Scaling Delineation of Precipitation and Evaporation with Wavelet Analysis at Anyang City in North Plain, China

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Abstract: Precipitation and evaporation are essential to water resources availability to economy and society development. Monthly, annual and growing seasonal departure from average and cumulative departure from average precipitation and evaporation series were analyzed using Morlet wavelet analysis at Anyang City in North Plain, China. The results indicated that monthly precipitation and evaporation varies 53 and 46 years. The annual precipitation and evaporation have 52 years scales and the precipitation and evaporation in the growing seasons have 44 and 45 years.

Key words: Anyang, departure from mean, cumulative departure from mean, morlet wavelet analysis, precipitation, evaporation

INTRODUCTION

Anyang city is situated in the semi-arid and semi-humid North Plain of China. Due to population growth, increasing water demand and the possible impacts of climate change, there are growing concerns over shortages of water resources to meet the needs of future economic and social development in the city (Shu et al., 2012; Shi et al., 2011). Precipitation is the crucial factor to assess water resources in a watershed or an area. Water resources play an essential role in economy and society development because water resources are the basic strategic natural resources for the nature and society development (Hu et al., 2010). Evaporation is an essential part of the water cycle and is important to water availability. Concerns have been raised over the quantity and quality of water resources and how precipitation and evaporation changes may affect water resources. There is a need for precipitation and evaporation studies to understand water resources availability and vulnerability.

A series of prior studies have examined climate issues in the North Plain at both large and local scales (Tao and Zhang, 2013). The current researches are focus on the factors of precipitation, temperature on the climate changes and few researches are made on the relations of precipitation and evaporation (Fang et al., 2010).

Recently, the wavelet analysis which is a relatively present development in signal processing that has also appeared as a tool used in trend analysis (Nalley et al., 2012). Wavelet analysis is better than traditional Fourier analysis in frequency analysis mainly because wavelet analysis is capable to describe local features of sign in time and frequency domains and conduct multiple scales analysis in sign analysis (Wang et al., 2011).

Wavelets are often irregular and asymmetric in shape, however, sine waves are the main functions used in Fourier analysis (Xu et al., 2009). The wavelet transform is capable to conduct these issues by decomposing a one-dimensional signal into two-dimensional time-frequency domains at the same time (Zhang et al., 2010). Wavelet analysis was employed to analyze climate change and variation, hydrologic factors changes in atmospheric and hydrology disciplines (Goodwin, 2008). Morlet wavelet analysis was used to handle local analysis in time and frequency domains to evaluate variation features in multiple scales within Anyang city for a 49 years precipitation time series.

METHODOLOGY AND DATA

In recent years, wavelet analysis which is a relatively current kind of time-frequency representation technique, has obtained rapid development and widespread

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application in different atmospheric and hydrologic disciplines (Chou, 2011). Based on non-orthogonal Morlet wavelet, its analysis characteristic in time-frequency domain and application in scales and trends identification (Rossi et al., 2011) are studied deeply and systemically in this study.

Wavelet transform means that function named $\psi(t)$ of mother wavelet is moved by a displacement named $\tau$ and multiplies the analytic function named $f(t)$ at multiple scales $a$. It can be described as Eq. 1:

$$C(a,\tau)=\frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} f(t)\psi\left(\frac{t-\tau}{a}\right) \, dt, a>0$$

(1)

where, $a$ refers to factors of scales; $\tau$ means factor of translation; $C$ is coefficient of wavelet; $\psi(t)$ means square summable function which refers to $\psi(t) \in L^2(R)$.

At the same time, the real part of Morlet wavelet is also one symmetrical wavelet function and the real part of wavelet coefficients could also contain the information of the strength and phase at different time location $\tau$. Without scale function, the Morlet wavelet function could be description by the following equation:

$$\psi(t)=a^{-\frac{1}{2}}e^{\frac{-t^2}{2}}\cos(\omega_0 t)$$

(2)

The precipitation and evaporation data in the Anyang city is observed from 1952-2000. Monthly precipitation and evaporation, annual precipitation and evaporation and growing seasonal (May-September) precipitation and evaporation were analyzed using wavelet analysis to examine precipitation variation from 1952-2000 at a total of 49 years which were 588 months. Firstly, monthly mean precipitation from January-December was calculated. Departure from average monthly precipitation series were obtained by monthly precipitation minus each mean monthly precipitation in order to avoid effect of natural yearly cycle. The annual and growing seasonal precipitation time series were handled at the same way to the monthly precipitation.

