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Research Article
Constructions of Ternary Z-complementary Sequences

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Abstract
Sequences over the alphabet {-1,0,1} are called ternary sequences. Including the conventional ternary complementary sequences as special cases, the aperiodic ternary Z-complementary sequences are brought forward and may be used as an alternative of ternary complementary sequences in many engineering applications. The elementary transformations on ternary sequences and elementary operations on ternary Z-complementary sets are proposed. It is shown that aperiodic ternary Z-complementary pairs are better than aperiodic ternary complementary ones of the same length in terms of the number of them. In the end, constructions of ternary Z-complementary sets and their mates are given.

Key words: Aperiodic autocorrelation functions, mates, zero correlation zone, ternary sequences

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INTRODUCTION

An aperiodic ternary sequence set consists of k ternary sequences whose out-of-phase aperiodic autocorrelations sum to zero. When k = 2, the aperiodic ternary complementary set is called an aperiodic ternary complementary pair. Aperiodic ternary complementary pairs include binary complementary ones as special cases, which are called Golay complementary pairs, were originally considered by Golay in connection with his study of infrared spectrometry (Golay, 1961). Tseng and Liu (1972) investigated aperiodic binary complementary sets of k sequences with k>2. Sivaswamy (1978) studied multiphase complementary sets. Gavish and Lempel (1994) introduced ternary complementary Sequences. Complementary sequences were subsequently used in radar, synchronization, channel estimation and so on (Yuan *et al*., 2008; Spasojevic and Georghiades, 2001). In particular, modern application of Golay complementary pairs is in multicarrier communications, which have recently attracted much attention in wireless applications. Orthogonal frequency-division multiplexing is a method of transmitting data simultaneously over multiple equally-spaced carrier frequencies, using Fourier transform processing for modulation and demodulation. However, the lengths of the known binary complementary pair are $2^a10^b26^c$ for all non-negative integers a, b and c, which are very limited. Quadriphase complementary pair of length 7, 9, 15 and 17 do not exist. Aperiodic binary Z-complementary sequences are presented firstly (Fan *et al*., 2007). A Z-complementary set is defined as a sequence set if the sum of out-of-phase aperiodic autocorrelation functions with a certain region around the in-phase position is zero. Such a region is called Zero Correlation Zone (ZCZ). When the width of ZCZ is equal to the sequence length, such a Z-complementary, set is reduced to a complementary set. Aperiodic quadriphase Z-complementary sequences are studied later (Li *et al*., 2010). Li *et al*. (2011) studied the existence problem of binary Z-complementary pairs with ZCZ widths 2, 3, 4, 5 and 6. An upper bound on zero correlation zones of odd-length binary Z-complementary pairs is proposed and a new construction of binary Z-complementary pairs is presented. Some constructions of Z-complementary sequences are given by Li *et al*. (2014) and Liu *et al*. (2014). In this study, however, interest focuses on the ternary Z-complementary sequences.

MATERIALS AND METHODS

The ternary sequence consists of 1, -1 and 0. Let, $a = (a_0, a_1, ..., a_{N-1})$ and $b = (b_0, b_1, ..., b_{N-1})$ be ternary sequences of length N. The aperiodic correlation function of a and b is given as:

$$C_{a,b}(\tau) = \begin{cases} \sum_{n=0}^{N-1-\tau} a_n b_{n+\tau}, & 0 \le \tau \le N-1 \\ \sum_{n=0}^{N-1+\tau} a_{n-\tau} b_n, & 1-N \le \tau < 0 \\ 0, & |\tau| \ge N \end{cases} \quad (1)$$

generally, the aperiodic function of ternary sequences in the range $0 \le \tau \le N-1$, i.e., in Eq. 2:

$$C_{a,b}(\tau) = \sum_{n=0}^{N-\tau-1} a_n b_{n+\tau}, \quad 0 \le \tau \le N-1 \quad (2)$$

When, a = b is considered, the above equation of the aperiodic correlation function of ternary sequences becomes aperiodic autocorrelation function (AACF), which is denoted by $C_a(\tau)$. When, $a \ne b$, the above equation of the aperiodic correlation function of ternary sequences becomes aperiodic cross-correlation function (ACCF).

