Comparison between ARX and FIR Decorrelation Models in Detecting Model-plant Mismatch

Nur Hidayah Kamal Iqbal, Nooryusmiza Yusoff and Lemma Dendena Tufa
Department of Chemical Engineering, Universiti Teknologi PETRONAS, Bandar Seri Iskandar, Tronoh, 31750, Perak, Malaysia

Abstract: Partial correlation analysis is used in detecting the model-plant mismatch as it will give accurate location of mismatched submodel. In the decorrelation step of the observed input and model residual from the other affecting inputs, model is identified to capture the dynamic relationship of the input-output data. In this study, the use of Auto Regressive with Exogenous Inputs (ARX) and Finite Impulse Response (FIR) model structures are compared to investigate the effectiveness of detecting the gain mismatch using different model structures. The optimal numbers of parameters of the estimated models also plays a significant role in the detection of mismatched submodel. In this research, the use of the ARX model structure outperforms the FIR model structure in detecting the single submodel mismatch which successfully demonstrated to the process system of Wood and Berry distillation column.

Key words: Decorrelation models, ARX, FIR, model-plant, submodel

INTRODUCTION

Model Predictive Control (MPC) plays a significant role in process industries and widely used. MPC term refers to an acceptable range of control methods which explicitly use a process model to obtain the control signal by minimizing an objective function (Camacho and Bordons, 2003). The calculations are based on current measurements and predictions of the future values of the outputs over a finite horizon (Seborg et al., 2004). The ability of MPC controller to predict future outputs accurately relies on the goodness of the process models. Over time, changes in plant dynamics and nonlinearities in the process broaden the discrepancies between the model and process and it is classified as Model-plant Mismatch (MPM). Any discrepancies between process model and real plant beyond acceptable threshold are not tolerable. MPM will contribute in the degradation of MPC performance which in need for expensive re-identification of the new process model. Thus, it is significant to detect the exact location of submodel mismatch and concentrate on that particular subsystem/submodel for re-identification process (Badwe et al., 2009).

Thorough research has been done over last three decades since 1980s to improve the performance of MPC controllers (Harris, 1989) and its future has thrived ever since. A method of error detection in frequency domain with the introduction of sinusoidal test signals was introduced by Ji et al. (2012). The difference between the estimated frequency response and actual response from MPC controller at three points are employed to form model error matrix index.

Mismatch detection using partial correlation analysis was introduced by Badwe et al. (2009) by employing Manipulated Variables (MVs) and model residuals from the closed-loop operational data. The authors also concluded that the use of partial correlation outperformed the regular correlation analysis as it gave misleading location of the mismatched subsystem. The partial correlation analysis between the MVs and model residuals was performed by controlling the effects of other inputs. The effects of other inputs are decorrelated from the observed input and model residual to obtain absolute correlation. However, the authors did not acknowledge the specific decorrelation model used to identify the dynamic relationship between the observed input to the others which affect the absolute correlation of the observed input and model residual. Carlsson (2010) used the method proposed by Badwe et al. (2009) which controlling the effect of previous inputs by demonstrating the method on both linear and nonlinear model-plant mismatch. Badwe et al. (2010) continued their previous research on MPM to investigate its impact to control quality which is based on setpoint directions. The research found that set point direction also gave a significant impact on the mismatch detection in quality control. Other research for MPM detection was performed by Kano et al. (2010) based on statistical variable

Corresponding Author: Nur Hidayah Kamal Iqbal, Department of Chemical Engineering, Universiti Teknologi PETRONAS, Bandar Seri Iskandar, Tronoh, 31750, Perak, Malaysia Tel: +60197305996
selection method. In this study, a score is proposed to measure the significance of the mismatch in submodel. From the demonstration, significant mismatch introduced in submodel gave significant score compared to low level of mismatch introduced at other submodels. This work proved that the method used is superior to other methods of Multivariable Regression Analysis (MRA), Partial Least Square (PLS) and Correlation Analysis (CA). Nafiu and Yusoff (2011) investigated the effect of MPM to the performance of an MPC scheme commissioned on Wood-berry column. They concluded that gain mismatches contributed significantly to the deterioration of MPC performance. Therefore, the current work attempts to detect the gain parameter mismatch by employing partial correlation analysis between the MVs and model residuals controlling for other affecting inputs.

