Journal of Applied Sciences

ISSN 1812-5654

Journal of Applied Sciences 14 (1): 66-71, 2014
ISSN 1812-5654 / DOI: 10.3923/jas.2014.66.71

Segmentation of MRI Brain Images Using FCM Improved by Firefly Algorithms

[1]Waleed Khamees Alomoush, [1]Siti Norul Huda Sheikh Abdullah,
[1]Shahnorbanun Sahran and [2]Rizuana Iqbal Hussain
[1]Pattern Recognition Research Group, Center for Artificial Intelligence Technology,
Faculty of Information Science and Technology, Universiti Kebangsaan Malaysia,
43600, Bangi, Selangor D.E., Malaysia
[2]Department of Radiology, Universiti Kebangsaan Malaysia Medical Centre, 56000 Cheras,
Kuala Lumpur, Malaysia

Abstract: Fuzzy clustering algorithms suffer from some weakness. The main weakness including the inclination to be trapped in local optima and vulnerable to initialization sensitivity. This study proposed a new approach called (FFCM) to solve Fuzzy C-Means (FCM) initialization problem using firefly algorithm to find optimal initial cluster centers for the FCM, thus improve all applications related fuzzy clustering such as image segmentation. The new approach (FFCM) has been evaluated in MRI Brain segmentation problem using simulated brain dataset of McGill University and MRI real images from IBSR center benchmark datasets. The experiments indicate encouraging results after applying (FFCM) and compared the outcomes with FCM random initialize cluster center.

Key words: Fuzzy c-means, firefly algorithm, images segmentation

INTRODUCTION

The process of digital image analysis comprises a lot of steps, of which image segmentation is one of the most significant and intricate phase. In this phase, images are segmented into many components and each component will comprise identical attributes. Nevertheless, it is possible to model image segmentation as clustering problem which has resulted in the development of a number of clustering algorithms (Omran *et al*., 2005). However, in reality, the collection of data presents ambiguous limits among clusters. Generally, the MRI images have unclear and patchy limits. In this perspective, there has been remarkable prospect for fuzzy clustering, due to its capability of handling with unclear and patchy data characteristics (Alia *et al*., 2009).

Consequently, Fuzzy C-Means algorithm (FCM) (Bezdek, 1981) has been widely and successfully employed in several applications to address image segmentation problems (Kang *et al*., 2009). On the other hand, the preliminary FCM algorithm has several constraints, including the inclination to be trapped in local optima and vulnerable to initialization sensitivity (Pham *et al*., 2007). Therefore, researchers all over the world have triggered intense investigations to enhance the algorithm. In general, the reason for these problems is the exposure of greedy search behavior by the FCM which just promises to generate local optima (Karaboga and Ozturk, 2011; Hassanzadeh and Meybodi, 2012). Consequently, the selection of inappropriate preliminary cluster centers might usually cause undesired clustering outcomes. Nevertheless, this serious issue can be addressed by combining FCM algorithm with one of the metaheuristic search optimization algorithm which might yield global optimal solution (Kao *et al*., 2008; Puchinger and Raidl, 2005). However, there is no evidence to claim that evolutionary algorithms always produce precise solutions; however they typically produce substantial or near-optimal solutions (Puchinger and Raidl, 2005).

The last decades, many algorithms of metaheuristic search have been applied with FCM algorithm to found optimal cluster centers. These algorithms explore all search space in the problem to determined possible solutions (Abraham *et al*., 2008). These algorithms include bees optimization (Karaboga *et al*., 2012), harmony search (Ingram and Zhang, 2009), ant colony algorithm (Kanade and Hall, 2007), simulated annealing (Selim and Alsultan, 1991), genetic algorithm (Sheikh *et al*., 2008; Hall *et al*., 1999), tabu search (Al-Sultan and Fedjki, 1997), firefly algorithm (Hassanzadeh and Meybodi, 2012) and particle swarm

Corresponding Author: Waleed Khamees Alomoush, Pattern Recognition Research Group,
Center for Artificial Intelligence Technology, Faculty of Information Science and Technology,
Universiti Kebangsaan Malaysia, 43600, Bangi, Selangor D.E., Malaysia

(Li *et al*., 2007). Also, according to the authors' knowledge, this is the first time to apply Firefly-FCM algorithm to image segmentation.