The scales where the peak appeared are considered as the main scales, i.e., the main cycles. The wavelet coefficient variance is the integral of the wavelet coefficient module squares at the time domain and its computation formula is given by the Eq. 3:

$$Var(a) = \int |C(a,\tau)|^2 \, dt$$

(3)

The wavelet analysis is conducted by MATLAB 7.0 and the following context presented the results about the wavelet analysis of the precipitation in Anyang city.

RESULTS AND DISCUSSION

Features of monthly precipitation and evaporation: The cumulative precipitation departure (CRD) from average (mean or normal) precipitation is a concept sometimes used by hydrologists or meteorologists to characterize precipitation trends. Where the result is mathematically negative, it is often termed a cumulative precipitation deficit. The Cumulative Evaporation Departure (CED) from average evaporation is developed from the case of CRD. According to the observed monthly precipitation and evaporation in the years from 1952-2000, the average monthly precipitation and evaporation are 47.6 and 161.1 mm, respectively. Furthermore, average monthly precipitation and evaporation are 47.6 and 161.1 mm, respectively. Furthermore, the cumulative departure curves of the monthly precipitation and evaporation in Anyang, China, are plotted in the Fig. 1. From the results in the Fig. 1, the evaporation cumulative departure in 1952-1958, is fluctuated in small ranges without any abrupt increase and decrease trends. Thus, this conclusion indicates that the monthly evaporation in the observed years is normal without exceptional

![Fig. 1: Cumulative departure curves of monthly precipitation and evaporation in the years 1952-2000, Anyang, China](image_url)
changes. In the period of 1958-1963, the monthly evaporation cumulative departure is increased in the linear way and large monthly evaporation appeared in these years. Meanwhile, the monthly evaporation in the years 1968-1975, was increased slowly as one whole, indicating that the monthly evaporation is larger than the average monthly evaporation. However, the monthly evaporation is fluctuated at the average monthly evaporation in the years of 1975-1984 and the evaporation is lower than the its average values for the period of 1984-2000 as one whole.

According to the monthly cumulative precipitation departure as plotted in Fig. 1, the monthly precipitation in Anyang, 1952-1958, is mainly fluctuated at its average monthly average values in the period of 1952-1958. However, the monthly precipitation in 1958-1959, is mainly higher than its monthly average which leads to the ascending of its cumulative departure. At the same time, the corresponding monthly evaporation is also higher than the monthly average evaporation. In the years of 1963-1965, the larger monthly precipitation appeared in special months, when rainstorm and other extreme precipitation climate took places and the monthly evaporation is far smaller than the its average values in the same term. The monthly precipitation cumulative departure curves become more flat without large fluctuation in the years of 1965-1978 and the monthly evaporation in the same observed time changed also in one small ranges. From the years of 1978-2000, the monthly precipitation cumulative departure is persistingly in the decreased trend and the observed precipitation is higher than the monthly average precipitation data when the deficiency of rainfall appeared. The monthly evaporation in the same term is also smaller than its average values. By comparing the cumulative monthly precipitation and evaporation curves, they have the same change trends with strong relativity in the observed years from 1952-2000, Anyang, China. This means that the precipitation trend in Anyang is determined by that of its corresponding evaporation.

In the followings, the Morlet wavelet is used to conduct multiple scale wavelet analysis to the cumulative departure curves of monthly precipitation and evaporation in the years 1952-2000, where about 8192 wavelet scales are used to figure out the possible features in the observed data. Then, their wavelet coefficient energy contour and corresponding wavelet coefficient variance with different wavelet scales are plotted in Fig. 2 and 3, respectively.

According to Fig. 2 and 3, the similarity from the monthly precipitation and evaporation wavelet coefficient energy contours could be figured out with that of their wavelet coefficient variance. This indicates that the strong relativity is existing between the monthly precipitation and monthly evaporation. The two contours

![Fig. 2(a-b): Wavelet analysis results of monthly cumulative evaporation departure in 1952-2000, Anyang, China](image-url)
have common strong wavelet coefficient energy rally points at the scale 512 and at the time of 1975 year. At the same time, one small wavelet coefficient energy rally point is located at the same scale and the years from 1989-2000. These wavelet coefficient energy rally points are similar to each other in the respect of sizes and locations and the strong relativity between the monthly precipitation and monthly evaporation cumulative departures.

