Journal of Applied Sciences

ISSN 1812-5654

OPEN ACCESS Journal of Applied Sciences

ISSN 1812-5654
DOI: 10.3923/jas.2016.98.102

Research Article
Temperature Fields Near Maximum Friction Surfaces in Pressure-Dependent Plasticity

[1]Alexandrov Sergei, [2]Iskakbayev Alibay and [2]Teltayev Bagdat

[1]Institute for Problems in Mechanics, Russian Academy of Sciences, Russia
[2]Kazakhstan Highway Research Institute, Almaty, Kazakhstan

Abstract
In many cases the modeling of machining and deformation processes may be based on a rigid plastic material model. The yield criterion of some plastically incompressible metallic materials depends on the hydrostatic stress. The double shearing model can be used to describe the behavior of such materials. As a rule, friction occurs in machining and deformation processes. One of widely used friction law is the maximum friction law. In the case of the double shearing model this law demands that the friction surface coincides with an envelope of characteristics. The present study deals with the effect of this friction law on the temperature field in the vicinity of the friction surface. The study is restricted to stationary planar flows. In particular, the behavior of the temperature field near the maximum friction surface is found from asymptotic analysis of the systems of equations of thermoplasticity. It is shown that the plastic work rate follows an inverse square root rule near maximum friction surfaces and thus, approaches infinity at the surface. Since, the plastic work rate is involved in the heat conduction equation, the temperature and the heat flux must be describable by nondifferentiable functions where the singular behavior of the plastic work occurs. It is hypothesized that the asymptotic analysis performed can be useful for predicting the generation of white layers in machining processes and fine grain layers in deformation processes.

Key words: Friction surface, temperature field, friction law, asymptotic analysis, plastic work rate

Received: November 03, 2015 Accepted: January 13, 2016 Published: February 15, 2016

Citation : Alexandrov Sergei, Iskakbayev Alibay and Teltayev Bagdat, 2016. Temperature fields near maximum friction surfaces in pressure-dependent plasticity. J. Applied Sci., 16: 98-102.

Corresponding Author: Teltayev Bagdat, Kazakhstan Highway Research Institute, Almaty, Kazakhstan

Competing Interest: The authors have declared that no competing interest exists.

Data Availability: All relevant data are within the paper and its supporting information files.

INTRODUCTION

The temperature of the metal rises during plastic deformation because of the heat generated by mechanical work. Under certain conditions, this temperature rise is of considerable importance. From the point of view of plasticity theory, the first systematic study on temperature distributions in metal forming processes has been conducted by Tanner and Johnson (1960). A recent review of the literature on this topic has been provided by Hadala (2013). A review of the literature on heat generation and temperature prediction in metal machining processes has been given by Abukhshim *et al*. (2006). In contrast to these works, the present paper deals with the effect of singular velocity fields on the temperature rise within a narrow layer in the vicinity of frictional interfaces. This research is motivated by numerous experimental observations that demonstrate that a narrow layer with drastically modified microstructure is generated near such interfaces. This layer is usually called white layer in papers devoted to machining processes and fine grain layer in papers devoted to metal forming processes. A complete review of results on white layer/fine grain layer generation published before 1987 has been presented by Griffiths (1987). According to this review article there are three main contributory mechanisms responsible for white layer generation. Two of these mechanisms are; (a) Mechanism of rapid heating and quenching and (b) Mechanism of intensive plastic deformation. The latter can be described by means of the strain rate intensity factor introduced by Alexandrov and Richmond (2001). This factor is the coefficient of the leading singular term in a series expansion of the equivalent strain rate near maximum friction surfaces. This expansion shows that the equivalent strain rate is infinite at the friction surface. Therefore, the strain rate intensity factor controls the magnitude of the equivalent strain rate in its vicinity. A necessary condition for the existence of the strain rate intensity factor is the maximum friction law. This boundary condition is often adopted at the tool-chip interface (at least, over a portion of this interface) in machining processes (Ng *et al*., 1999; Karpat and Ozel, 2006; Ramesh and Melkote, 2008; Lalwani *et al*., 2009; Akbar *et al*., 2010; Molinari *et al*., 2012; Chen *et al*., 2014; Agmell *et al*., 2014). This zone is usually called the sticking zone. The existence of such zones has been reported in deformation processes as well (Kim and Ikeda, 2000; Wideroe and Welo, 2012; Sanabria *et al*., 2014a, b). It is evident that the strain rate intensity factor controls the intensity of heat generation by plastic deformation and moreover, this intensity approaches infinity as the normal distance to the friction surface approaches zero. This should affect the distribution of temperature in the vicinity of frictional interfaces and therefore, the generation of white layer/fine grain layer under thermally dominant conditions. Because of the singular nature of the plastic work rate distribution in the vicinity of maximum friction surfaces, an asymptotic analysis of equations is required for the development of efficient numerical methods to accurately predict the distribution of temperature and as a result, the conditions of phase transformations near surfaces with high friction. Such an analysis is performed in the present paper for the double shearing model proposed by Spencer (1964). The result for the classical rigid perfectly plastic model is obtained as a particular case.