Therefore, this study explore firefly algorithm to FCM in a gray MRI images segmentation domain since this domain is a very complicated domain especially to natural difficulties of MRI images segmentation.

This study has explored the efficiency of the Firefly algorithm in generating near-optimal initial cluster centers for FCM algorithm, for assuring to generate outcomes of superior and constant image segmentation. The most recent algorithm inspired by the biological behavior of fireflies is the Firefly Algorithm (FA) (Yang, 2010).

FIREFLY SEARCH ALGORITHM

Fireflies are the rarest insects with the natural capacity of being illumine in dark with flickering and glowing biological lights. Firefly Algorithm (FA) has been motivated by the biological behavior of fireflies (Yang, 2010); generally FA employs the following three rules: (1) Fireflies are unisexual, therefore every individual firefly will be fascinated to the other, irrespective of its gender, (2) The unique feature of glowing light of fireflies will attract preys and (3) The attraction of fireflies is proportionate to their brightness which makes the less brighter firefly to move towards more brighter glowing firefly (Fister *et al*., 2013).

The population-based Firefly Algorithm (FA) is capable of discovering the global-optima of objective functions, depending on the intelligence of the swarm (Senthilnath *et al*., 2011); moreover FA also examines the foraging behavior of fireflies. In the FA, the physical entities are arbitrarily spread in the search space, in this case the physical gentility is fireflies which have a substance known as luciferin that makes the fireflies glow in dark and generally luciferin will discharge light that is proportional to this value. As mentioned earlier, the fireflies with slightly dimmer light will be attracted towards the brighter individuals; nevertheless the degree of attraction will reduce if the distance between those fireflies increases. On the other hand, if any firefly fails to find another firefly that is brighter than itself, then the former will travel arbitrarily. When FA is employed to solve clustering problems, the cluster centers are the decision variables and the objective function is associated with the value of all Euclidean distance training set cases in an N-dimensional space (Karaboga and Ozturk, 2011).

Depending on this objective function, in the beginning, all the agents (fireflies) will be arbitrarily spread all over the search space. The following are the two stages of FA: The first stage is the difference in the intensity of light, where the intensity of light is associated with the objective values (Yang, 2008). Therefore in case of maximization/minimization problem, a firefly with higher or lower intensity will entice another individual with higher or lower intensity. Let us presume that, there is a swarm of n agents (fireflies), where x_i signifies the solution of a firefly i, whilst its fitness value is signified by $f(x_i)$; furthermore the current position i of its fitness value f(x) is determined by the brightness I of a firefly (Yang, 2008):

$$I_i = f(x_i),\ 1 \le i \le n \tag{1}$$

The next stage is movements towards attractive fireflies: The attractive force of firefly is proportionate to the intensity of light witnessed by nearby fireflies (Yang, 2008). Every single firefly possesses its unique attraction β which indicates the power of attraction over individuals of the swarm. Nevertheless, the attractiveness β will change with the distance r_{ij} between two fireflies i and j at locations x_i and x_j, respectively as the following:

$$r_{ij} = \|x_i - x_j\| \tag{2}$$

The attractiveness function β(r) of the firefly is established by:

$$\beta(r) = \beta_0 e^{-\gamma r^2} \tag{3}$$

where, β_0 is the attractiveness at r = 0 and γ is the coefficient of ingestion of light. The motion of a firefly i from the position x_i which is attracted to another much more attractive (brighter) firefly j at position x_j has been established as below:

$$x_{i(t+1)} = x_{i(t)} + \beta_0 e^{-\gamma r_{ij}^2}\left(x_i - x_j\right) + \propto\left(\text{rand} - \frac{1}{2}\right) \tag{4}$$

A comprehensive explanation of FA can be referred to Yang (2010).