In the decorrelation process of the observed input from other affecting inputs, identification using Auto Regressive with Exogenous Inputs (ARX) and Finite Impulse Response (FIR) models are employed. The effectiveness of these models during the decorrelation step in detecting the MPM is investigated. The identification process to develop model which capture the dynamic relationship between the input-output data is significant to be implemented in the decorrelation step of the methodology proposed by Badwe et al. (2009). ARX and FIR models are the most commonly used linear-time invariant system (Lemma et al., 2010) and most of system identification research evolved from these simple models. Equation 1 represents the ARX model structure:

$$y(k) = \frac{B(q)}{A(q)} u(k) + \frac{1}{A(q)} e(k)$$

where, Eq. 2-3:

$$A(q) = 1 + a_q q^{-1} + a_2 q^{-2} + \ldots + a_n q^{-n}$$

$$B(q) = b_1 q^{-1} + b_2 q^{-2} + \ldots + b_m q^{-m}$$

A(q) and B(q) consist of a set of model parameters predicted using the input-output data. The model parameters are estimated using MATLAB’s arx function. The size of ARX model depends on the number of poles, na and the number of zeros, nb. The system delay is defined as, nk. For easier notation, nk = 1 is mostly used (Ljung, 1999).

As shown in Eq. 4, the FIR model differs from the ARX model due to the omission of parameter A(q):

$$y(k) = B(q)u(q) + e(q)$$

where, Eq. 5:

$$B(q) = b_1 q^{-1} + b_2 q^{-2} + \ldots + b_m q^{-m}$$

The size of FIR model is described by the model order, m. FIR model parameters B(q) are estimated using the least-square method as shown in Eq. 6:

$$\theta = (X^T X)^{-1} X^T y$$

where, \( \theta \) is the vector of the model parameters, y is the N-th output data sequence and X is the regressor matrix which is arranged in a particular form as shown by Eq. 7:

$$X = \begin{bmatrix}
    u(m) & u(m-1) & \ldots & u(1) \\
    u(m+1) & u(m) & \ldots & u(2) \\
    \vdots & \vdots & \ddots & \vdots \\
    u(N-1) & u(N-2) & \ldots & u(N-m)
\end{bmatrix}$$

**METHODOLOGY**

**IMC structure analysis:** Figure 1 represents the closed-loop system based on the Internal Model Control (IMC) arrangement. The controller Q represents the MPC controller used in the process system. The relations between the inputs and model residuals to the setpoint and unmeasured disturbance are represented by Eq. 8 and 9 (Badwe et al., 2009). \( S_i \) and \( S_u \) are the input sensitivities affected by the setpoints and unmeasured disturbances, respectively as shown in Eq. 8, 9:

$$e(k) = [I+\Delta Q]^{-1} \Delta r(k) + [I+\Delta Q]^{-1} v(k)$$

$$u(k) = Q[I+\Delta Q]^{-1} \Delta r(k) - Q[I+\Delta Q]^{-1} v(k)$$

where, \( Q[I+\Delta Q]^{-1} \Delta r(k) = S_i \) and \( Q[I+\Delta Q]^{-1} v(k) = S_u \).

The presence of unmeasured disturbances, \( v(k) \), in the input, u(k) and model residual, e(k), may disturb the true correlation as mismatch may be falsely detected. Therefore, each of the inputs and model residuals must be free from the effect of unmeasured disturbance.

**Case study: Wood-berry column:** Wood and Berry model of a distillation column (Wood and Berry, 1973) for

![Fig. 1: Closed-loop system based on IMC structure](image)
separation of methanol-water mixture is used as a test bed. The actual model consists of two Manipulated Variables (MVs), two Controlled Variables (CVs) and a measured disturbance (D). The objective is to control the composition of methanol at the distillate (X_D) and bottoms (X_B) (measured in wt.% methanol) by manipulating the reflux flow rate (R) and steam flow rate (S) (measured in lb min\(^{-1}\)). The steady state values for the X_D, X_B, R and S are 96.0 and 0.5 wt.%, 1.95 and 1.71 lb min\(^{-1}\), respectively. After removing the disturbance effect, the process model relating the CVs to the MVs is shown in Eq. 10:

\[
\begin{bmatrix}
X_D(s) \\
X_B(s)
\end{bmatrix} = \begin{bmatrix}
12.8e^{-3} & -18.9e^{-5} \\
6.6e^{-7} & -19.4e^{-5}
\end{bmatrix} \begin{bmatrix}
R(s) \\
S(s)
\end{bmatrix}
\]  

