Simulation of the Potato Osmo-dehydration Process
Using Equation Oriented Approach

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Abstract: A mathematical model was developed in order to predict the main variables involved in potato osmo-dehydration process. The model consider the effects of shrinkage, the variation of apparent diffusion coefficients and osmotic solution properties. The initial conditions of the main process variables needed to solve the corresponding differential equations were obtained from the literature. A numerical algorithm was developed for solving these equations. The validity of the model was demonstrated using the experimental data from the literature and the satisfactory agreement was found between the experimental data and the model predictions.

Key words: Osmo-dehydration process, modeling, potato, shrinkage, water loss, solid gain, simulation

INTRODUCTION

Osmotic processes are an integrated part of the extraction processes in which water is removed from food stuffs at relatively low temperatures well around 20-50°C. The application range of osmotic processes is rather wide based on phase contact type and can be categorized in two main groups, a) Liquid–liquid Osmotic Dehydration (LLOD) (i.e.; suitable for milk, whey, syrups, fruit juices and vegetable juices, b) Solid–liquid Osmotic Dehydration (SLOD) (i.e.; suitable for mushroom, seafood, meat fruits and etc.)[1-3].

Current interest in the osmotic processes mostly comes from the quality improvement of product combined with the energy saving. A large number of basic experimental works were conducted in the osmotic processes in order to understand the effect of process variables on osmotic operations, underlying mechanisms in osmotic processes and transport phenomena. Although, the mathematical modeling plays an important role in optimization, design and simulation of osmotic processes, but the amount of work conducted on this issue is rather small. The proposed models in the osmo-dehydration process can be classified into three main groups as the following:

1. Models based on the first order kinetics or algebraic equations obtained from solving second Fick’s law for short contact times using the lumped-modeling approach. In this approach, all independent variables vary as a function of time and the effect of the food shape is not included and only the water loss (wl) and solid gain (sg) are predictable[4-6].

2. Models based on the osmotic mass transfer in cellular scale. These models not only predict macroscopic variables (i.e.; wl, sg) but also calculate cellular evolutions (microscopic variables) such as variations of trans-membrane flux (based on non-reversible thermodynamics), extra-cellular volume and bulk velocity[4-6].

3. Models based on the Fick’s second law. In this approach, food stuff is considered as a porous and homogeneous medium where food external surface area is considered as a mass transfer area. These models take into account the food shape, aiming to predict process rate and local concentration profiles[4-6].

The objective of the present study was to develop a mathematical model and a simulation algorithm as a suitable way to design and optimize the potato
osmo-dehydration process. Furthermore, the volume and mass of the osmotic solution and process main variables may also be estimated. Hence, an attempt was made to develop a model which includes several features that were neglected or simplified in the previous studies as outlined above, in order to produce a predictive tool. This study presents the details of the modeling approaches taken to obtain a process simulation program for the potato osmo-dehydration process.

**MODELING APPROACH**

Mathematical models can be useful in all phases of food engineering, from research and development to plant operations, and even in business and economic studies. The most important goals of developing a mathematical model for food processing systems are exploring the effects of different operating conditions for optimization and control studies, adding in scale up calculations, process design simulation and control, troubleshooting and de-bottlenecking. Such goals could also largely be applied to osmo-dehydration process. The proposed model in this section is able to calculate Wt, sg, potato volume and mass, osmotic solution volume, osmotic solution mass and local concentration profiles during the osmo-dehydration process. The model depends on size and shape of potato, mass ratio of osmotic solution to potato, temperature, concentration and composition of potato and osmotic solution. For all modeling purposes, one should first start with the model hypotheses, which are presented below.

**Model hypotheses:** The model presented in this section for the potato osmo-dehydration process is based on the following assumptions:

- The process is isothermal
- The osmotic solution is considered well mixed in most treatments. Since external resistance is considered negligible as compared to the internal resistance, this hypothesis seems justified. Based on this hypothesis, the species concentration on the food surface is considered equal to the concentration of osmotic solution.
- Two simultaneous counter-current phenomena are assumed in modeling: water diffusion into the osmotic solution and the osmotic solutes diffusion into the potatoes.
- Initial water and soluble solids in the potato cubes are considered uniform.
- Volume variation of fiber in the potato is assumed negligible. Since, the initial amount of fiber volume in potato is very small (~1-5%) and its contribution to mass transfer is insignificant, therefore this hypothesis is justified[25].