PLASTIC WORK RATE NEAR MAXIMUM FRICTION SURFACES

The double shearing model has been proposed by Spencer (1964). It is known that this model is described by a hyperbolic system of equations. Therefore, the definition for the maximum friction law is that the friction surface coincides with an envelope of characteristics. This definition is valid for the regime of sliding. In what follows, planar flows are considered. Let ω be a tool surface (curve in planes of flow) where the maximum friction law is satisfied (Fig. 1). Introduce an orthogonal curvilinear coordinate system (α, β) such that the curve β = 0 coincides with ω as in Eq. 1. It is assumed that the β-lines are straight. Then, the scale factor of these coordinate lines is $H_\beta = 1$. With no loss of generality it is possible to take the scale factor of the α-lines as:

$$H_\alpha = 1 + \frac{\beta}{R(\alpha)} \quad (1)$$

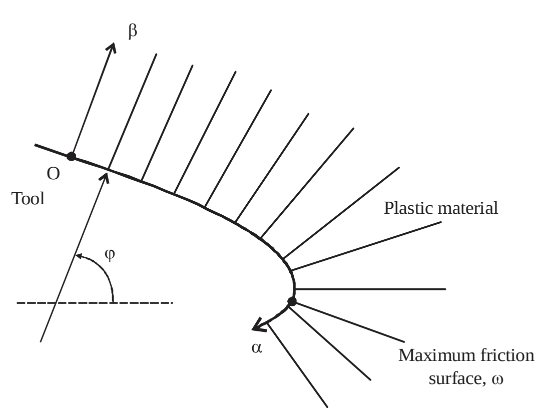

Fig. 1: Coordinate system in the vicinity of friction surface

where, $R(\alpha)$ is the radius of curvature of the line $\beta = 0$. This radius is defined as $R(\alpha) = -\Delta\alpha/\Delta\varphi$ (Fig. 1). A distinguished feature of the double shearing model is that the shear strain rate $\xi_{\alpha\beta}$ approaches infinity near the maximum friction surface (if the regime of sliding occurs) and follows an inverse square root rule (Alexandrov and Lyamina, 2002). In particular:

$$\left|\xi_{\alpha\beta}\right| = \frac{D}{\sqrt{s}} + o\left(\frac{1}{\sqrt{s}}\right) \tag{2}$$

as $s \to 0$. Here, D is the strain rate intensity factor and s is the normal distance to the friction surface. Equation 2 is also valid for rigid perfectly plastic materials (Alexandrov and Richmond, 2001). Equation 2 is in qualitative agreement with experimental observations that plastic strain is localized and therefore, white layers/fine grain layers are generated near frictional interfaces.

It is evident that conventional numerical methods are not capable to predict the correct distribution of the velocity field where Eq. 2 is satisfied. In particular, using ABAQUS a ring upsetting process has been analyzed by Chen *et al.* (1998). All the finite element analyses presented in this paper have failed to converge in the case of the maximum friction law. In the case of coupled thermo-plastic problems, the same difficulty has been reported by Rebelo and Kobayashi (1980). To predict an accurate distribution of temperature in the vicinity of maximum friction surfaces, the corresponding asymptotic expansion should be derived.