CLUSTERING WITH FUZZY C-MEANS

Clustering is a unsupervised learning approach which is capable of partitioning identical data objects (patterns) based on some level of similarity. It increases the similarity of objects with in a group and decreases the similarity among the objects between various groups (Jain *et al*., 1999; Bsoul and Mohd, 2011). Clustering algorithm for grouping fuzzy data is carried out on a collection of n objects (pixels) $\{x_1, x_2, ..., x_n\}$ and each of these objects is $x_i \in \Re^d$ a characteristic vector, which consists of d real-valued dimensions that reveal the characteristics of the object depicted by x_i. A fuzzy membership matrix referred to as fuzzy partition $U = [u_{ij}]_{c \times n}$ ($U \in M_{fcn}$ as in Eq. 5):

$$M_{fcn}\left\{U \in \Re^{c\times n}\left| \begin{matrix}\sum_{j=1}^{c} U_{ij} = 1,\ 0 < \sum_{j=1}^{n} U_{ij} < n \\ \text{and } U_{ij} \in [0, 1];\ 1 \le j < c;\ 1 < i \le n\end{matrix}\right.\right\} \quad (5)$$

represents the fuzzy clusters c of the objects, where, signifies the fuzzy membership of the ith object to the jth fuzzy cluster. For example, each and every data object is related to a specific (probably zero) degree of every single fuzzy cluster. Fuzzy C-Means algorithm (FCM) is a repetitive technique which capable to locally reduce the following objective functions:

$$j_m = \sum_{i=1}^{c}\sum_{i=1}^{n} u_{ij}^m \|x_i - v_j\|^2 \quad (6)$$

where, $\{v_j\}_{j=1}^{c}$ (the centroids of the clusters c) which indicates the standards of inner-product ||.|| (e.g., euclidean distance) from the data point x_i to the jth cluster center; furthermore the parameter $m\epsilon[1, \infty)$, is a distort proponent of each fuzzy membership which ascertains the level of fuzziness of the ensuing classification.

The following are the summary of FCM steps. Choose the number of fuzzy clusters c.

Choose initial cluster centers $v_1, v_2,..., v_c$.

Estimate the components of the fuzzy partition matrix:

$$U_{ij} = \frac{1}{\sum_{k=1}^{c}\left(\frac{\|x_i - v_j\|}{\|x_i - v_k\|}\right)^{\frac{2}{m-1}}} \quad (7)$$

Calculate the cluster centers:

$$v_j = \frac{\sum_{i=1}^{n} u_{ij}^m . x_i}{\sum_{i=1}^{n} u_{ij}^m} \quad (8)$$

Repeat 3rd and 4th steps until the number of iterations (t) surpasses the set limit, or a termination criterion is met:

$$\|v_{new} - v_{old}\| < \varepsilon \quad (9)$$

where, $\varepsilon<0.001$.

PROPOSED METHOD

Here, an investigated the performance of the firefly algorithm in terms of obtaining near-optimal cluster centers values in the initialization phase in FCM algorithm. Here, new proposed a clustering approach which consists of two phases. In the first phase, the firefly examines the search space of the given dataset, to determine the near-optimal cluster centers. The centers value determine by the (FA) are then evaluated by (Rm) clustering validity index.

In the second phase, the outcome of the first phase is used to initialize the Fuzzy C-Means algorithm. The main significance of this approach to solve problem of the inclination to be trapped in local optima and vulnerable to initialization sensitivity clustering.

Identifying near-optimal cluster centers using firefly search: The cluster centers of the provided dataset are encoded by each and every Firefly Population Search (FPS) vector. The solution vector can be defined as in Eq. 10:

$$a = \begin{pmatrix} v_1 \ \ v_2 \ \ v_3 \\ a_1a_2...a_d,\ a_1a_2...a_d,\ a_1a_2...a_d \end{pmatrix} \quad (10)$$

where, a_i is a numerical characteristic which explains a cluster center and $a_i\epsilon A$, where A is the collection feasible array of each and every pixel attribute. Consequently, each cluster center v_i is defined by d numerical feature-($a_1, a_2,..., a_d$). As a result, every single vector has an actual size of (c×d), where, c represents given number of clusters and d indicates the number of feature sample outlining the given dataset.

