frac{\partial E_1}{\partial p_1} \]

and let:

\[ \frac{\partial E_1}{\partial p_1} = 0 \]

computing the optimal reaction function to get the following results:

\[ \frac{\partial E_1}{\partial p_1} = 1 + a - b + \frac{p_2 - 2p_1}{t(1 - a - b)} + \frac{c_1}{t(1 - a - b)} = 0 \]

\[ \frac{\partial E_2}{\partial p_2} = 1 + a - b + \frac{p_1 - 2p_2}{t(1 - a - b)} + \frac{c_2}{t(1 - a - b)} = 0 \]

Solving the simultaneous equations:

\[ p_1^* = \frac{c_1 + 2c_2}{3} + \frac{t(1 - a - b)(3 + a - b)}{3} \]

Here, \((p_1^*, p_2^*)\) is the Nash equilibrium point.

**RESULTS**

**Case 1:** It's locating at both ends of \([0, 1]\):

\[ p_1^* = \frac{c_1 + 2c_2}{3} + t \]

\[ p_2^* = \frac{2c_1 + c_2}{3} + t \]

\((p_1^*, p_2^*)\) is the Nash equilibrium point who is corresponding to the equilibrium outcome.

**Case 2:** It's locating at any position in the interval \([0, 1]\):

\[ p_1^* = \frac{2c_1 + c_2}{3} + \frac{t(1 - a - b)(3 - a + b)}{3} \]

\[ p_2^* = \frac{c_1 + 2c_2}{3} + \frac{t(1 - a - b)(3 - a + b)}{3} \]

\((p_1^*, p_2^*)\) is the Nash equilibrium point.

**GAME ANALYSIS BETWEEN THE DEVELOPER AND THE CONSUMER**

Real estate developers choose to develop various commercial housing and consumers who are in a passive position choose their favorite commercial housing. Consumers can be divided into three groups: investment buyers, speculative buyers and autonomy buyers. No matter what kind of consumers they belongs to, consumers must buy a house first in terms to investing, trading or living. So all the consumers can be seen as the same in the game (He, 2006).

**Model assumption:** Real estate developers develop various commercial housing. It can be renewable energy construction building, or traditional construction building. And then consumers can be grouped into two kinds: One prefer renewable energy construction building, the other prefer traditional construction building. Developers have two strategies: obtain high valence via strengthening propaganda or not. Consumers also have two strategies: buy or not (Xiang and Zhang, 2012). Developers don’t know consumers’ preferences for energy-efficient
buildings. And consumers don’t know the reliability of the developers’ description of the renewable energy construction building.

The basic motive force of developers is obtaining more profit. For the consumers who are in demand, if developers develop traditional construction and the profit margin between cost and price is set at 1, then the profit is 1. If the building is renewable energy construction building, the price increase with the cost’s increasing. Taking preferential policy for renewable energy construction into consideration, the profit can be set at 2. If they sell the building via strengthening propaganda, they get more profit because of lower cost, set at 4. If the consumers do not have any demand, no matter what building structure, what publicity measures were taken, the building can not be sold out and the profit is set at -1. Considering the cost factor, the profit is set at -2 for renewable energy construction, at -3 for developers who develop traditional construction building and take strengthening propaganda and at -4 for renewable energy construction building (Li, 2005).

Consumers are divided into three groups: investment buyers, speculative buyers and autonomy buyers. For speculative buyers and investment buyers, they would sell or buy building only if they may obtain profit. For consumers with demand, if they buy traditional construction building with lower price, they pay equivalent cost after the deal, so the profit is 0. Price and profit are the key point for all consumers. Then assume that the profit which is brought by renewable energy construction building to autonomy buyers is 2 but for investment buyers and speculative buyers because they buy it not for living. And assume that the profit brought by price is 2 and the profit is 1 when it meets consumers’ preferences (Xiang and Wang, 2012) The profit in different situations can be shown as follows:

For autonomy buyers with preferences: The profit of renewable energy construction building with high price is: profit of preference + profit of renewable energy construction = 1 + 2 = 3.

The profit of renewable energy construction building with low price is: profit of preference + profit of renewable energy construction + profit of price = 1 + 2 + 2 = 5.

The profit of traditional construction building with high price is: profit of price-profit of preference = 2-1 = 1.

The profit of renewable energy construction building with low price is: profit of renewable energy construction - profit of price - 4.

