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Research Article
Grey Stochastic Multi-criteria Decision-making Approach for Information System Evaluation

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Abstract

For grey stochastic multi-criteria decision-making problem with criterion value as extended grey number, the study proposes grey stochastic multi-criteria decision-making approach based on Hausdorff distance. First, it provides definition and operation rule of extended grey number stochastic variable and expectation, then obtains expectation decision matrix about grey number based on grey decision matrix and natural state probability. Second, the study calculates distance between the various solutions and positive and negative ideal solutions, respectively by combining weight vector of various criteria and ultimately determines the relative closeness degree and sorts the solution based on the value. Finally, through information system evaluation, the study results verify feasibility and effectiveness of the proposed method.

Key words: Grey stochastic, multi-criteria decision-making, extended grey numbers, Hausdorff distance, information system

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INTRODUCTION

Multi-criteria decision-making (MCDM), as an important part of modern decision-making science, emphasizes on solving limited situation decision-making problem under multi-criteria circumstance. Its theory and methods have been widely applied to social life, engineering design, system engineering and management science, etc. In real life, due to complexity of the external environment, ambiguity of objective things by themselves and limitations of human knowledge, there are many uncertainties in the decision-making process. Therefore, decision-making information in actual decision-making problems usually has such uncertainties as fuzziness, randomness or grayness, etc.

Grey stochastic MCDM problem has two characteristics of grayness and randomness. The relevant studies progresses slowly with relatively few studies results obtained. Present, studies of this aspect has attracted positive attention of experts and scholars all over the world, for instance, Yalcin *et al.* (2012) proposed a new financial performance evaluation approach to rank the companies of each sector in the Turkish manufacturing industry. For this purpose, a hierarchical financial performance evaluation model is structured based on the AFP and VFP main-criteria and their sub-criteria. Krohling and de Souza (2012) proposed a hybrid approach combining prospect theory and fuzzy numbers to handle risk and uncertainty in MCDM problems. Wang *et al.* (2013) defined possibility degree of grey stochastic variable expectation, studied stochastic MCDM problem with weight not completely certain and with criterion value as interval grey number. Boran (2011) proposed the integration of intuitionistic fuzzy preference relation aiming to obtain weights of criteria and intuitionistic fuzzy TOPSIS method aiming to rank alternatives for dealing with imprecise information on selecting the most desirable facility location. Mousavi *et al.* (2013) developed a new fuzzy grey multi-criteria group decision making model to solve evaluation and selection problems under uncertainty in real-life situations. Luo *et al.* (2008), based on relative membership degree of ideal matrix, explored risk multiple criteria group decision-making problem with weight information unknown and with criterion value as interval grey number. The above-mentioned methods have provided some research ideas to solve MCDM problems. However, it can be found that there is relatively little study on stochastic MCDM problem with criterion value as extended grey number that considers criterion natural state. However, in the actual decision-making problems, it is relatively difficult for decision makers to accurately predict the occurrence probability of event or natural state. Thus, this study proposes the corresponding decision-making approach to meet the needs of such decisions.

MATERIALS AND METHODS

Preliminaries

Extended grey number: Grey number refers to number (Liu *et al.*, 1999) only with approximate range known but not the exact value, which can effectively measure the grayness of things. In practice, the value of grey numbers is limited to a certain interval or a general set of numbers, usually denoted as "$\otimes$".

Definition 1: Assume $\otimes$ is a grey number, D is a collection that covers $\otimes$, then:

- If D is an interval, then $\otimes$ can be called interval grey number, denoted as $\forall\otimes\Rightarrow d^*\in [a, b]$ or $\otimes = [a, b]$
- If D is a discrete set, then $\otimes$ can be called discrete grey number, denoted as $\forall\otimes\Rightarrow d^*\in D$, $D = \{d_1, d_2, ..., d_n\}$ or $\otimes = \{d_1, d_2, ..., d_n\}$

where, the value of interval grey number can be compared with probability degree of interval grey number. To better describe the grayness of decision-making information, extended grey number that combines discrete grey number and continuous grey number can be used (Dalalah *et al.*, 2011).