For the purpose of delineating the connection between the clustering and image segmentation, it is possible to map every single pixel in an image as a sample or data point in the clustering sector, whilst it is also possible to map the image regions as clusters or classes. In case of a 256×256 image, there will be 65536 pixels (data points). For instance, in a given gray image with three distinct regions (e.g., WM, GM and CSF in brain MRI image), eight bit depth and three features (e.g., intensity value, entropy and energy) which illustrate each and every pixel, the probable degree of pixel intensity value pertaining to the depth of image will be in interval $\epsilon[0, 255]$ and the deterioration and energy features can be illustrated by interval $\epsilon[0, 10]$. Therefore, the firefly vector could be such as (5, 2.5, 2.6, 30, 6.2, 2.1, 80, 2.3, 1.3), in which the first three digits (5, 2.5, 2.6) signify the cluster center values for the 1st image region in each image sequence and the next three digits (30, 6.2, 2.1) indicate the cluster center values for the 2nd image region, whereas the final cluster center (80, 2.3, 1.3) indicate the 3rd image spot. Immediately after firefly sets the factors (ALPHA, GAMMA, DELTA, FPS, MAXg), the initialization step of FPS is examined. All the cluster centers in all the solution vectors in FPS might be randomly initialized from their image attributes data range. Soon after the FPS is loaded with initialized solution vectors, the fitness value will be measured for all the solution vectors in FPS by an objective function. Later the FPS vectors will be reorganized in a descending order of the objective function value after choosing minimal values from the solutions with objective function. The FA finds near optimal cluster centers that guarantee fuzzy c-means reaches to the near global optima and does not trapped in local optima. Hence, it leads to improve FCM and replaces the traditional method of multiple random initialization when it determines the cluster center.

Objective function: The fitness degree the solution is indicated by the assessment (fitness value) of each FPS vector. In this study, we have used an enhanced version of the conventional FCM objective function (Hathaway and Bezdek, 1995) as the cluster centers are just employed in the evolving process of the first stage of FFCM. The enhanced FCM objective function depends merely on the calculations of cluster centers, whereas, the membership matrix U as in standard objective function is not employed and paid out for the required changes. According to Hathaway and Bezdek (1995) standard and reformulated objective functions are comparative in general, however the later it becomes less complicated. Consequently, the time consumed for determining the objective function for each solution vector in FPS is minimized. The reformulated version of the FCM's objective function is as follows:

$$R_m = \sum_{i=1}^{n}\left(\sum_{j=1}^{c} D_{ji}^{\frac{1}{1-m}}\right)^{1-m} \quad (11)$$

where, D_{ij} is $\|x_i-v_j\|$ the distance from pixel intensity x_i to the jth cluster center, m is the fuzziness of the resulting classification and it is set m = 2 and n is the total number of pixels in the given image. In this case, we have measured the aggregate of the distances between all the pixels in the given image with each cluster center which have been produced from the new solution of firefly vector. However, firefly will attempt to reduce this value of R_m in order to accomplish the preferred near-optimal solution, or to fulfill the stopping criterion.

EXPERIMENTAL RESULTS

Here, explain the evaluation of the proposed solution (FFCM) and compared the outcomes with the results acquired from the conventional FCM clustering algorithm, such as the random initialization technique of FCM.

Data set: The data set collection of different MRI of images for the purpose of illustrating the efficacy of the proposed FFCM algorithm. The first group comprises three normal simulated T1-weighted MRI brain images (T1WI) which were acquired from simulated brain dataset of McGill University (BrainWeb, 2003). Nevertheless, the size of all these images is 217×181 with 8 bit gray scale level. The images in the first group have various degree of complication in terms of MRI features such as, noise and intensity inhomogeneity.