From the cumulative departure curves of the monthly precipitation and evaporation in the years from 1952-2000, Anyang and their wavelet analysis variance curves, the main wavelet analysis scales of monthly precipitation is about 630 months, i.e., 35 years. This result could be found in Fig. 4 while the wavelet analysis scales of monthly evaporation cumulative departures is about 556 months, i.e., 46 years. By the wavelet analysis of monthly precipitation cumulative departure, the wavelet coefficient real part curve at the typical scale of 630 months is described in Fig. 4 while the corresponding wavelet coefficient real part curve for the monthly evaporation cumulative departure is plotted in Fig. 4 at the typical scale 556 months. Obviously, the two wavelet coefficient curves are period signal with the cycles 53 and 46 years, respectively. With the consideration about the differences of the two cumulative departure, their cycles are approximate and this also indicates the internal relativity between the monthly evaporation and monthly precipitations.

Fig. 3(a-b): Wavelet analysis results of monthly cumulative precipitation departure in 1952-2000, Anyang, China

Fig. 4: Wavelet coefficient real part of the monthly precipitation and evaporation cumulative departure at typical scale 630 months in the years from 1952-2000, Anyang, China
**Features of annual precipitation and evaporation departures:** As known by everyone, the departure curves could reflect the relative changes between the observed values and their averages and the fluctuation magnitude at the average for the observed variables could be disclosed. According to the observed monthly precipitation and evaporation in the years from 1952-2000, Anyang, the corresponding annual precipitation and evaporation with 49 years could be evaluated and their departure curves are further plotted in Fig. 5, respectively. For the annual precipitation departure curves, the annual precipitation at the years 1952-1965 had large fluctuations while the magnitude of annual precipitation in the years 1965-2000 became smaller. Here, for the annual evaporation departure curve, the annual evaporation in the year from 1952-1975 had large fluctuations while the fluctuation magnitude in 1975-2000 is also small. The maximal evaporation and the minimal evaporation in the observed 49 years for Anyang area appeared at the years of 1965 and 1964, respectively. At the same time, the maximal precipitation in 1963 is the biggest observed data in the years from 1952-2000. By comparing the two departure curves, the fluctuation magnitudes in recent 49 years presented the progress from great to small.

Then, the Morlet wavelet analysis was conducted on the annual precipitation and evaporation departure curves in the years from 1952-2000, Anyang, where the minimal scale 2 years and the maximal scale 1024 years were used. According to the wavelet coefficients at every scale from 2 and 1024, the coefficient energy contours are plotted in Fig. 6 and 7. Obviously, the similarity of the two contours could be found and there are one distinct wavelet energy

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![Graph showing annual precipitation and evaporation departures](image)

**Fig. 5:** Annual precipitation and evaporation departure in the years from 1952-2000, Anyang, China

![Graph showing wavelet analysis results](image)

**Fig. 6(a-b):** Wavelet analysis results of annual precipitation departure in 1952-2000, Anyang, China
rally points and wavelet energy fillers. For the wavelet energy rally area of annual precipitation, it is located at the areas of main scale 15 and observed years 1958 while the wavelet energy rally area for the annual evaporation is located at the range of wavelet main scale 15 and observed year 1965. There is about 7 year difference between the two energy rally areas in the direction of observed years. By comparing the fillers in the contours of annual precipitation and evaporation, the filler area in the contour of annual precipitation is located in the area of main scale 15 and observed 1984 while the filler area in the contour of annual evaporation is located in the area of main scale 15 and observed 1990. There is about the difference of 6 years between the two fillers. For the wavelet energy rally areas and filler areas at the low scales, they are similar to each other in the aspect of appearing sequence and shapes and the time differences about 6 years exists. By comparing the wavelet energy contours of annual precipitation and evaporation in the years of 1952-2000, they have the similar change features and time difference of about 6 years.

According to the wavelet coefficient variance curves for the annual precipitation and evaporation in the years of 1952-2000, two maximal points could be found at the position of scale 15 years. Therefore, the wavelet coefficient real part curves of annual precipitation and evaporation at the scale 15 years, are presented in Fig. 8. For the wavelet coefficient real part curve of annual...
precipitation, its maximal value appeared at the 1958 year while the minimal value at 1984. At the same time, good period feature could be found obviously for the whole wavelet coefficient real part curve with the cycle 52 years. For the annual evaporation wavelet coefficient real part curve, its maximal value appears at 1964 year while the minimal value at 1990. As the annual precipitation wavelet coefficient real part curve, the real part curve for the annual evaporation has the similar cycle feature, i.e., 52 years. By comparing the two wavelet coefficient real part curves for the annual precipitation and evaporation departures, the time differences for the maximal points and minimal points are 6 years, respectively. Therefore, the annual precipitation and evaporation in the years of 1592-2000, Anyang, are seasonal with the cycle 52 years and about 6 year time difference.