Definition 2: If D is a set of a series of interval grey numbers, then $\otimes$ can be called extended grey number, denoted as:

$$\otimes = \bigcup_{i=1}^{n} (a_i, b_i)$$

Where:

$$[a_i, b_i]\cap[a_j, b_j] = \varnothing(i\neq j),\ a_i\leq b_i\ (i = 1, 2, ..., n)$$

Denote set of all extended grey numbers as $R(\otimes)$.

Extended grey number distance and expectation: Grey number distance describes the degree of separation between two grey numbers, which plays an important role in description of distance between criterion evaluation value and ideal value. In view of current study (Lin *et al.*, 2008) on definition of interval grey number distance and considering that the theory does not fit extended grey number, this study gives definition of extended grey number distance.

Definition 3: If:

$$\otimes_1 = \bigcup_{i=1}^{n}[a_i, b_i], \otimes_2 = \bigcup_{j=1}^{m}[c_j, d_j] \in R(\otimes), a_i \le b_i \ (i = 1, 2, ..., n), c_j \le d_j \ (j = 1, 2, ..., m)$$

then Hausdorff distance between extended number $\otimes_1$ and $\otimes_1$ is (Wang and Wang, 2014):

$$D(\otimes_1, \otimes_2) = \max\{h(\otimes_1, \otimes_2), h(\otimes_2, \otimes_1)\} \quad (1)$$

Where:

$$h(\otimes_1, \otimes_2) = \max_{i=1}^{n}\min_{j=1}^{m} \| \otimes x_i - \otimes y_j \|$$

is Hausdorff distance between $\otimes_1$ and $\otimes_2$, $\otimes x_i = [a_i, b_i]$, $\otimes y_j = [c_j, d_j]$ $(i = 1, 2, ..., n, j = 1, 2, ..., m)$. $\|\cdot\|$ represents any norm, such as L_p.

When $\|\cdot\|$ is L_p:

$$\| \otimes x_i - \otimes y_j \| = \sqrt[p]{|a_i - c_j|^p + |b_i - d_j|^p}$$

Thus obtain:

$$D(\otimes_1, \otimes_2) = \max\{\max_{i=1}^{n}\min_{j=1}^{m} \sqrt[p]{|a_i - c_j|^p + |b_i - d_j|^p}, \max_{j=1}^{m}\min_{i=1}^{n} \sqrt[p]{|c_j - a_i|^p + |d_j - b_i|^p}\} \quad (2)$$

where, p = 1, 2, ..., l, l tends to + ∞.

Definition 4: Extended grey number random variable is a set of random variables composed of a limited number of different extended grey numbers $\otimes$, denoted as $\xi(\otimes)$. Its probability distribution is shown in Table 1, which can also be denoted with probability distribution function $f(\xi(\otimes))$.

In Table 1, $\otimes_l$ is the value of extended grey number random variable $\xi(\otimes)$ at occurrence of the i-th state,

$$\otimes_i \in \bigcup_{i=1}^{n}[\underline{x}_i, \overline{x}_i], \underline{x}_i \le \overline{x}_i 1 \le i \le n$$

$p_i = 1$, n is the probability at occurrence of the i-th state, which meets $\sum_{i=1}^{n} p_i = 1$, n is the number of possible values for extended grey number random variables (Marques *et al.*, 2011). Probability distribution function $f(\xi(\otimes))$ is $f(\xi(\otimes) = \otimes_l) = p_i$.