The second group comprises three normal and up normal MRI real images which were acquired from IBSR center for morphometric analysis, massachusetts general hospital repository (IBSR, 2005).

DISCUSSION

The excellence of the solution created is assessed with regards to the intent function. The tests are meant to analyze the efficiency of firefly search in obtaining suitable preliminary cluster centers for the FCM algorithm, as against the conventional random initialization technique employed to select cluster centers. The outcomes for the FCM with firefly search initialization are designated as FFCM, whereas the outcomes for the FCM with random initialization are proclaimed as (FCM). This study has employed the fitness function value as a measure of the goodness of clustering. It has also documented the typical and the standard deviation results of FCM and FFCM for about 50 trials. Additionally, all the tests were carried out on an Intel Core5Duo 2.5 GHz machine, with 4 GB RAM; and the codes are written using (MATLAB 2010). Additionally, the firefly variables are set as follows, FPS = 300, α = 0.00556, β = 0.98, γ = 20.48 and the maximum number of iterations mxg = 1000. Table 1 illustrates the above mentioned outcomes, where the 1st column and 2nd column depict the average objective function and standard deviation value for FFCM and FCM correspondingly.

These outcomes indicate that, the majority of the analyzed images show case considerable enhancements regarding minimization of objective function values, when using the proposed algorithm FFCM as against FCM. The bold items in Table 1 signify equivalent or superior outcomes of FFCM as against FCM.

In addition and with more detailed investigation of these results, it is justifiable that FFCM is more constant and reliable than the outcomes of FCM pertaining to the acquired standard deviation measurements. Figure 1

Table 1: Average and standard deviation after applying FFCM and FCM subsequently

	FFCM		FCM	
MRI	AVG Rm	SD±	AVG Rm	SD±
S0	68730248	885809.1	74313563	11301955
S040	74998354	478511.8	85187983	11770134
S30	79030282	21794694.0	79319368	8836978
R01	4372070	97024.0	7030067	8786194
R02	13629834	2257487.0	26269632	30746101
R03	18967849	3701929.0	30594362	48865833

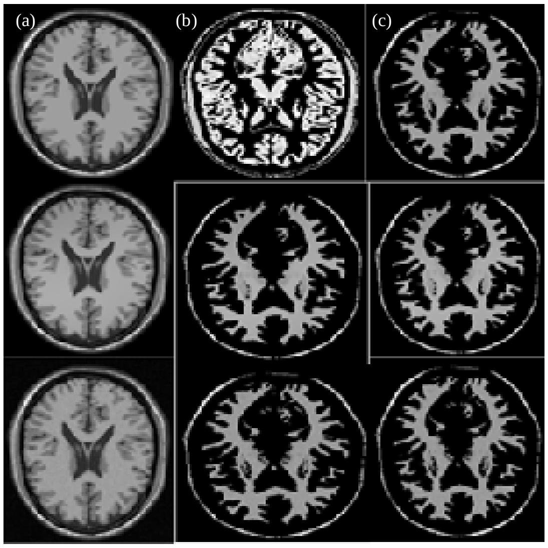

Fig. 1(a-c): (a) Illustrates the all normal simulated original images, (b) From Brain Web benchmark dataset after applying FCM and (c) FFCM image segmentation methods subsequently

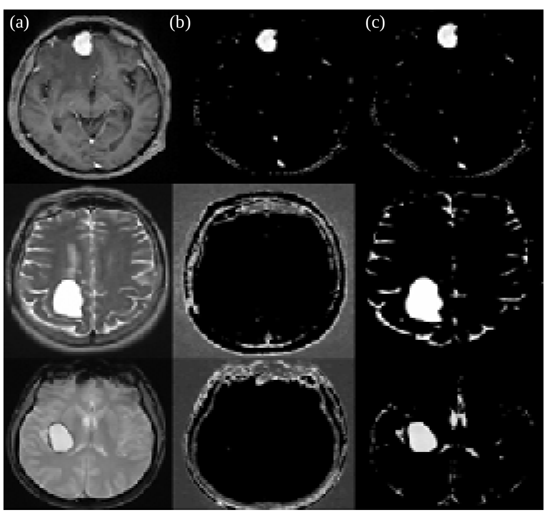

Fig. 2(a-c): (a) Illustrates the all abnormal real from IBSR center images (b) After applying FCM and (c) FFCM, respectively