(10)

The 2×2 Multi-Input-Multi-Output (MIMO) process is simulated in MATLAB-Simulink environment. A sampling time of 1 min is employed to produce 2000 samples. In the decoupling step, the first 1000 samples are used for model identification and the rest for model validation. Pseudo-random Binary Sequence (PRBS) signals are generated using the id input function to excite the process. The band and amplitudes of the signals are \([0.0004]\) and \([-3.0.3]\), respectively.

For the design and tuning of the MPC controller used in the simulation, the control and prediction horizons are 20 and 30, respectively. The bounds of input rates and outputs are \([-10 10; -10 10]\) and \([-96 4; -0.5 99.5]\), respectively. The weighting matrices for the inputs, input rates and outputs are diag [0.0], diag [0.1 0.1] and diag [1.5 1.5], respectively. During simulation, unmeasured disturbances of zero mean normally distributed white noise are added as the output disturbances with \(\sigma_{e_{d1}} = 0.001\) and \(\sigma_{e_{d2}} = 0.0001\).

**Partial correlation analysis:** Partial correlation analysis is performed for each process submodel implemented in the controller, Q. In the analysis for m×n MIMO system, correlation at \(Q_i\) submodel is between \(i\)-th model residual and \(j\)-th input which result in a correlation matrix as shown in Eq. 11:

\[
\text{Corr}(e, u) = \begin{bmatrix}
\text{Corr}(e_i, u_1) & \ldots & \text{Corr}(e_i, u_m) \\
\text{Corr}(e_j, u_1) & \ldots & \text{Corr}(e_j, u_m)
\end{bmatrix}
\]  

(11)

The methodology used in this present study was proposed by Badwe et al. (2009). Here, the partial correlation is analyzed in dynamic sense between the observed input and model residual by using an unknown dynamic decorrelation model obtained using the Prediction Error Method (PEM). In the present study, decoupling models in the forms of ARX and FIR are employed to investigate the effect of model structure and model order selections on the MPM detection.

The flow diagram of the detection of MPM using partial correlation analysis is illustrated in Fig. 2. In the first step, 3σ control limits are established. Then, gain mismatch is introduced to one of the process submodels. The MPM is detected using the partial correlation analysis which can be described as follows:

- The MVs and model residuals data are chosen in which it contain sufficient setpoint excitation in the process.
- The disturbance free component of MVs are found by the Eq. 12 below:

\[
u(k) = S_u u(k) - S_v u(k) - u'(k) + u''(k)
\]  

(12)

where \(u'\) and \(u''\) are the components of \(u\) which contain effects of setpoints and disturbances, respectively. Both components are uncorrelated. \(S_u\) is obtained using Eq. 13 and \(u'\) is predicted using Eq. 13 and 14:

\[
u(k) = S_u u(k) + S_v u(k)
\]  

(13)

\[
u'(k) = S_u u(k)
\]  

(14)

where, \(u'\) is the component of \(u\) which is freed from the effects of disturbances. In this study, \(u'\) and \(u''\) are affected by both \(r_i(k)\) and \(r_s(k)\).

- The \(u''\) component is decorrelated from the effects of other MVs. The ARX and FIR models are identified between \(u''\) and other MVs, as shown in Eq. 15:

\[
u''(k) = G_u u''(k) + v_u(k)
\]  

(15)

where, \(v_u\) is the free disturbance component of other MVs and \(v_u\) is the component of \(u''\) that is uncorrelated from other MVs. The estimated of \(v_u\) which is the component of observed MV (\(u\)) that is free from effects of disturbances and other MVs, is obtained using Eq. 16:

\[
u''(k) = G_u u''(k) - v_u(k) - G_u u''(k)
\]  

(16)

- The red-dashed box in Fig. 2a indicates the steps where ARX and FIR decorrelation models are identified to obtain decorrelated inputs and model residuals data.
Fig. 2(a-b): (a) Flow diagram for the detection of model-plant mismatch using partial correlation analysis, modified from (Badwe et al., 2009) and (b) Flow diagram for the identification of the optimal model of the predicted data. This figure contains comprehensive explanation of the red-dashed box in Fig. 2a.