Since the normal strain rates $\xi_{\alpha\alpha}$ and $\xi_{\beta\beta}$ are finite at the maximum friction surface, it follows from Eq. 2 and the choice of the coordinate system that the plastic work rate is given by:

$$W = 2\tau_f \left|\xi_{\alpha\beta}\right| + o\left(\frac{1}{\sqrt{\beta}}\right) \tag{3}$$

as $\beta \to 0$. Here τ_f is the friction stress. Since τ_f is finite, it is seen from Eq. 3 that the plastic work rate approaches infinity in the vicinity of maximum friction surfaces. On the other hand, the plastic work rate is involved in the heat conduction equation. In particular:

$$\frac{\Delta T}{\Delta t} = a\Delta T + \frac{\gamma W}{\rho C_V} \tag{4}$$

where, T is the temperature, *a* is the thermal diffusivity, ρ is the density, C_V is the specific heat at constant volume and γ is a numerical factor that determines the fraction of the plastic work rate released as heat. Also, $\frac{\Delta T}{\Delta t}$ denotes the convected derivative and Δ the Laplace operator. It is assumed that a and C_V are both constant. The constitutive equations dictate that ρ is constant as well.

TEMPERATURE FIELD NEAR MAXIMUM FRICTION SURFACES IN STATIONARY FLOW

Using Eq. 1 and the equation $H_\beta = 1$ the Laplace operator is written as:

$$\Delta T = \frac{R}{(R+\beta)}\frac{\partial}{\partial\alpha}\left[\frac{R}{(R+\beta)}\frac{\partial T}{\partial\alpha}\right] + \frac{\partial^2 T}{\partial\beta^2} + \frac{1}{(R+\beta)}\frac{\partial T}{\partial\beta} \tag{5}$$

Since the flow is stationary, the convected derivative is given by:

$$\frac{\Delta T}{\Delta t} = \frac{Ru_\alpha}{(R+\beta)}\frac{\partial T}{\partial\alpha} + u_\beta\frac{\partial T}{\partial\beta} \tag{6}$$

where, u_α and u_β are the components of the velocity vector in the coordinate system chosen. Substituting Eq. 5 and 6 into Eq. 4 and using Eq. 3 yield:

$$\frac{Ru_\alpha}{(R+\beta)}\frac{\partial T}{\partial\alpha} + u_\beta\frac{\partial T}{\partial\beta} = a\left\{\frac{R}{(R+\beta)}\frac{\partial}{\partial\alpha}\left[\frac{R}{(R+\beta)}\frac{\partial T}{\partial\alpha}\right] + \frac{\partial^2 T}{\partial\beta^2} + \frac{1}{(R+\beta)}\frac{\partial T}{\partial\beta}\right\} + \frac{2\gamma\tau_f\left|\xi_{\alpha\beta}\right|}{\rho C_V} + o\left(\frac{1}{\sqrt{\beta}}\right) \tag{7}$$

as $\beta \to 0$. It is reasonable to assume that the derivatives $\partial T/\partial\alpha$ and $\partial^2 T/\partial\alpha^2$ are bounded along the maximum friction surface. Then, it is seen from Eq. 7 that:

$$\frac{\partial T}{\partial\beta} = O\left(\frac{1}{\sqrt{\beta}}\right) \text{ or } \frac{\partial^2 T}{\partial\beta^2} = O\left(\frac{1}{\sqrt{\beta}}\right) \tag{8}$$

as $\beta \to 0$. In Eq. 8 $\partial^2 T/\partial\beta^2 = O(\beta^{-3/2})$ as $\beta \to 0$. This contradicts Eq. 7. Therefore, it follows from Eq. 7 and 8 that the asymptotic representation of the temperature field in the vicinity of maximum friction surfaces is:

$$T = T_f(\alpha) - \frac{q_f(\alpha)}{k}\beta - \frac{4\gamma\sigma_s D(\alpha)}{3a\rho C_V}\beta^{3/2} + o\left(\beta^{3/2}\right) \tag{9}$$

as $\beta \rightarrow 0$. Here, k is the coefficient of thermal conductivity, T_f is the temperature at the friction surface and q_f is the component of the heat flux vector normal to the friction surface. Both T_f and q_f are in general functions of α.