Table 1: Probability distribution of extended grey number random variable $\xi(\otimes)$

$\xi(\otimes)$	$\otimes_1$	$\otimes_2$	...	$\otimes_l$	...	$\otimes_n$
p	p_1	p_2	...	p_i	...	p_n

Definition 5: Assume $\xi(\otimes)$ is an extended grey number random variable and then $\sum_{i=1}^{n} p_i \times \otimes_i$ can be called expectation of extended grey random variable, denoted to be:

$$E(\xi(\otimes)) = \sum_{i=1}^{n} p_i \times \otimes_i$$

Grey stochastic multi-criteria decision-making approach: For stochastic MCDM problem with criterion value as extended grey number, assume that $A = \{A_1, A_2, \dots, A_m\}$ is a scheme set, $B = \{B_1, B_2, \dots, B_n\}$ is a mutually independent set of criteria, criterion weight vector $w = \{w_1, w_2, \dots, w_n\}$, which satisfies $\sum_{j=1}^{n} w_j = 1$, $w_j \ge 0$ (j = 1, 2, ..., n). Due to uncertainty of decision-making environment, solution has s kinds of natural state in various criteria, the state set of, which is $\theta = \{\theta_1, \theta_2, ..., \theta_s\}$. Denote the probability at occurrence of the t-th state ($t \le s$) as P_t. The value of solution A_i at the j-th criterion is extended grey number random variable u_{ij}, whose value at the t–th state is extended grey number $\otimes u_{ij}^t$, denoted as:

$$\otimes u_{ij}^t = \bigcup_{k=1}^{l}[a_{ijk}^t, b_{ijk}^t]$$

and thus, obtain decision matrix $R^t = \{u_{ij}^t\}_{m \times n}$ (Nayagam *et al.*, 2011).

When the various criteria weights are known, the decision-making approach is the best solution or sorting to determine solution set, whose decision-making procedure is as follows:

Step 1: Normalization approach of decision matrix. To eliminate the influence of criteria on decision-making results due to different dimensions, decision matrix R^t can be normalized (Li *et al.*, 2007). In MCDM problems, the common types include efficiency and cost type. For efficiency criterion, the greater the value, the better, while for cost criterion, the smaller the value, the better

Efficiency criterion value is:

$$\otimes r_{ij}^t = \frac{\otimes u_{ij}^t}{b_{ijk}^{t\ (max)}} = \bigcup_{k=1}^{l}[\frac{a_{ijk}^t}{b_{ijk}^{t\ (max)}}, \frac{b_{ijk}^t}{b_{ijk}^{t\ (max)}}] \quad (3)$$

Cost criterion value is:

$$\otimes r_{ij}^{t} = \frac{a_{ijk}^{t\ (min)}}{\otimes u_{ij}^{t}} = \bigcup_{k=1}^{l}[\frac{a_{ijk}^{t\ (min)}}{b_{ijk}^{t}}, \frac{a_{ijk}^{t\ (min)}}{a_{ijk}^{t}}] \quad (4)$$

Where:

$$b_{ijk}^{t\ (max)} = \max_{1\le k\le l;1\le i\le m} b_{ijk}^{t},\ a_{ijk}^{t\ (min)} = \min_{1\le k\le l;1\le i\le m} a_{ijk}^{t}$$

Corresponding to various criteria, standardization decision matrix of *s* natural state is $G^{t} = \{\otimes r_{ij}^{t}\}_{m\times n}$.

Step 2: Determine expectations. According to grey decision matrix G^t and probability P_t of natural state t, calculate expectation of each solution at various state based on definition 5 and thereby obtain expectation decision matrix $G = \{\otimes r_{ij}\}_{m\times n}$

Calculation formula of expectation is:

$$E(\xi(\otimes r_{ij})) = \sum_{t=1}^{s} p_t \times \otimes r_{ij}^{t} \quad (5)$$

Step 3: Determine positive ideal solution and negative ideal solution

Positive ideal solution A^+ is:

$$\begin{cases} A^{+} = (A_1^{+}, A_2^{+}, \cdots, A_n^{+}) \\ A_j^{+} = [\max_{i=1,2,\cdots,m} \underline{r}_{ij1}, \max_{i=1,2,\cdots,m} \overline{r}_{ijh}] \end{cases} \quad (6)$$