The profit of traditional construction building with high price is: Profit of renewable energy construction + profit of price - 3.

For speculative buyers and investment buyers with preferences: The profit of traditional construction building with low price is: Profit of price-profit of preference = 2-1 = 1.

The profit of renewable energy construction building with low price is: Profit of price-profit of renewable energy construction = -2-2 = -4.

The profit of renewable energy construction building with high price is: Profit of price - 2.

For speculative buyers and investment buyers without preferences: The profit of renewable energy construction building with high price is zero.

The profit of renewable energy construction building with low price is: profit of price-profit of renewable energy construction = -2-2 = -4.

The profit of traditional construction building with high price is: profit of price - profit of renewable energy construction = 2-1 = 1.

This can be shown as Table 1.

All these data above can be explained rationally. For example, the developers sell renewable energy construction building with a low price and buyers with preference buy it at a low price, the profit can be expressed as (2, 5). Developing renewable energy construction building run a higher risk than developing traditional construction building and invest more, so the profit for developer is lower. This is shown as 2<3. But for buyers with preference, they get more booms. This is expressed as 5>3. For buyers without preference, what they care is price, not renewable energy construction, so their profit is 4, between 2 and 5.

Developers have two types: \( T_1 = \{ A_1, A_2 \} \) Renewable energy construction building marked as \( A_1 \) traditional construction building marked as \( A_2 \). The buyers also have two types: \( T_2 = \{ B_1, B_2 \} \). It call buyers with preference \( B_1 \),

<table>
<thead>
<tr>
<th>Developers</th>
<th>Consumers</th>
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<tbody>
<tr>
<td></td>
<td>Favorism</td>
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<tr>
<td></td>
<td>Buying</td>
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<tr>
<td>Renewable energy construction</td>
<td>High price</td>
</tr>
<tr>
<td></td>
<td>Low price</td>
</tr>
<tr>
<td>Traditional construction</td>
<td>High price</td>
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<td></td>
<td>Low price</td>
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</tbody>
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and without preference $B_i$. Developers' strategy set is $S_1 = \{H, L\}$, buy a house with high price is H, at low price is L. Buyers' strategy set is $S_2 = \{Y, N\}$. To buy marked as Y and what buy not is N.

Now assume that the probability of buyers with preference to buy renewable energy construction building is 0.25 and to buy traditional construction building is 0.25, too. The probability of buyers without preference both to buy renewable energy construction building and to buy traditional construction building is 0.25. That is $P(A_1, B_1) = P(A_1, B_2) = P(A_2, B_1) = P(A_2, B_2) = 0.25$. It will analysis which strategy should the buyers or developers take in the following part of the study.

**MODEL ANALYSIS**

**Methods:** Based on the Bayesian principle and assumptions obtained:

$$P(B_1 | A_1) = \frac{P(A_1, B_1)}{P(A_1, B_1) + P(A_2, B_1)} = \frac{0.25}{0.25 + 0.25} = \frac{1}{2}$$

$$P(B_2 | A_1) = \frac{P(A_1, B_2)}{P(A_1, B_1) + P(A_2, B_2)} = \frac{0.25}{0.25 + 0.25} = \frac{1}{2}$$

$$P(B_1 | A_2) = \frac{P(A_1, B_1)}{P(A_1, B_1) + P(A_2, B_1)} = \frac{0.25}{0.25 + 0.25} = \frac{1}{2}$$

$$P(B_2 | A_2) = \frac{P(A_1, B_2)}{P(A_1, B_1) + P(A_2, B_2)} = \frac{0.25}{0.25 + 0.25} = \frac{1}{2}$$

And also:

$$P(A_1 | B_1) = P(A_2 | B_2) = P(A_1 | B_2) = P(A_2 | B_1) = \frac{1}{2}$$

The hybrid strategy is $(x_i, 1-x_i) \in [0,1]$ when the developer is type $A_0$, $(x_i, 1-x_i) \in [0,1]$ when the type is $A_0$. The hybrid strategy for buyers is $(y_i, 1-y_i) \in [0,1]$ when the type is $B_1$ and $(y_i, 1-y_i) \in [0,1]$ when type is $B_2$.