illustrates that all normal simulated original images (Fig. 1a) from benchmark dataset (BrainWeb, 2003), after applying FCM (Fig. 1b) and FFCM (Fig. 1c) image segmentation methods subsequently. Figure 2 illustrates the all abnormal real original images (Fig. 2a) from IBSR center (IBSR, 2005) after using FCM and FFCM methods, respectively.

CONCLUSION

This research propose a new firefly algorithm-based fuzzy clustering algorithm. This algorithm consists of 2 stages. In the first stage, a near-optimal value of a predefined number of clusters are indentified; whereas in the second stage, the output of the first stage is used to initialize the FCM, where the later, it performs the clustering process. This algorithm triumphs over the most crucial disadvantage of conventional FCM algorithm, such as the initialization sensitivity and its resultant local optima problem. The proposed algorithm (FFCM) is employed as an image segmentation method and the outcomes from the simulated and real MRI brain images indicate the efficiency of the proposed algorithm as against the randomly initialized FCM algorithm.

ACKNOWLEDGEMENT

This research is based on two research grants from Ministry of Science, Technology and Innovation, Malaysia entitled “Sciencefund 01-01-02-SF0694 Spiking-LVQ Network For Brain Tumor Detection” and Arus Perdana Fund AP-2012-019 entitled “Automated Medical Imaging Diagnostic Based on Four Critical Diseases: Brain, Breast, Prostate and Lung Cancer”. We have obtained Ethics approval entitled “FF-342-2012 Imaging Processsing of the Brain Tumor Using LVQ Classifier” from PPUKM or UKM Medical Center, Malaysia for collecting and conducting experiments on MRI Brain Tumor patient’s record.

REFERENCES

Abraham, A., S. Das and S. Roy, 2008. Swarm intelligence algorithms for data clustering. In: Soft Computing for Knowledge Discovery and Data Mining, Maimon, O. and L. Rokach (Eds.). Springer, USA., ISBN-13: 9780387699349, pp: 279-313.

Al-Sultan, K.S. and C.A. Fedjki, 1997. A tabu search-based algorithm for the fuzzy clustering problem. Pattern Recogn., 30: 2023-2030.

Alia, O.M., R. Mandava, D. Ramachandram and M.E. Aziz, 2009. Harmony search-based cluster initialization for fuzzy c-means segmentation of MR images. Proceedings of the IEEE Region 10 Conference, January 23-26, 2009, Singapore, pp: 1-6.

Bezdek, J.C., 1981. Pattern Recognition with Fuzzy Objective Function Algorithms. 1st Edn., Kluwer Academic Publishers, Norwell, MA, USA.

BrainWeb, 2003. BrainWeb: Simulated brain database. Mcconnell Brain Imaging Centre Montreal Neurological Institute Mcgill University. http://www.bic.mni.mcgill.ca/brainweb.

Bsoul, Q.W. and M. Mohd, 2011. Effect of ISRI stemming on similarity measure for Arabic document clustering. Proceedings of the 7th Asia Information Retrieval Societies Conference on Information Retrieval Technology, December 18-20, 2011, Dubai, UAE., pp: 584-593.

Fister, I., I. Fister Jr., X.S. Yang and J. Brest, 2013. Swarm and evolutionary computation. Swarm Evolutionary Comput., 10.1016/j.swevo.2013.06.001

Hall, L.O., I.B. Ozyurt and J.C. Bezdek, 1999. Clustering with a genetically optimized approach. IEEE Trans. Evol. Comput., 3: 103-112.