Negative ideal solution A^- is:

$$\begin{cases} A^{-} = (A_1^{-}, A_2^{-}, \cdots, A_n^{-}) \\ A_j^{-} = [\min_{i=1,2,\cdots,m} \underline{r}_{ij1}, \min_{i=1,2,\cdots,m} \overline{r}_{ijh}] \end{cases} \quad (7)$$

Step 4: Calculate distance between various solution and positive, negative ideal solution

The distance between A_i and A^+ is:

$$d_i^{+}(A_i, A^{+}) = \sum_{j=1}^{n} w_j D_1(\otimes r_{ij}, A_j^{+}) \quad (8)$$

The distance between A_i and A^- is:

$$d_i^{-}(A_i, A^{-}) = \sum_{j=1}^{n} w_j D_1(\otimes r_{ij}, A_j^{-}) \quad (9)$$

Where:

$$D_1(\otimes r_{ij}, A_j^{+})$$

is the distance between $\otimes r_{ij}$ and A_j^{+}, $D_1(\otimes r_{ij}, A_j^{-})$ is the distance between $\otimes r_{ij}$ and A_j^{-}.

Step 5: Calculate relative closeness degrees K_i and sort the solution

$$K_i = \frac{d_i^{+}(A_i, A^{+})}{d_i^{+}(A_i, A^{+}) + d_i^{-}(A_i, A^{-})} \quad (10)$$

where, the smaller the value K_i is, the better the solution is.

RESULTS AND DISCUSSION

Ren and Gao (2010), for MCDM problem with criteria weight information incomplete and with criterion value as normally distributed random variables, proposed a stochastic MCDM based on interval arithmetic. Zhou *et al.* (2015) defined possibility degree and distance formula of extended grey number, studied uncertain MCDM problem with solution criterion value as extended grey number and proposed a multiple criteria decision-making approach with uncertain probability based on Hurwicz. The study results prove feasibility and effectiveness of this approach, from the computational analysis step and process, compared to method used in the reference literature (Krohling and de Souza, 2012; Wang *et al.*, 2013; Ren and Gao, 2010), the study proposed approach can better meet practical needs, more in line with actual situation of MCDM problem and with stronger operability.

Illustrative example: The decision maker chooses information management systems providers from four optional companies (A_1, A_2, A_3 and A_4). The decision maker evaluates each company from the four criteria: B_1 is system reliability and adaptability, B_2 is system flexibility, B_3 is control ability, B_4 is equipment cost. Under criterion B_1, B_2, B_3, solution corresponds to three different natural states. Natural state probability p = (0.3, 0.4 and 0.3), while B_4 will not vary with state change. Each criterion weight vector given by decision makers is w = (0.1, 0.3, 0.4 and 0.2) (Zhou *et al.*, 2015). In each state, evaluation information is given in the form of extended grey number random variable and its decision-making data is shown in Table 2-4. Determine best information system provider to be chosen by the decision maker.

Table 2: Decision matrix R^1 at good state

	B_1	B_2	B_3	B_4
A_1	[0.5, 1.0]∪[1.2, 1.5]	[6.5, 7.0]	[5.5, 7.0]	[8.5, 9.5]
A_2	[1.5, 2.0]	[7.5, 8.5]	[5.5, 6.0]∪{6.5}	[6.5, 7.5]
A_3	[2.5, 2.7]∪{3.0}	[3.5,4.0]∪{4.5}	[7.5, 8.5]∪{9.0}	[7.5, 8.5]
A_4	[1.5, 2.0]	[4.5, 5.0]	[9.5, 10.0]	[5.5, 6.5]

Table 3: Decision matrix R^2 at moderate state

	B_1	B_2	B_3	B_4
A_1	[1.5,2.0]	[7.5,8.0]∪[8.5,9.0]	[4.5,5.0] [6.5,7.5]	[8.5,9.5]
A_2	[2.5,3.0]∪[3.5,4.0]	[6.0,8.5]∪{9.0}	[6.0,7.5]	[6.5,7.5]
A_3	[0.5,1.0]	[2.5,3.5]	[9.5,10.5]	[7.5,8.5]
A_4	[1.0,2.0]	[5.5,6.0]	[8.5,9.0]	[5.5,6.5]