- Model residual in consideration, $e_i$, is decorrelated from other MVs ($u'$) by employing ARX and FIR model structures, similar to step 3 (Eq. 17):

  $$ r_e(k) = Ge_x\hat{u}(k) + e_x(k) \quad (17) $$

  where, the estimate of $e_x$ is obtained by:

  $$ \hat{e}_p(k) = e_r(k) - Ge_x\hat{u}(k) \quad (18) $$

- Correlation between $e_i$ and $\hat{u}$ is evaluated in which indicates the presence of MPM in the $u_i$-$y_i$ channels. The nearer the correlation function to the absolute value of 1, the more significant is the mismatch.
Fig. 3(a-b): (a) Baseline partial correlation detection shows the minimum allowable mismatch to the system and (b) G_{11} submodel is zoomed in to show the UCL and LCL.

However, the ARX and FIR model structures are compared in the decorrelation steps to investigate the effect of different model structure and number of parameters on the mismatch detection. The optimal decorrelation model structures are identified as shown in the flow diagram in Fig. 3. The Percentage Prediction Error
(PPE) is calculated to assess the accuracy of the identified decorrelation model structure which best explain the input-output data. In other words, the squared-deviations of predicted data are compared to that based on the mean of the original data as shown in Eq. 19:

\[
(PPE)_i = \frac{\sum (y_i(k) - \hat{y}_i(k))^2}{(\hat{y}_i(k) - \bar{y}(k))^2} \times 100
\]  

A lower PPE value indicates more accurate model identification whereas a higher PPE value indicates the closeness of the model to the mean of the original data.

RESULTS AND DISCUSSION

In this study, single and multiple gain mismatch(es) are introduced to investigate how the decorrelation models used, ARX and FIR, affect the detection of gain mismatches introduced in the Wood-berry column system.

In the detection of MPM using this 2×2 case study, the data are predicted from the identified decorrelation models which are:

- The disturbance free components the Mvs: \( \bar{u}_1 \) and \( \bar{u}_2 \)
- The decorrelated MV components which is freed from other MV: \( \hat{e}_u \) and \( \hat{e}_u \)
- The decorrelated model residuals which is freed from other MV: \( \hat{e}_i \) and \( \hat{e}_i \)

Therefore, the PPE value is calculated in each step until the optimal order is obtained to ensure that the mismatch is detected at the exact submodel.

Baseline correlation due to unmeasured disturbance: To establish a baseline, no mismatch is introduced at any of the submodels. The correlations between the MVs and residuals exist due to the presence of unmeasured disturbance only. The upper and lower control limits (UCL and LCL) of 0.0671 and -0.0671, respectively are estimated using 'crosscorr' function in MATLAB based on 3σ confidence interval from the mean of the data. The UCL and LCL are represented by the blue lines in all submodel cross-correlation plots in Fig. 3. If the red matchsticks exceed the control limits, the MPM is deemed detected.

Single submodel mismatch: An overestimated gain mismatch of 30% is introduced at \( G_{ii} \) submodel. From Fig. 4 and 5, the MPM is detected when both ARX and FIR decorrelation models are used. However, the degrees of detection vary between the two.