Based on the assumptions presented here, the details of the modeling approach will be given here.

**Modeling procedure:** The system of the governing equations for osmo-dehydration process is presented bellows:

For the specie j, the Fick's second law is used to estimate the concentration profiles and driving force.

\[
\frac{\partial \rho_j}{\partial t} = D \frac{\partial^2 \rho_j}{\partial x^2}
\]  

(1)

The mass variation of specie j within potato during osmo-dehydration process is:

\[
\frac{dm_{ij}}{dt} = n_j A_p
\]  

(2)

The potato volume and mass is calculated from these equations:

\[
\frac{dm_{ip}}{dt} = \sum_{j=1}^{n} dm_{ij}
\]  

(3)

and

\[
\frac{dV_p}{dt} = \sum_{j=1}^{n} dm_{ij} V_i
\]  

(4)

The mass of component j in the osmotic solution is:

\[
\frac{dm_{ip}}{dt} = \frac{-dm_{ip}}{dt}
\]  

(5)

The osmotic solution volume and mass are calculated using the following expressions:

\[
\frac{dV_i}{dt} = \frac{-dV_{ip}}{dt}
\]  

(6)

\[
\frac{dm_{ip}}{dt} = \frac{-dm_{ip}}{dt}
\]  

(7)

Equation 5-7 indicate that the changes of the osmotic solution are the inversion of changes in the potato variables.
Amount of the mass flux passed through the interface between product and osmotic solution is

\[ n_j = \pm D_{ij} \frac{\rho_j |Z_{+ \Delta z} - \rho_j |Z_{- \Delta z}}{\Delta z} \]  

(8)

Where, \( \rho_j |Z_{+ \Delta z} \) is mass concentration at the interface and \( \rho_j |Z_{- \Delta z} \) is mass concentration in its neighborhood point. \( \Delta z \) is distance between two neighborhood points. The positive sign in Eq. 8 is related to the water transfer and the negative sign is related to the osmotic agents transfer. The total volume changes during the osmo-dehydration process is attributed to changes in solid matrix (sm) including insoluble solid, liquid phase (lp) including water and soluble solids and gas phase (gp) including filled air in the space between wall cell and membrane

\[ \Delta V_p = \Delta V^{lp} + \Delta V^{gp} + \Delta V^{sm} \]  

(9)

In fact, it is possible to assume that \( \Delta V^{sm} \) is negligible since the initial volume of the solid matrix is very small (about 1-5 %). The changes in gp in the case of some fruits (such as apple) are significant but in potato it would be negligible. In result, gas phase contribution to potato volume changes is considered negligible. Therefore, Eq. 9 is simplified to:

\[ \Delta V_p = \Delta V^{lp} \]  

(10)

The shrinkage quantity of potato is calculated from Eq. 4. The transfer area and the half thickness of potato cube are calculated using the following expressions:

\[ Z = 0.5 \sqrt{V} \]  

(11)

\[ A = 6 \sqrt[3]{V} \]  

(12)

The initial conditions for the above-mentioned equations are described as the followings:

\[ m_{p, j}(0, Z) = cte \]  

(13)

\[ V_{p, j}(0, Z) = cte \]  

(14)

\[ \rho_{jp}(0, Z) = \frac{m_{jp}(0, Z)}{V_{jp}(0, Z)} \]  

(15)

The boundary conditions for the above-mentioned equations are:

\[ \frac{\partial \rho_{jp}}{\partial Z} = 0 \quad \text{at} \quad Z = 0 \]  

(16)

\[ \rho_{jp} = \frac{m_{jp}(t)}{V_{jp}(t)} \quad \text{at} \quad Z = L \]  

(17)

Due to a limited mass ratio of solution to potato (\( R = 4 \)) components concentration of osmotic solution varies as a function of time (Eq. 17). The apparent diffusion coefficient used in the modeling is defined as:

\[ D_j = \frac{D_{ij}}{\mu_j} \]  

(18)

Where, \( \mu_j \) is the relative kinematics viscosity of the solution within potato. The apparent diffusion coefficient at the initial time (\( D_i \)) may be obtained from the literature and is shown in Table 1.

