Drainage Pipe Network Optimization Design Based on Branch-bound Method

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Abstract: The cost functions of the hydraulic parameters design are analyzed, both in single pipe section and many pipe sections, based on the standard prescriptive fixed constraints. The flow diagram of the drainage pipe network parameter optimization design based on branch-bound method is presented and the characteristics of this application are summarized. The method in this paper is different from traditional branch-bound method. With this method, some subsets of solution space are decreased, so the search range is reduced.

Key words: Branch-bound method, drainage pipe network, parameter optimization design, cost functions

INTRODUCTION

The drainage system of towns is used to prevent flood disaster and water pollution and it is absolutely necessary to carry out healthy society circle. The well service of the drainage system is important to help towns implement the sustainable development and the scientific development. However, the drainage system design is crucial decision-making activity of the system work normally. The development level of the town, the denizen's healthy and quality of life, the investment condition of the town and the sustainable development of the economic and even the security of the town, are decided by the drainage system design rationality (Liu, 2008). Thereby, the drainage system design rationality is emphasis in city infrastructure management. Hereinto, pipe parameters design (Wang et al., 2010) is a significant part in pipe network system design.

In the pipe parameters optimization design, the pipe diameter, buried depth (slope) and different pipe section parameters of the appointed underground sewage pipeline system are optimized (Deng and Laibin, 2003). Usually, there are many combinations of pipe diameter and buried depth for a long large sewage network. Essentially it is a mixed integer programming for pipe parameters optimization design (Chen and Lou, 2008) of pipeline layout fixed which continuous range of slope is calculated by discrete standard pipe diameter and search the optimal solution in this range. As a result of the drainage pipe network is the tree structure, the drainage pipe network parameter optimization design (Liu and Lili, 2008) is a problem which ergodics all the values that satisfy each constraints and the objective function in the tree. Whatever, branch-bound method is widely applied in searching tree structure and the basic idea of this algorithmic is searching every feasible solution spaces and during the searching, the solution spaces are incessant divided into smaller subset. In each subset, the lower bound and upper bound of the solutions are calculated, in this way, the branches which had exceeded the known feasible solution are decreased, the searching range is reduced, sequentially.

ANALYSIS OF THE DRAINAGE PIPE NETWORK COST FUNCTIONS

In order to reduce the branches in the drainage pipe network parameter optimization design as much as possible it is necessary to analysis the relationship between the cost of the drainage pipe network and the hydraulic parameters.

In the pipe diameter and buried depth optimization design, the results show that the cost function is defined by pipe diameter and buried depth, satisfying the following:

\[ F = f(d, h) \]

Zhang (1993). Actually, for a pipe section design, when the flow has been defined, the pipe diameter \( d \) and buried depth \( h \) which satisfy the standard requirement are many group. If \( d \) is selected larger, \( h \) should be smaller, that is the pipe cost is higher but the buried pipe cost is lower. Whereas, if the pipe cost is lower, the buried pipe cost will be higher. Thus, there is a pair of \( d \) and \( h \), which can ensure the lowest cost (Hao et al., 2010).

MAIN DRAINAGE PIPE NETWORK CONSTRAINT CONDITIONS

Design full degree: The constraint condition of full degree is:

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Buried depth of pipeline: The constraint condition of sewage pipeline buried depth is:

\[ H_{\text{min}} \leq H \leq H_{\text{max}} \]

According to the specification, minimum thickness of covering soil should be: under the sidewalk 0.6 m, roadwa 0.7 m. In general, the drainage pipeline should be buried below the frost line but when the area or similar areas have shallow experience and take corresponding measures, that also can be buried above the frost line, the shallow value shall be according to the area experience.

Analysis of pipe section hydraulic parameters: For a single pipe section, the average buried depth \( h_b \) can be approximated by:

\[ \frac{h_b}{D} = \left( h_b / D \right)_{\text{avg}} \leq \frac{h_b}{D} \leq \left( h_b / D \right)_{\text{max}} \]

\[ h_b = \left( h_1 + h_2 \right) / 2 \]

Let \( L \) define the length and \( i \) define the slope, so that:

\[ H_j = h_j + L \cdot i_j \]

\[ h_j = (2h_1 + L \cdot i_1) / 2 \]

\[ F_j = f(d, i, h) \]

Also, according to the actual situation, the cost function becomes larger with \( d \), the slope \( i \), and the buried depth \( h \).

Analysis of many pipe sections cost function: For many pipe sections system, the design of different pipe section can interact each other; the downriver results depend on the upstream pipe bottom elevation. However, the upstream pipe bottom elevation depends on the upstream buried depth. As the whole system, the many pipe sections optimization depends on the suitable pipe diameter \( d \) and buried depth \( h \), so that the sum cost of the upstream pipe design and the downriver design pipe is lowest.

Figure 2 shows the two pipe sections system consists of downriver pipe design and the upstream pipe design, the whole cost can be calculated by:

\[ F_{13} = F_{13;1} + F_{23} \]
Fig. 2: Two pipe sections system

\[ F_{13} = F_{12} + F_{23} \]
\[ = f_i(d_{12}, i_{12}, h_i) + f_i(d_{23}, i_{23}, h_i) \]
\[ = f_i(d_{12}, i_{12}, h_i) + f_i(d_{23}, i_{23}, h_i) \]

Therefore, if \( F_{12} > F_{13} \) and \( h_i > h_b \), it is known \( F_{13} \) and \( h_i \) are in direct proportion and \( F_{12} > F_{13} \), hence \( F = F_{13} + F_{23} + F_{3b} = F_{9} \).

Consequently, during searching, if \( F = F_{13} + F_{23} + F_{3b} = F_{9} \), we can obtain that the cost of buried depth \( a \) is higher than the cost of buried depth \( b \) and the branches of feasible solution \( b \) are decreased.