Table 4: Decision matrix R^3 at poor state

	B_1	B_2	B_3	B_4
A_1	[0.5,1.5]	[8.5,9.0]	[6.5,8.0]	[8.5,9.5]
A_2	{3.5}	[8.5,9.0]	[7.5,8.0]	[6.5,7.5]
A_3	[3.5,4.0]	[3.5,4.0]	[9.0,10.0]	[7.5,8.5]
A_4	{3.0}∪[3.5,4.0]	[7.5,8.0]∪{9.0}	[9.5,10.0]∪{10.5}	[5.5,6.5]

Table 5: Normalized decision matrix G^1 at good state

	B_1	B_2	B_3	B_4
A_1	[0.167, 0.333]∪[0.400, 0.500]	[0.765, 0.824]	[0.550, 0.700]	[0.579, 0.647]
A_2	[0.500, 0.667]	[0.882, 1]	[0.550, 0.600]∪{0.650}	[0.733, 0.846]
A_3	[0.833, 0.900]∪{1}	[0.412, 0.471]∪{0.529}	[0.750, 0.850]∪{0.900}	[0.647, 0.733]
A_4	[0.500, 0.667]	[0.529, 0.588]	[0.950, 1]	[0.846, 1]

Table 6: Expectation decision matrix G

	B_1	B_2	B_3	B_4
A_1	[0.2375,0.4625]	[0.8461, 0.9471]	[0.5221, 0.7243]	[0.5789, 0.6471]
A_2	[0.6625, 0.8625]	[0.8147, 1.0000]	[0.6079, 0.7093]	[0.7333, 0.8462]
A_3	[0.5625, 0.7000]	[0.3513, 0.4477]	[0.8440, 0.9557]	[0.6471, 0.7333]
A_4	[0.4750, 0.7000]	[0.6533, 0.7431]	[0.8802, 0.9429]	[0.8462, 1.0000]

- In the above criteria, system reliability and adaptability, flexibility and control ability belong to efficiency criteria, cost of equipment belongs to cost criterion. According to formula 3 and 4, normalize decision matrix R^1 and obtain normalized decision matrix G^1, as shown in Table 5

Similarly, for normalized decision matrix G^2, G^3 at moderate or poor constractible state, due to limited space, its operation process will not be repeated.

- According to natural state probability p = (0.3, 0.4 and 0.3) and operation rule of extended grey number, calculate expectation with formula 5 and obtain expectation decision matrix $G = \{\otimes r_{ij}\}_{3\times 3}$, the result of which is shown in Table 6
- According to formula 6 and 7, calculate the positive ideal solution and negative ideal solution

A^+ = ([0.6625, 0.8625], [0.8461, 1.0000], [0.8802, 0.9557], [0.5789, 0.6471])

A^- = ([0.2375, 0.4625], [0.3513, 0.4477], [0.5221, 0.7093], [0.8462, 1.0000])

- According to the formula 8 and 9, calculate the distance between various solutions and positive, negative ideal solution, respectively, knowing that each criterion weight vector w = (0.1, 0.3, 0.4 and 0.2)

The distance between A_i and A^+ is:

$$d_1^+(A_1, A^+) = 0.2448,\ d_2^+(A_2, A^+) = 0.2067$$

$$d_3^+(A_3, A^+) = 0.2780,\ d_4^+(A_4, A^+) = 0.2148$$

The distance between A_i and A^- is:

$$d_1^-(A_1, A^-) = 0.3054,\ d_2^-(A_2, A^-) = 0.3471$$

$$d_3^-(A_3, A^-) = 0.2690,\ d_4^-(A_4, A^-) = 0.3313$$

- Calculate relative closeness degree based on formula 10

$$K_1 = 0.4449,\ K_2 = 0.3733,\ K_3 = 0.5082,\ K_4 = 0.3934$$

Thereby, obtain $K_2<K_4<K_1<K_3$, so sorting result of various solutions is as follows: $A_2>A_4>A_1>A_3$. Therefore, it can be known that best provider of information management system is A_2. The result is basically consistent with conclusion of literature (Zhou *et al*., 2015), which proves feasibility and effectiveness of this study, from the computational analysis step and process, it can be seen that, compared to method used in the reference literature, the proposed approach can better meet practical needs, more in line with actual situation of MCDM problem and with stronger operability.