According to Table 1, when the developer's type is $A_0$, the gains matrix with two types of buyers can be shown as:

$$A_0 = \begin{pmatrix} 3 & -4 \\ 2 & -2 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 4 & -3 \\ 1 & -1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 4 & -3 \\ 1 & -1 \end{pmatrix}$$

The expected revenue is:

$$E_1(x_i, y_i, y_j) = \frac{1}{2} (x_i, 1-x_i)A_0(x_i, 1-y_i) + \frac{1}{2} (x_i, 1-x_i)A_2(y_i, 1-y_j)$$

For Bayesian static game $G = [N, (T_i), P, (S_i(t_i)), (u_i)]$, if $s_i(t_i), ... s_i(t_n), ... s_i(t_k)$ is a hybrid combination and for each $i \in N_i$, the Bayesian equilibrium necessary and sufficient conditions for $s_i \in S_i$, $t_i \in T_i$ is:

$$\sum_{s_i \in S_i} p_i(t_i, s_i)u_i(s_i(t_i), s_i) \geq \sum_{s_i \in S_i} p_i(t_i, s_i)u_i(s_i(t_i), s_i)$$

The necessary and sufficient conditions for Bayesian Nash Equilibrium is:

$$E_1(x_i = 0, y_i, y_j) \leq E_1(x_i, y_i, y_j)$$

$$E_2(x_i = 1, y_i, y_j) \leq E_1(x_i, y_i, y_j)$$

Based on the Eq. 1 and inequalities above obtained:

$$\begin{cases} x_i = 0, \\ 0 < x_i < 1, \\ x_i = 1, \end{cases}$$

$$3y_i + 3y_j \leq 4$$

$$3y_i + 3y_j = 4$$

According to Table 1, when the developer's type is $A_0$, the gains matrix with two types of buyers can be shown as:

$$E_2(x_i, y_i, y_j) = \frac{1}{2} (x_i, 1-x_i)A_0(y_i, 1-y_i) + \frac{1}{2} (x_i, 1-x_i)A_2(y_i, 1-y_j)$$

The necessary and sufficient conditions for Bayesian Nash Equilibrium is:

$$E_2(x_i = 0, y_i, y_j) \leq E_2(x_i, y_i, y_j)$$

$$E_2(x_i = 1, y_i, y_j) \leq E_2(x_i, y_i, y_j)$$

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Based on the Eq. 2 and inequalities above obtained:

\[
\begin{align*}
\begin{cases}
x_2 = 0, \\
0 < x_1 < 1, \\
x_1 = 1. \\
\end{cases}
\end{align*}
\]  

\[(\text{ii})\]

\[
\begin{align*}
5y_1 + 5y_2 & \leq 4 \\
5y_1 + 5y_2 & = 4 \\
5y_1 + 5y_2 & \geq 4
\end{align*}
\]

According to Table 1, when the buyers’ type is \(B_1\), the gains matrix with two types of developers can be shown as:

\[
B_{11} = \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}
\]

\[
B_{12} = \begin{pmatrix} -3 & 0 \\ 0 & 1 \end{pmatrix}
\]

The expected revenue is:

\[
E_{11}(x_1, x_2, y_1 = 0) = \frac{1}{2}(x_1, 1 - x_2)B_{11}(y_1, 1 - y_2)T
\]

\[
T + \frac{1}{2}(x_1, 1 - x_2)B_{12}(y_1, 1 - y_2)T
\]

\[(3)\]

The necessary and sufficient conditions for Bayesian Nash Equilibrium is:

\[
E_{11}(x_1, x_2, y_1 = 0) \leq E_{11}(x_1, x_2, y_2)
\]

\[
E_{11}(x_1, x_2, y_1 = 1) \leq E_{11}(x_1, x_2, y_2)
\]

Based on the Eq. 3 and inequalities above obtained:

\[
\begin{align*}
\begin{cases}
y_1 = 0, \\
0 < y_2 < 1, \\
y_1 = 1. \\
\end{cases}
\end{align*}
\]  

\[(\text{iii})\]

\[
2x_1 + x_2 \geq 4 \\
2x_1 + x_2 = 4 \\
2x_1 + x_2 \leq 4
\]

According to Table 1, when the buyers’ type is \(B_2\), the gains matrix with two types of developers can be shown as:

\[
B_{21} = \begin{pmatrix} 2 & 4 \\ 0 & -4 \end{pmatrix}
\]