A ternary sequences set $\{a_i, 1 \le i \le P\}$ of length N is called ternary Z-complementary sets (TZCS), which is denoted by $TZCS^{Z}_{P,N}$, if:

$$\sum_{i=1}^{P} C_{a_i}(\tau) = \begin{cases} NP, & \tau = 0 \\ 0, & 0 < \tau < Z \end{cases} \quad (3)$$

where, Z is the length of ZCZ. Above equation of TZCS includes the conventional ternary complementary set as a special case when Z = N. The TZCS becomes a ternary Z-complementary pair (TZCP) when, P = 2.

A ternary Z-complementary set $\{a_1, a_2, ..., a_p\}$ is called a ternary Z-complementary mate (mutually uncorrelated ternary Z-complementary set) of another ternary Z-complementary set $\{b_1, b_2, ..., b_p\}$, if:

$$\sum_{i=1}^{P} C_{a_i b_i}(\tau) = 0, \ 0 \le \tau < Z \quad (4)$$

When, Z = N, the ternary Z-complementary mate becomes the conventional ternary complementary mate. A class of M ternary Z-complementary mates, each set with P ternary sequences of length N and zero relation zones Z, is denoted by $TZCS^{M,Z}_{P,N}$, where, M is an integer larger than one.

Elementary operations of TZCP: Transformations of individual four-level sequence were given by Li and Hao (2010). Similarly, individual ternary sequence transformations are defined in Lemma 1 to provide the basis for the study of TZCS.

Lemma 1: Let $a = (a_0, a_1, ..., a_{N-1})$ be a ternary sequence of length N. The following transformations of individual ternary sequence preserve the aperiodic auto-correlation function:

(i) $a_n \rightarrow a_{N-n-1}$ (n = 0, 1, 2, ..., N-1), which is denoted by $\bar{a}$
(ii) $a_n \rightarrow ca_n$ (n = 0, 1, 2, ..., N-1, $c \in \{\pm 1\}$), which is denoted by ca

Above transformations of individual ternary sequence was defined as elementary transformations on ternary sequences based on aperiodic autocorrelation function. When a ternary sequence can be obtained from another ternary sequence via the successive application of the elementary transformations, the two ternary sequences are proved to be equivalent based on aperiodic autocorrelation function.

Theorem 1: Let a_1, a_2 be a Z-complementary pair of ternary sequences. The following operations also yield ternary Z-complementary pairs:

(i) Interchange, namely, $(a_1, a_2) \rightarrow (a_2, a_1)$ (ii) Reverse, namely, $(a_1, a_2) \rightarrow (\bar{a}_1, a_2)$
(ii) Scalar multiply, namely, $(a_1, a_2) \rightarrow (ca_1, a_2)$, while c generally equals 1 or -1
(iii) Negating elements with odd index in a_1 and a_2, denoted by $(\hat{a}_1, \hat{a}_2)$, namely, $(a_1, a_2) \rightarrow (\hat{a}_1, \hat{a}_2)$

The above operations are called to be elementary operations on ternary Z-complementary pair. It is clear that the above elementary operations on ternary Z-complementary pair keep ternary Z-complementary property, because all sequences in the set of ternary sequences denoted by $\{a_1, \bar{a}_1, -a_1, -\bar{a}_1\}$ aperiodic auto-correlation function, so do those ternary sequences in the set $\{a_2, \bar{a}_2, -a_2, -\bar{a}_2\}$ and $C_{\bar{a}1}+C_{\bar{a}2}=0$ is deduced from $C_{a1}+C_{a2}=0$.