CONCLUSION

For grey stochastic MCDM problem with criterion value as extended grey number, the study provides Hausdorff distance formula of extended grey number, proposes grey stochastic MCDM study based on Hausdorff distance, discusses in detail its implementation steps and verifies feasibility and rationality of the proposed approach with sample calculation analysis. The decision-making approach proposed in this study is very effective for dealing with decision-making problem with both extended grey number and randomness. The solution sorting process takes full account of natural state probability corresponding to various criteria and enhances scientific and rationality of the study. The decision-making study has good value in application promotion and actual decision-making and can be widely applied in the fields of project evaluation, supply chain management and investment decision.

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REFERENCES

Boran, F.E., 2011. An integrated intuitionistic fuzzy multi criteria decision making method for facility location selection. Math. Comput. Applic., 16: 487-496.

Dalalah, D., M. Hayajneh and F. Batieha, 2011. A fuzzy multi-criteria decision making model for supplier selection. Expert Syst. Applic., 38: 8384-8391.

Krohling, R.A. and T.T.M. de Souza, 2012. Combining prospect theory and fuzzy numbers to multi-criteria decision making. Expert Syst. Applic., 39: 11487-11493.

Li, G.D., D. Yamaguchi and M. Nagai, 2007. A grey-based decision-making approach to the supplier selection problem. Mathe. Comput. Modell., 46: 573-581.

Lin, Y.H., P.C. Lee and H.I. Ting, 2008. Dynamic multi-attribute decision making model with grey number evaluations. Expert Syst. Applic., 35: 1638-1644.

Liu, S.F., T.B. Guo and Y.G. Dang, 1999. Grey System Theory and Its Applications. Science Press, Beijing, China.

Luo, D., L. Zhou and D.X. Luo, 2008. Grey multi-attribute risk group decision-making method. Syst. Eng. Electron., 30: 1674-1678.

Marques, G., D. Gourc and M. Lauras, 2011. Multi-criteria performance analysis for decision making in project management. Int. J. Project Manage., 29: 1057-1069.

Mousavi, S.M., F. Jolai, R. Tavakkoli-Moghaddam and B. Vahdani, 2013. A fuzzy grey model based on the compromise ranking for multi-criteria group decision making problems in manufacturing systems. J. Intell. Fuzzy Syst., 24: 819-827.

Nayagam, V.L.G., S. Muralikrishnan and G. Sivaraman, 2011. Multi-criteria decision-making method based on interval-valued intuitionistic fuzzy sets. Expert Syst. Applic., 38: 1464-1467.

Ren, J. and Y. Gao, 2010. Stochastic multi-criterion decision-making method based on interval operation. Syst. Eng. Electron., 32: 308-312.

Wang, J.Q., H.Y. Zhang and S.C. Ren, 2013. Grey stochastic multi-criteria decision-making approach based on expected probability degree. Scientia Iranica, 20: 873-878.

Wang, J.Q. and D.D. Wang, 2014. Stochastic multi-criteria decision-making method based on Hausdorff distance of extended grey numbers. Control Decis., 29: 1823-1827.

Yalcin, N., A. Bayrakdaroglu and C. Kahraman, 2012. Application of fuzzy multi-criteria decision making methods for financial performance evaluation of Turkish manufacturing industries. Expert Syst. Applic., 39: 350-364.

Zhou, H., J.Q. Wang and D.D. Wang, 2015. Grey stochastic multi-criteria decision-making approach based on Hurwicz with uncertain probability. Control Decis., 30: 556-560.