The expected revenue is:

\[
E_{11}(x_1, x_2, y_1 = 0) = \frac{1}{2}(x_1, 1 - x_2)B_{11}(y_1, 1 - y_2)T
\]

\[
T + \frac{1}{2}(x_1, 1 - x_2)B_{22}(y_1, 1 - y_2)T
\]

\[(4)\]

The necessary and sufficient conditions for Bayesian Nash Equilibrium is:

\[
E_{21}(x_1, x_2, y_1 = 0) \leq E_{21}(x_1, x_2, y_2)
\]

\[
E_{21}(x_1, x_2, y_1 = 1) \leq E_{21}(x_1, x_2, y_2)
\]

Based on the Eq. 4 and inequalities above obtained:

\[
\begin{align*}
\begin{cases}
y_2 = 0, \\
0 < y_2 < 1, \\
y_2 = 1. \\
\end{cases}
\end{align*}
\]  

\[(\text{iv})\]

\[
3x_1 + 2x_2 \geq 4 \\
3x_1 + 2x_2 = 4 \\
3x_1 + 2x_2 \leq 4
\]

**Results:** If \((x_1, 1 - x_1), (x_1, 1 - x_2), (x_1, 1 - x_2), (y_1, 1 - y_2)\) is Bayesian Nash Equilibrium for the game, the necessary and sufficient conditions must meet i, ii, iii and iv. Both \(x_1, x_2\) have three cases, so get \(C_i \times C_j = 9\) cases:

1. \((x_1, x_2) = (0, 0)\)
2. \((x_1, x_2) = (0, 1)\)
3. \((x_1, x_2) = (1, 0)\)
4. \((x_1, x_2) = (1, 1)\)
5. \((x_1, x_2) = ((0, 1), (0, 1))\)
6. \((x_1, x_2) = (0, (0, 1))\)
7. \((x_1, x_2) = (1, (0, 1))\)
8. \((x_1, x_2) = ((0, 1), 0)\)
9. \((x_1, x_2) = ((0, 1), 1)\)

In the following part it will analysis which case could meet i, ii, iii and iv.
When $x_1, x_2, y_1, y_2$ are both between [0,], only $x_1 = 1$ can meet inequality ii and $y_1 = 1$ can meet inequality iii. So, 1, 3, 5, 6, 7 and 8 are excluded.

In case 2, it get $y_2 = 1$ when $x_1 = 0, x_2 = 1$ according to iv but this cannot meet inequality i.

In case 4, it get $y_2 = 0$ when $x_1 = 1, x_2 = 1$ according to iv, this also meet inequalities i, ii, iii and iv.

In case 9, it get:

$$y_2 = \frac{1}{3}$$

when $y_1 = 1$ according to i, this also meet inequalities i, ii, iii and iv.

So, the Bayesian Nash Equilibrium for this game is:

$$(0, 0, 1, 0, 1, 0, 0, 0, 0)$$

Or:

$$(x_1, 1-x_2, 0, 0, 0, 0, 0, 0, 0)$$

(Li et al., 2012)

CONCLUSION

When the two developers are located at both ends of the city, the equilibrium value is involved in $t$ and $(c_i-c_j)$, the larger value $(c_i-c_j)$ results in more profit differences between the two developers. The higher the travel cost are, the larger building structural difference is. Furthermore, it leads to more obvious real estate contrast. The two properties can’t substitute for each other very well and the developer strengthens monopoly power to the consumers nearby, this phenomenon causes more fierce competition between developers. So developers should take into account in the round and select the type of building structures suitable for the development of enterprises.

When the two developers are located at anywhere in addition to endpoints of the city, the results calculated with this model are universal. Game analysis for any two developers just needs to distribute the two developers in the normalized interval [0, 1] and apply specific data to compare, then obtain the equilibrium value and select building structures.

Developers should always pay attention to the latest policy, situation in the industry and seize every opportunity that is conducive to the development of enterprises when they choose the building structure. For example, the country released some preferential subsidy policies in order to promote the application of renewable energy in the construction (Peng and Fang, 2003).

If the building is renewable energy construction, the developer can obtain earnings either taking strategy with high price or strategy with low price, if the building is traditional construction, the developer will take strategy with high price for profit (Zhang and Zhang, 2010).

For buyers with preference, they would buy renewable energy construction buildings certainly. But for buyers without preference, they may not buy any buildings, or 1/3 of them would choose to buy and 2/3 of them choose to buy nothing.

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