Growing seasonal precipitation and evaporation departure features: As the important season in 1 year for the plant growth in the agricultural activities, the changes of precipitation and evaporation are vital to the economics and societies developments. For the Anyang cities, its growth season is mainly in the months of May-September. According to the observed monthly precipitation and evaporation data in the years from 1952-2000, the growing seasonal precipitation and evaporation could be evaluated and their departure curves are also plotted in Fig. 9. In the years of 1952-2000, the growing seasonal average precipitation is about 460 mm while the growing seasonal evaporation is about 1170 mm. Furthermore, the maximal evaporation appeared at the year 1965 while the minimal evaporation appeared at the year 1954. However, the growing seasonal precipitations appeared at the year 1964. During the period of 1952-2000, the fluctuation magnitude is slight large for growing seasonal evaporation while the fluctuation magnitude of growing seasonal precipitation is slight small with comparison to that of growing seasonal evaporation.

In order to obtain more information from the growing seasonal precipitation and evaporation in the years of 1952-2000, Anyang, the precipitation and evaporation departure curves were wavelet analyzed by Morlet wavelets, where about 1024 scale wavelets were used to analyzing the departure curves. Then, the wavelet coefficient energy contours were plotted in Fig. 10 and 11.

From the wavelet coefficient energy contour of growing seasonal evaporation departure, there are one large wavelet energy Rally area and one wavelet energy filler area which are located in the area of scale 32 and 1964 year and the area of scale 32 and 1987 year, respectively. They are symmetrically distributed in the contours with the time 1975 year as their boundary. On March 25, 2014 at same time, typical several wavelet energy rally areas with small sizes are also equably distributed at the horizontal of scale 8 year which indicated the maximal points of growing seasonal evaporations.

Compared with the growing seasonal evaporation departure wavelet energy contour, the wavelet energy contour of growing seasonal precipitation contour presents out-of-order in the years 1952-2000. Although there exists one large wavelet energy rally area and filler area, their sizes and shapes are not centered as that of growing seasonal evaporation contour. They are equably distributed in the horizontal line at the scale 5 years and the wavelet energy rally area is obviously spread in the scale direction. However, the wavelet energy filler areas are obviously spread in the time direction. This indicated that the growing seasonal precipitation in the year 1952-1965 has great fluctuation while with small

![Graph showing the growing seasonal evaporation and precipitation departure curves in the years of 1952-2000, Anyang, China](image-url)

**Fig. 9:** Growing seasonal evaporation and precipitation departure curves in the years of 1952-2000, Anyang, China
Fig. 10(a-b): Wavelet coefficient energy contour for growing seasonal evaporation departure curve in the year 1952-2000, Anyang, China

Fig. 11(a-b): Wavelet coefficient energy contour of growing seasonal precipitation departure in the years 1952-2000, Anyang, China

fluctuation magnitude in the period of 1970-2000. At the horizontal line of scale 6 year, there are several small wavelet energy rally areas, filler areas which could reflect the maximal and minimal points of growing precipitation departure wavelet coefficient energy contour.

In the years of 1952-2000, the wavelet coefficient variance of the growing seasonal precipitation and evaporation departures are plotted in Fig. 11 and 12. The maximal points in the growing seasonal precipitation wavelet coefficient variance curve, are located at the year of 2 and 3 while the maximal points in the evaporation and precipitation wavelet coefficient variance curve are scales of 4 and 45, scales of 3, 9 and 44, respectively. Thus, the wavelet coefficient real part curves at these
Fig. 12: Growing seasonal evaporation departure wavelet coefficient real part curves at typical scales 3, 9, 44 years, 1952-2000, Anyang, China

Fig. 13: Growing seasonal precipitation departure wavelet coefficient real part curves at typical scales 4 and 45 years, 1952-2000, Anyang, China

typical scales for the precipitation and evaporation in the growing season, are plotted in Fig. 12 and 13, respectively. Obviously, the wavelet coefficient real part curves at the scales of 44 and 45, have the similar periods for the precipitation and evaporation in the growing seasons. This conclusion indicated that the annual precipitation and evaporation had the features with good cycles.

CONCLUSIONS

- Monthly precipitation and evaporation departure from average time series analyzed by wavelet analysis indicated that the real part coefficients have periodic characteristics. The monthly precipitation and evaporation have the long term scales at about 53 and 46 year, respectively.
- Based on the annual precipitation and evaporation data, the results indicated that annual precipitation and evaporation at about 52 years variation with 6 years lag.
- The wavelet coefficient real part curves at the scales of 44 and 45 years, have the similar periods for the precipitation and evaporation in the growing seasons.

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