The relative kinematic viscosity of the solution was calculated from empirical correlation reported by Chen et al.

\[ \mu_j = 1 + \sum_{i=1}^{n} (\Psi j - w(T_j)) \prod_{i=1}^{n} [\mu_{i} \exp(\text{kg} m^2)] \]  

(19)

where:

\[ \Psi j = \exp(-\frac{m_{ij}}{gT_h + h}) \]  

(21)

\[ w(T_j) = \exp(-3.7377T_j) \]  

(22)

\[ T_h = \frac{T(K)}{273.1} \]  

(23)

Where, \( T(K) \) is temperature of process (in K). The constant parameters (e, f, g, h, k, n, q) in Eq. 19 and 21 are shown in Table 2.

**Computer simulation:** MATLAB is used as simulation tool to solve the corresponding equations. MATLAB is a high-performance language for technical computing, which is distinguished by its ability to perform all the calculations in matrix form, its large library of built-in functions and its rich graphical visualization tools. The solution algorithm for the modeling is presented in Fig. 1, along with the corresponding numerical methods applied. The simulation program consists of one script m.file as
main file (including initialization, calculation and data exploitation) and several functions in which different variables (i.e., apparent diffusion coefficients, transfer area and position direction) are calculated and the resulting differential equations are solved numerically.

**RESULTS AND DISCUSSION**

To validate the proposed model, the experimental data were taken from the literature on potato square cubes of 1 cm in a well mixed solution of sucrose-sodium chloride 45-15% by weight at 23°C (14, 15).

As shown in Fig. 2, there is a good agreement between predicted values and experimental data for \( w_l \) and \( s_g \). At the beginning of the process, these variables encounter a significant increase due to the highest level of the driving force and apparent diffusion coefficients. Figure 3 shows a satisfactory agreement between the predicted values for soluble solids mass fraction in osmotic solution and potato. As indicated in this Fig. 3, from the mass balance point of view, the developed model is quite suitable. \( w_l \) and \( s_g \) are in good agreement with experimental data and as mentioned previously, Eq. 10 remains valid for potato volume variation (Fig. 4 and 5). Because of the lack of experimental data in the case of variations of osmotic solution volume and osmotic

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**Table 1:** Apparent diffusion coefficient at the initial time

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Water</th>
<th>Sodium chloride</th>
<th>Sucrose</th>
</tr>
</thead>
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<tr>
<td>( D^* 10^9 )</td>
<td>1.1</td>
<td>4.5</td>
<td>0.68</td>
</tr>
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**Table 2:** Constant parameters used in Eq. 19 and 21

<table>
<thead>
<tr>
<th>Solvent</th>
<th>c</th>
<th>f</th>
<th>g</th>
<th>H</th>
<th>k</th>
<th>n</th>
<th>q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sodium chloride</td>
<td>0.120</td>
<td>-0.44</td>
<td>-3.713</td>
<td>2.792</td>
<td>2.393</td>
<td>0.10</td>
<td>0.67</td>
</tr>
<tr>
<td>Sucrose</td>
<td>0.73</td>
<td>1.1</td>
<td>8.345</td>
<td>-7.042</td>
<td>0.49</td>
<td>1.10</td>
<td>0.688</td>
</tr>
</tbody>
</table>

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Fig. 2: Variation of \( w_l \) and \( s_g \) versus time in the potato osmo-dehydration process.

Fig. 3: Variation of \( s_c \) (for potato) and \( s_c \) (for sucrose) versus time during the osmo-dehydration process.
solution mass, predicted quantities were only presented in Fig. 5. However, as indicated in Fig. 5, the mass variation and volume variation of the osmotic solution are equal to inversion of potato mass and volume variation, respectively. This means that the overall trend remains valid. Figure 5 also shows that the parameter changes in quantity of osmotic solution mass to volume for R=4 are significant. This point should be carefully considered in the osmotic dehydration model development. As shown in Fig. 2, 4 and 5, mass loss and volume variation were reached to a higher level after 5 h contact time in contactor as compared to 20 h. This indicates that after reaching the water to equilibrium conditions, the sugar and salt gain continue to increase in the potato tissue to a level, which causes water flow acceleration in the reverse direction from osmotic solution to potato. Due to