To the best of authors' knowledge, no publication devoted to the asymptotic behavior of the temperature field in the vicinity of maximum friction surfaces in the case of the double shearing model is available. Moreover, it is evident that no direct experimental verification of the asymptotic solution of Eq. 9 is possible. However, experimental results reported by Jaspers and Dautzenberg (2002), Lo and Lin (2002), Sartkulvanich *et al*. (2005), Liljerehn *et al*. (2009), Bahi *et al*. (2012) and Molinari *et al*. (2012) demonstrate the importance of the precise determination of temperature distributions in the vicinity of frictional interfaces. Usually, finite element modeling is used to find such distributions (Lo and Lin, 2002; Sartkulvanich *et al*., 2005) among many others. It is evident from Eq. 9 that traditional finite element methods are not capable to predict the correct distribution of temperature in the vicinity of maximum friction surfaces in the case of the double shearing model. The main result of the present study can be used to develop the extended finite element method for this purpose. An overview of extended finite element method method is presented in Fries and Belytschko (2010).

A particular solution for non-stationary rigid perfectly plastic flow has been found by Alexandrov and Miszuris (2015). This solution is singular. The singularity in the temperature field is caused by the singularity in the velocity field. In the present study, the double shearing model has been investigated. However, the velocity field is also singular for the double slip and rotation model (Alexandrov and Harris, 2006), models of anisotropic plasticity (Alexandrov and Jeng, 2013) and viscoplastic models (Alexandrov and Mustafa, 2013, 2015). The main result of the present study can be extended to these rigid plastic models without any conceptual difficulty.

CONCLUSION

The asymptotic analysis of the heat conduction equation in conjunction with the double shearing model has been performed in the vicinity of the surfaces with maximum friction. A distinguished feature of solutions for the model chosen is that the plastic work rate follows an inverse square root rule near maximum friction surfaces. Therefore, it may be advantageous to adopt this model to describe the generation of narrow white/fine grain layers in the vicinity of frictional interfaces. For, the temperature rise due to the singular behavior of the plastic work rate may be of considerable importance in this respect. In order to accurately predict the distribution of temperature near maximum friction surfaces and the asymptotic expansion given in Eq. 9 should be used in numerical codes.

ACKNOWLEDGMENT

The first author acknowledges support from grant RFBR-14-01-00087.

REFERENCES

Abukhshim, N.A., P.T. Mativenga and M.A. Sheikh, 2006. Heat generation and temperature prediction in metal cutting: A review and implications for high speed machining. Int. J. Mach. Tools Manuf., 46: 782-800.

Agmell, M., A. Ahadi and J.E. Stahl, 2014. Identification of plasticity constants from orthogonal cutting and inverse analysis. Mech. Mater., 77: 43-51.

Akbar, F., P.T. Mativenga and M.A. Sheikh, 2010. An experimental and coupled thermo-mechanical finite element study of heat partition effects in machining. Int. J. Adv. Manuf. Technol., 46: 491-507.

Alexandrov, S. and O. Richmond, 2001. Singular plastic flow fields near surfaces of maximum friction stress. Int. J. Non-Linear Mech., 36: 1-11.

Alexandrov, S. and D. Harris, 2006. Comparison of solution behaviour for three models of pressure-dependent plasticity: A simple analytical example. Int. J. Mech. Sci., 48: 750-762.

Alexandrov, S. and Y. Mustafa, 2013. Singular solutions in viscoplasticity under plane strain conditions. Meccanica, 48: 2203-2208.

Alexandrov, S. and Y.R. Jeng, 2013. Singular rigid/plastic solutions in anisotropic plasticity under plane strain conditions. Continuum Mech. Thermodyn., 25: 685-689.

Alexandrov, S. and W. Miszuris, 2015. Heat generation in plane strain compression of a thin rigid plastic layer. Acta Mechanica, (In Press). 10.1007/s00707-015-1499-8

Alexandrov, S. and Y. Mustafa, 2015. Quasi-static axially symmetric viscoplastic flows near very rough walls. Applied Math. Modell., 39: 4599-4606.

Alexandrov, S.E. and E.A. Lyamina, 2002. Singular solutions for plane plastic flow of pressure-dependent materials. Doklady Phys., 47: 308-311.