When a ternary Z-complementary pair can be obtained from another ternary Z-complementary pair by finite applications of elementary operations on the ternary Z-complementary pair, the two ternary Z-complementary pairs are said to be equivalent. All ternary Z-complementary pairs which are equivalent form an equivalent class. Any element chosen from the equivalence class is defined as a generator of the equivalence class and we also call it a generator of ternary Z-complementary pairs hereafter. So, a set of nonequivalent generators describes the set of ternary Z-complementary pairs with same fixed length. Similarly, the elementary operations on ternary Z-complementary pair can be generalized to ternary Z-complementary sets.

Based on computer search, Table 1 presents a generator of ternary Z-complementary pairs for length $N \le 10$ and in each case, its Z is N-1. Ternary Z-complementary pairs with ternary complementary ones are compared in Table 2.

As with ternary Z-complementary pairs, other TZCS can be obtained from a TZCS by elementary operations on TZCS.

Constructions of TZCS: Three constructions of quadriphase Z-complementary sets (Li *et al.*, 2010) similarly are applicable to TZCS. Besides, three recursive methods of constructing TZCS are presented and they are obtained by modifying or improving the original methods of quadriphase Z-complementary sets (Li *et al.*, 2010). Different from those conventional construction methods using quadriphase complementary sets as a basic starter, the proposed ones are based on TZCS. The constructed TZCS preserve the original ternary Z-complementary properties and have even better ternary Z-complementary properties.

Construction 1 of TZCS: Let, $TZCS^{Z}_{2P,N} = \{A_i, 1 \le i \le 2P\}$ be a TZCS with 2P ternary sequences, each of length N. Then a new TZCS with 2P ternary sequences of length 2N, which is denoted by:

$$TZCS^{Z}_{2P,2N} = \{B_i, 1 \le i \le 2P\}$$

is obtained by Eq. 5 and 6 below:

Table 1: Generators of ternary Z-complementry pairs

N	Z	Examples of generators	Summed AACF
3	2	(1, 1, -1; 1, 0, -1)	(5, 0, -2)
4	3	(1, 1, 1, -1; 1, -1, 0, 0)	(6, 0, 0, -1)
5	4	(1, 1, 1, -1, 1; 1, 1, 0, -1, 1)	(9, 0, 0, 0, 2)
6	5	(1, 1, 0, -1, 1, -1; 1, 0, 1, 1, 0, -1)	(9, 0, 0, 0, 0, -2)
7	6	(1, 1, 1, 1, -1, 1, -1; 1, 1, -1, -1, 1, 0, 0)	(12, 0, 0, 0, 0, 0, -1)
8	7	(1, 1, 1, 1, -1, 0, -1, 1; 1, 1, 0, -1, 1, 1, -1, 1)	(14, 0, 0, 0, 0, 0, 0, 2)
9	8	(1, 1, 1, 1, 1, -1, -1, 0, 1; 1, -1, 1, -1, 1, 1, -1, 0, 1)	(18, 0, 0, 0, 0, 0, 0, 0, 2)
10	9	(1, 1, 1, 0, 0, 1, -1, -1, 1, 0; 1, 1, -1, -1, 0, -1, 1, -1, 0, -1)	(15, 0, 0, 0, 0, 0, 0, 0, 0, -1)

Table 2: Comparing ternary Z-complementary pairs with complementary ones

N	No. of ternary complementary pairs	No. of ternary Z-complementary pairs with Z = N-1
3	49	125
4	140	258
5	335	787
6	762	1184
7	1613	2889
8	3256	4318
9	6283	9023
10	11526	12668

$$B_{2k-1} = A_{2k-1}A_{2k} \ (k=1, 2, \cdots, P) \quad (5)$$

$$B_{2k} = A_{2k-1}(-A_{2k}) \ (k=1, 2, \cdots, P) \quad (6)$$

where, $A_{2k-1}A_{2k}$ denotes the concatenation of the two ternary sequences A_{2k-1} and A_{2k}. A TZCS of length N. The 2^r can be formed from a starting TZCS of length N by using the recursive method r times.