the use of solution concentration change instead of initial concentration of solution for the surface concentration value of product in this model, model prediction is very close to experimental data. In order to optimize the mass ratio of osmotic solution to product the curve of Wt and sG (water loss and solid gain after four hours) versus R was plotted in Fig. 6. Lenart and Flink[14,15] reported the optimal value for R at the range of 4-6, while Fig. 6 indicates the optimal value around 4, since the variation of Wt is negligible as R passes 4. The calculation of the osmotic solution to product mass ratio is very important for the determination of the optimized dimensions of the osmotic unit operation devices. Figure 7 shows the water profile in cubes potatoes. The prediction of moisture content distribution is very useful in the study of chemical reactions that occur during osmotic dehydration, where reactions generally depend on moisture content. One of the irreversible changes that
accompany the osmotic dehydration of potato is non-
enzymatic browning, which can lead to loss of protein
biological value. The prediction of the extent of non-
enzymatic browning that occurs in food dehydration
should be based upon local rather than average
conditions of moisture content in the foodstuff and
therefore, the results of this study could be very helpful
in this way for future modeling works.

CONCLUSIONS

A model was developed for the osmotic dehydration
of potato based on the set of differential equations. The
model considers the shrinkage of potato, variations of
apparent diffusion coefficients and osmotic solution
volume, which are partly neglected in the existing
models in the literature. The resulting model was used to predict
the performance of the osmo-dehydration process in
potato in terms of potato volume variation and osmotic
solution properties. The agreement between the model
predictions and the experimental data are rather
satisfactory, but more experimental data are still needed
from the industrial and pilot plants in order to confirm the
applicable range of the proposed SLOD model and to
make it more comprehensive and robust. However, the
model can now be used to represent SLOD units in
various extraction processes in food industries. The
framework developed in this study could also be used
develop a new model in order to predict the chemical
changes occurs during the osmo-dehydration processes
for various fruits.

ACKNOWLEDGMENT

The financial support of the University of Tehran is
gratefully acknowledged.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$\mu$</td>
<td>kinematics viscosity (m$^2$/s)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Mass concentration (kg/m$^3$)</td>
</tr>
<tr>
<td>$v$</td>
<td>specific volume (m$^3$/kg)</td>
</tr>
<tr>
<td>$A$</td>
<td>transfer area (m$^2$)</td>
</tr>
<tr>
<td>$D_a$</td>
<td>apparent diffusion coefficient (m$^2$/s)</td>
</tr>
<tr>
<td>$K$</td>
<td>gas volume coefficient (-)</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass (kg)</td>
</tr>
<tr>
<td>$m_v$</td>
<td>Mass variation (kg/s)</td>
</tr>
<tr>
<td>$N$</td>
<td>number of components</td>
</tr>
<tr>
<td>$R$</td>
<td>osmotic solution to product mass ratio</td>
</tr>
<tr>
<td>$\delta_g$</td>
<td>solid gain</td>
</tr>
<tr>
<td>$s$</td>
<td>Mass fraction of soluble solids (-)</td>
</tr>
<tr>
<td>$v_v$</td>
<td>volume variation (m$^3$/s)</td>
</tr>
<tr>
<td>$w$</td>
<td>water loss (-)</td>
</tr>
<tr>
<td>$Z$</td>
<td>thickness of the transfer pathway (m)</td>
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<table>
<thead>
<tr>
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<th>Definition</th>
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<tbody>
<tr>
<td>subscript</td>
<td>j species no,j</td>
</tr>
<tr>
<td></td>
<td>0 initial condition</td>
</tr>
<tr>
<td></td>
<td>p product</td>
</tr>
<tr>
<td></td>
<td>r relative</td>
</tr>
<tr>
<td></td>
<td>s solution</td>
</tr>
<tr>
<td></td>
<td>w water</td>
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<table>
<thead>
<tr>
<th>Superscript</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>gp</td>
<td>gas phase in potato</td>
</tr>
<tr>
<td>lp</td>
<td>liquid phase in potato</td>
</tr>
<tr>
<td>0</td>
<td>initial time</td>
</tr>
<tr>
<td>Sm</td>
<td>solid matrix in potato</td>
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REFERENCES