Bahi, S., M. Nouari, A. Moufki, M. El Mansori and A. Molinari, 2012. Hybrid modelling of sliding-sticking zones at the tool-chip interface under dry machining and tool wear analysis. Wear, 286-287: 45-54.

Chen, G., J. Li, Y. He and C. Ren, 2014. A new approach to the determination of plastic flow stress and failure initiation strain for aluminum alloys cutting process. Comput. Mater. Sci., 95: 568-578.

Chen, J.S., C. Pan, C.M.O.L. Roque and H.P. Wang, 1998. A Lagrangian reproducing kernel particle method for metal forming analysis. Comput. Mech., 22: 289-307.

Fries, T.P. and T. Belytschko, 2010. The extended/generalized finite element method: An overview of the method and its applications. Int. J. Numer. Methods Eng., 84: 253-304.

Griffiths, B.J., 1987. Mechanisms of white layer generation with reference to machining and deformation processes. J. Tribol., 109: 525-530.

Hadala, B., 2013. Implementation of the heat balance in the finite element solution to the temperature field of the plastically deformed material. Int. J. Thermal Sci., 71: 172-181.

Jaspers, S.P.F.C. and J.H. Dautzenberg, 2002. Material behaviour in conditions similar to metal cutting: Flow stress in the primary shear zone. J. Mater. Process. Technol., 122: 322-330.

Karpat, Y. and T. Ozel, 2006. Predictive analytical and thermal modeling of orthogonal cutting process-Part I: Predictions of tool forces, stresses and temperature distributions. J. Manuf. Sci. Eng., 128: 435-444.

Kim, Y.T. and K. Ikeda, 2000. Flow behavior of the billet surface layer in porthole die extrusion of aluminum. Metall. Mater. Trans. A, 31: 1635-1643.

Lalwani, D.I., N.K. Mehta and P.K. Jain, 2009. Extension of Oxley's predictive machining theory for Johnson and Cook flow stress model. J. Mater. Process. Technol., 209: 5305-5312.

Liljerehn, A., V. Kalhori and M. Lundblad, 2009. Experimental studies and modeling of heat generation in metal machining. Machin. Sci. Technol., 13: 488-515.

Lo, S.P. and Y.Y. Lin, 2002. An investigation of sticking behavior on the chip-tool interface using thermo-elastic-plastic finite element method. J. Mater. Process. Technol., 121: 285-292.

Molinari, A., R. Cheriguene and H. Miguelez, 2012. Contact variables and thermal effects at the tool-chip interface in orthogonal cutting. Int. J. Solids Struct., 49: 3774-3796.

Ng, E.G., D.K. Aspinwall, D. Brazil and J. Monaghan, 1999. Modelling of temperature and forces when orthogonally machining hardened steel. Int. J. Mach. Tools Manuf., 39: 885-903.

Ramesh, A. and S.N. Melkote, 2008. Modeling of white layer formation under thermally dominant conditions in orthogonal machining of hardened AISI 52100 steel. Int. J. Mach. Tools Manuf., 48: 402-414.

Rebelo, N. and S. Kobayashi, 1980. A coupled analysis of viscoplastic deformation and heat transfer-II: Applications. Int. J. Mech. Sci., 22: 707-718.

Sanabria, V., S. Mueller and W. Reimers, 2014a. Microstructure evolution of friction boundary layer during extrusion of AA 6060. Procedia Eng., 81: 586-591.

Sanabria, V., S. Mueller, S. Gall and W. Reimers, 2014b. Investigation of friction boundary conditions during extrusion of aluminium and magnesium alloys. Key Eng. Mater., 611-612: 997-1004.

Sartkulvanich, P., T. Altan and A. Gocmen, 2005. Effects of flow stress and friction models in finite element simulation of orthogonal cutting-a sensitivity analysis. Mach. Sci. Technol., 9: 1-26.

Spencer, A.J.M., 1964. A theory of the kinematics of ideal soils under plane strain conditions. J. Mech. Phys. Solids, 12: 337-351.

Tanner, R.I. and W. Johnson, 1960. Temperature distributions in some fast metalworking operations. Int. J. Mech. Sci., 1: 28-44.

Wideroe, F. and T. Welo, 2012. Conditions for sticking friction between aluminium alloy AA6060 and tool steel in hot forming. Key Eng. Mater., 491: 121-128.