Model identification procedures are repeated until the optimal structure is obtained. For the identification using ARX model, the calculations of the PPE values with different numbers of parameters are as depicted in

Fig. 4(a-d): Mismatch detection at \( G_{ii} \) submodel using optimal ARX model structure in the decorrelation procedure
Table 1: Percentage Prediction Error (PPE) values of different model parameters in the detection of 30% gain mismatch at $G_1$ submodel using ARX decorrelation model

<table>
<thead>
<tr>
<th>Iteration</th>
<th>$\hat{u}_1$</th>
<th>$\hat{u}_2$</th>
<th>$\hat{e}_1$</th>
<th>$\hat{e}_2$</th>
<th>$\hat{e}_3$</th>
<th>$\hat{e}_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>137.00 [4 4 1]**</td>
<td>58.10 [4 4 1]</td>
<td>88.38 [2 2 1]</td>
<td>54.780 [2 2 1]**</td>
<td>45.62 [4 4 1]</td>
<td>100.00 [5 2 1]</td>
</tr>
<tr>
<td>3</td>
<td>87.82 [7 7 1]*</td>
<td>21.51 [6 6 1]</td>
<td>72.31 [5 5 1]*</td>
<td>2.715 [8 8 1]*</td>
<td>29.45 [7 7 1]*</td>
<td>99.96 [8 8 1]*</td>
</tr>
<tr>
<td>4</td>
<td>13.79 [8 8 1]</td>
<td>32.04 [7 7 1]</td>
<td>166.57 [6 6 1]</td>
<td>0.020 [10 10 1]*</td>
<td>52.11 [8 8 1]</td>
<td>99.98 [10 10 1]</td>
</tr>
</tbody>
</table>

*Optimal model parameters selected which show significant detection of the gain mismatch, **False detection at other submodel or no detection at the $G_1$ submodel

Table 1 whereby the experiment is iterated until the optimal model parameters are obtained. In some cases, the PPE values exceed 100%. This is unacceptable because it shows that the predicted model is very poor compared to the validation data. In the first iteration, predicted values of $\hat{u}_1$ and $\hat{e}_n$ with [4 4 1] and [2 2 1] parameter structure (marked as ** in the table), respectively, show that the mismatch is not detected at $G_1$ submodel.

It can be seen from the Table 1 that to identify those six decorrelation models, different number of parameters gave different PPE and detection results. The same number of model parameters cannot be applied for all six decorrelation models as the input-output data used in model identification are also different. Hence, for every decorrelation model, optimal number of model parameters must be determined.

Therefore, from the result of the optimal model structures chosen for all six decorrelation models (marked as * in Table 1), the estimated model parameters corresponding to Eq 2 and 3 are shown in Table 2.

The same procedure is applied for the detection of 30% gain mismatch at $G_1$ submodel using FIR model structure. The experiment is iterated until optimal numbers of model parameters are obtained. Table 3 shows the calculated PPE value for different number of model parameters using FIR model structure in detecting the applied mismatch. From the table, most of the decorrelation models are poorly identified which can be observed from the calculated PPE value exceeding 50%. In addition, increasing the number of parameters only makes the prediction data worse than the validation data. Therefore, low numbers of parameters are chosen to be optimal in order to keep the PPE value at acceptable level. The optimal numbers of parameters chosen in detection of gain mismatch are marked as * in Table 3. The optimal estimated model parameters are shown in Table 4.

However, these optimal numbers of model parameters are unable to detect the exact location of the introduced mismatch, as shown in Fig. 5. There is a significant correlation between $MV_2$ and $e_2$ existed at $G_{12}$ submodel.
Table 2: Estimated model parameters for all six decorrelation models identified using ARX structure to detect 30% gain mismatch at $G_{3}$ submodel

<table>
<thead>
<tr>
<th>Predicted data</th>
<th>Model parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{u}_1$</td>
<td>$[0.2526 \ 0.6827 \ 0.2443 \ 0.22876 \ 0.0938 -0.3719 -0.0946]$</td>
</tr>
<tr>
<td>$A(q)$</td>
<td>$[0.3899 -0.1535 -0.08660 \ 1002 -0.0082 \ 0.0343 -0.0854]$</td>
</tr>
<tr>
<td>$B(q)$</td>
<td>$[0.00300 -0.0135 -0.0084 \ 0.01000 \ 0.0080 \ 0.0034 -0.0085]$</td>
</tr>
<tr>
<td>$\hat{u}_2$</td>
<td>$[1.0 \ -0.04970 \ 0.73900 \ 2.1240 \ 1.47110 \ 0.04101]$</td>
</tr>
<tr>
<td>$A(q)$</td>
<td>$[0.00004 -0.0564 -0.0043 \ 0.4962 \ 0.3880]$</td>
</tr>
<tr>
<td>$B(q)$</td>
<td>$[0.00045 -0.0056 -0.0050 \ 0.0405 \ 0.0387]$</td>
</tr>
</tbody>
</table>