In tree structure, the arrangements are 1-3 pipe section, 2-3 pipe section and 3-4 pipe section. They are showed in Fig. 3.

The whole cost is defined by \( F \), so it can be expressed by the following equation:

\[ F = F_{11} + F_{12} + F_{34} = f_i(d_{11}, i_{11}, h_i) + f_i(d_{12}, i_{12}, h_i) + f_i(d_{34}, i_{34}, h_i) \]

where, in \( f_i(d_{ij}, i_{ij}, h_i) \), the upstream pipe section of 3-4 pipe section consists of 1-3 pipe section and 2-3 pipe section. Based on drainage regulation, the upstream pipe bottom elevation of 3-4 pipe section \( h_i \) is defined by the lower of the downstream pipe bottom elevation of the 1-3 pipe section \( h_{13, low} \) and 2-3 pipe section \( h_{23, low} \):

\[ h_{i, low} = \min(h_{13, low}, h_{23, low}) \]

During the circulatory searching, we can compare the united cost of buried depth \( F_{11} + F_{12} + F_{34} \) and the upstream pipe bottom elevation min \((h_{11}, h_{23})\). Comparing the pipe diameters of a and b combination, if \( F_{13} + F_{23} > F_{13} + F_{3b} \) and min \((h_{13}, h_{23})\), the pipe diameters of b combination is deleted.

According to the "pruning" theoretical analysis and combined with traversal optimization algorithm steps, finally we obtained the following branch and bound method for pipeline parameter optimization calculation process (Fig. 4):

- In 16-15, 10-7 and other end of branch pipe sections are started with various standard pipe diameters
- Calculate the slope range according to the flow
- Calculate the minimum average buried depth according to the upstream pipe bottom elevation and slope range
- Calculate the cost function of pipe diameters and buried depth \( F_{13} \), \( F_{3b} \) Calculate the downstream pipe bottom elevation according to the upstream pipe bottom elevation and slope range \( h_{13}, h_{23} \)
- To get to the next level pipe section
- Judge the next level pipe is a node (upstream branch such as 14-12) or the transition point (upstream without branch such as 16-15)
- If it is the transition point
- Circulation in different pipe diameters, compared the average buried depth estimation cost \( F \) and the downstream pipe bottom elevation \( h_{down} \). For \( a, b \) two diameters' comparison, if \( F_{13} + F_{1b} \) and \( h_{1b} < h_{2b} \), we add a diameter to the alternative set and delete b diameter, if \( F_{13} + F_{1b} \) and \( h_{1b} > h_{2b} \), we add both a and b diameter to the alternative set. "pruning" operation according to upstream and downstream cost function
- According to the different diameter of the alternative set, we compute the downstream pipe bottom elevation and estimate the cost, compute \( F_{13} \), thus obtained \( F_{13} = F_{12} + F_{23} \). According to the comparison
of various diameter combinations if F_{15} > F_{1b} and h_{15} < h_{1b}, we add a diameter combination to the alternative set and delete b diameter combination; if F_{15} > F_{1b} and h_{15} > h_{1b}, we add both a and b diameter combination to the alternative set

- If it is a node
- Circulation in different pipe diameters, compared the combined depth estimation cost and the downstream pipe bottom elevation min (h_{15}, h_{1b}). For a, b two diameter combination comparison, if F_{15} + F_{15} > F_{1b} + F_{1b} and , we add a diameter combination to the alternative set and delete b diameter combination; if F_{15} + F_{15} > F_{1b} + F_{1b} and min(h_{15}, h_{1b}) < min(h_{1b}, h_{1b}), we add both a and b diameter combination to the alternative set. pruning” operation according to characteristics of tree structure

- According to the different diameter of the alternative set, we compute the downstream pipe bottom elevation and estimate the cost, compute F_{sa}, thus obtained F_{134} = F_{15} + F_{15} + F_{1b} + F_{1b}. Compared various diameter combinations until finding the diameter combination which has the smallest F_{134} = F_{15} + F_{15} + F_{1b}

- At the same time according to the constraints, delete part of pipe diameter combination scheme
- Layer-by-layer cycle until getting the optimal result.

The flow diagram of drainage pipe network parameter optimization design: According to hydraulic parameters analysis and the characteristics of branch-bound method, the flow diagram of drainage pipe network parameter optimization design is depicted in Fig. 5.

RESULTS

From the above flow diagram it shows that drainage pipe network optimization needs to calculate pipe diameters of each pipe section. In fact, the method in this study is different from traditional branch-bound method.

The final result of the drainage pipe network optimization is integer optimal in the network, so the computational complexity is higher than optimum path search.

The calculation of the drainage pipe network optimization is from the bottom to the top it means that

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![Flow diagram of drainage pipe network parameter optimization design](image-url)

Fig. 5: Flow diagram of drainage pipe network parameter optimization design
calculating the possibility of the each pipe diameters which is in the bottom branch firstly and then moving to top step by step, so cutting the possibility of the branch in the beginning is not allowed.

In the calculation of the drainage pipe network optimization, each pipe section should be calculated by branch-bound method, so the problem of pruning the branch is not existent. But the branch can be pruned by some constraints and optimization methods. Although, the branch can not be pruned completely, the possible range of the pipe diameter combination of each branch is less and the result is similar with pruning.

Although the calculation of the drainage pipe network optimization has many differences from branch-bound method, the width prior traversal method and the reducing possible pipe diameter value method have no essence difference. Therefore, in this searching method, the space complexity replaces time complexity and practice proves that this method can search the optimal result fast in the calculation of the drainage pipe network optimization design.

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