Hassanzadeh, T. and M.R. Meybodi, 2012. A new hybrid approach for data clustering using firefly algorithm and K-means. Proceedings 16th CSI International Symposium on Artificial Intelligence and Signal Processing, May 2-3, 2012, Shiraz, Fars, Iran, pp: 7-11.

Hathaway, R.J. and J.C. Bezdek, 1995. Optimization of clustering criteria by reformulation. IEEE Trans. Fuzzy Syst., 3: 241-245.

IBSR, 2005. Internet brain segmentation repository. Technical Report, Massachusetts General Hospital, Center for Morphometric Analysis.

Ingram, G. and T. Zhang, 2009. Overview of Applications and Developments in the Harmony Search Algorithm. In: Music-Inspired Harmony Search Algorithm, Geem, Z.W. (Ed.). Springer, Berlin/Heidelberg, pp: 15-37.

Jain, A.K., M.N. Murty and P.J. Flynn, 1999. Data clustering: A review. ACM Comput. Surveys, 31: 264-323.

Kanade, P.M. and L.O. Hall, 2007. Fuzzy ants and clustering. IEEE Trans. Syst. Man Cybernet., A Syst. Humans, 37: 758-769.

Kang, J., L. Min, Q. Luan, X. Li and J. Liu, 2009. Novel modified fuzzy c-means algorithm with applications. Digit. Signal Process., 19: 309-319.

Kao, Y.T., E. Zahara and I.W. Kao, 2008. A hybridized approach to data clustering. Expert Syst. Appl., 34: 1754-1762.

Karaboga, D. and C. Ozturk, 2011. A novel clustering approach: Artificial Bee Colony (ABC) algorithm. Applied Soft Comput., 11: 652-657.

Karaboga, D., B. Gorkemli, C. Ozturk and N. Karaboga, 2012. A comprehensive survey: Artificial Bee Bolony (ABC) algorithm and applications. Artif. Intelli. Rev., 10.1007/s10462-012-9328-0

Li, L., X. Liu and M. Xu, 2007. A novel fuzzy clustering based on particle swarm optimization. Proceedings of the 1st IEEE International Symposium on Information Technologies and Applications in Education, November 23-25, 2007, Kunming, China, pp: 88-90.

Omran, M., A.P. Engelbrecht and A. Salman, 2005. Particle swarm optimization method for image clustering. Int. J. Pattern Recogn. Artif. Intell., 19: 297-321.

Pham, D.T., S. Otri, A. Afify, M. Mahmuddin and H. Al-Jabbouli, 2007. Data clustering using the bees algorithm. Proceedings of the 40th CIRP International Seminar on Manufacturing Systems, May 30-June 1, 2007, Liverpool, UK.

Puchinger, J. and G.R. Raidl, 2005. Combining Metaheuristics and Exact Algorithms in Combinatorial Optimization: A Survey and Classification. In: Artificial Intelligence and Knowledge Engineering Applications: A Bioinspired Approach, Mira, J. and J. Alvarez (Eds.). Springer, Berlin, Germany, ISBN-13: 9783540263197, pp: 41-53.

Selim, S.Z. and K. Alsultan, 1991. A simulated annealing algorithm for the clustering problem. Pattern Recognit., 10: 1003-1008.

Senthilnath, J., S.N. Omkar and V. Mani, 2011. Clustering using firefly algorithm: Performance study. Swarm Evol. Comput., 1: 164-171.

Sheikh, R.H., M.M. Raghuwanshi and A.N. Jaiswal, 2008. Genetic algorithm based clustering: A survey. Proceedings of the 1st International Conference on Emerging Trends in Engineering and Technology, July 16-18, 2008, Nagpur, India, pp: 314-319.

Yang, X.S., 2008. Nature-Inspired Metaheuristic Algorithms. Luniver Press, UK., ISBN-13: 978-1905986101, Pages: 128.

Yang, X.S., 2010. Firefly algorithm, stochastic test functions and design optimisation. Int. J. Bio Inspired Comput., 2: 78-84.