Table 3: Percentage prediction error of different model parameters in the detection of 30% gain mismatch at $G_{3}$ submodel using FIR decorrelation model

<table>
<thead>
<tr>
<th>Iteration</th>
<th>$\tilde{u}_1$</th>
<th>$\tilde{u}_2$</th>
<th>$\tilde{u}_3$</th>
<th>$\tilde{u}_4$</th>
<th>$\tilde{u}_5$</th>
<th>$\tilde{u}_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>68.22 m = 2*</td>
<td>99.92 m = 2*</td>
<td>69.55 m = 5</td>
<td>10.28 m = 8</td>
<td>7.59 m = 5</td>
<td>100.05 m = 2*</td>
</tr>
<tr>
<td>2</td>
<td>107.58 m = 3</td>
<td>106.50 m = 3</td>
<td>69.39 m = 10*</td>
<td>8.12 m = 10</td>
<td>8.10 m = 8*</td>
<td>100.70 m = 5</td>
</tr>
<tr>
<td>3</td>
<td>142.40 m = 4</td>
<td>105.74 m = 4</td>
<td>69.38 m = 15</td>
<td>5.94 m = 15*</td>
<td>8.19 m = 10</td>
<td>101.96 m = 10</td>
</tr>
<tr>
<td>4</td>
<td>197.36 m = 8</td>
<td>230.61 m = 8</td>
<td>69.38 m = 20</td>
<td>5.32 m = 20</td>
<td>9.61 m = 20</td>
<td>102.28 m = 15</td>
</tr>
</tbody>
</table>

*Optimal model parameters selected which show significant detection of the gain mismatch

Table 4: Estimated model parameters for all six decorrelation models identified using FIR structure to detect 30% gain mismatch at $G_{3}$ submodel

<table>
<thead>
<tr>
<th>Predicted data</th>
<th>No. of model parameter</th>
<th>Model parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{u}_1$</td>
<td>m = 2</td>
<td>$B(q) = [0.4782 -0.4257]$</td>
</tr>
<tr>
<td>$\hat{u}_2$</td>
<td>m = 2</td>
<td>$B(q) = [4.7824 -4.2574]$</td>
</tr>
<tr>
<td>$\hat{u}_3$</td>
<td>m = 10</td>
<td>$B(q) = [-0.0127 \ 0.0317]$</td>
</tr>
<tr>
<td>$\hat{u}_4$</td>
<td>m = 15</td>
<td>$B(q) = [1.0652 \ 0.6445 \ 0.2587 \ 0.1037 \ 0.04123 \ 0.0155 \ 0.0037 \ -0.126 \ -0.02027 \ -0.0308]$</td>
</tr>
<tr>
<td>$\hat{u}_5$</td>
<td>m = 8</td>
<td>$B(q) = [0.0423 \ 0.0380 \ 0.0142 \ 0.0280 \ 0.0255 \ 0.0234 \ 0.0215 \ 0.01997 \ 0.0187 \ 0.0176 \ 0.0160 \ -0.0015 \ 0.0114 \ 0.0118]$</td>
</tr>
<tr>
<td>$\hat{u}_6$</td>
<td>m = 2</td>
<td>$B(q) = [6.0317 \ -0.2806 \ -2.3937 \ 0.3981 \ 1.4066 \ 0.5182 \ 1.9127 \ -1.1436]$</td>
</tr>
</tbody>
</table>

This wrong detection is due to the inability of the FIR model structure to accurately identify the decorrelation models using data from a closed-loop structure. In addition, the absence of error model in the FIR structure also contributes to the poorly predicted data. Hence, FIR model structure is not suitable to be used in the detection of model-plant mismatch using partial correlation analysis.

**CONCLUSION**

The selection of proper model structure and number of model parameters in the identification of the decorrelation model are important to detect MPM. ARX model is found to be superior to that of the FIR model when used in the decorrelation procedure.

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