Example 1: Let, $\{A_1, A_2, A_3, A_4\}$ be a TZCS

Where:
A_1 = {1, 1, 1, -1, 1}
A_2 = {1, 1, 0, -1, 1}
A_3 = {-1, -1, 0, 1, -1}
A_4 = {-1, 1, -1, -1, -1}

The sum of their aperiodic autocorrelation functions is {18, 0, 0, 0, 4}. Then, a new TZCS $\{B_1, B_2, B_3, B_4\}$ is obtained by using Eq. 5 and 6:

Where:
B_1 = {1, 1, 1, -1, 1, 1, 1, 0, -1, 1}
B_2 = {1, 1, 1, -1, 1, -1, -1, 0, 1, -1}
B_3 = {-1, -1, 0, 1, -1, -1, 1, -1, -1, -1}
B_4 = {-1, -1, 0, 1, -1, 1, -1, 1, 1, 1}

The sum of the new sequence aperiodic autocorrelation functions is {36, 0, 0, 0, 8, 0, 0, 0, 0, 0}. Namely, $TZCS^4_{4,5}$ is formed from $TZCS^4_{4,10}$ by using the recursive method once.

Construction 2 of TZCS: Given to be a ternary Z_1-complementary pair (A_1, A_2) of length N_1, its mate denoted by $(A_1^{'}, A_2^{'})$ and a ternary Z_2-complementary set with 2P ternary sequences of length N_2, which is denoted by $TZCS^{Z^2}_{2P, N2} = \{B_i, 1 \le i \le 2P\}$, a new TZCS with 2P sequences of length $2N_1N_2$ which is denoted by $TZCS^{Z}_{2P, 2N_1N_2} = \{T_i, 1 \le i \le 2P\}$ can be constructed by Eq. 7 and 8:

$$T_{2k-1} = (B_{2k-1} \otimes A_1) \ (B_{2k} \otimes A_1') \ (k=1, 2, \cdots, P) \quad (7)$$

$$T_{2k} = (B_{2k-1} \otimes A_2) \ (B_{2k} \otimes A_2') \ (k=1, 2, \cdots, P) \quad (8)$$

where, $Z = Z_1$ when, $Z_1 < N_1$ or $Z = Z_1Z_2$ when, $Z_1 = N_1$ and $\otimes$ denotes the Kronecker product.

Example 2: Let $\{B_1, B_2, B_3, B_4\}$ be a TZCS:

Where:
B_1 = {1, 1, 1, -1, 1}
B_2 = {1, 1, 0, -1, 1}
B_3 = {-1, -1, 0, 1, -1}
B_4 = {-1, 1, -1, -1, -1}

The sum of their aperiodic autocorrelation functions is {18, 0, 0, 0, 4}. Let (A_1, A_2) a ternary complementary pair, where, A_1 = {1, 1, -1} and A_2 = {1, 0, 1}. Its mate is $(A_1^{'}, A_2^{'})$, where A_1' = {1, 0, 1}, A_2' = {1, -1, -1}. Then new TZCS is obtained $\{T_1, T_2, T_3, T_4\}$ by 7 and 8:

Where:
T_1 = {1, 1, -1, 1, 1, -1, 1, 1, -1, -1, -1, 1, 1, 1, -1,1, 0, 1, 1, 0, 1, 0, 0, 0, -1, 0, -1, 1, 0, 1}
T_2 = {1, 0, 1, 1, 0, 1, 1, 0, 1, -1, 0, -1, 1, 0, 1, 1, -1, -1, 1, -1, -1, 0, 0, 0, -1, 1, 1, 1, -1, -1 }
T_3 = {-1, -1, 1, -1, -1, 1, 0, 0, 0, 1, 1, -1, -1, -1, 1, -1, 0, -1, 1, 0, 1, -1, 0, -1, -1, 0, -1, -1, 0, -1 }
T_4 = {-1, 0, -1, -1, 0, -1, 0, 0, 0, 1, 0, 1, -1, 0, -1, -1, 1, 1, 1, -1, -1, -1, 1, 1, -1, 1, 1, -1, 1, 1 }

The sum of the new sequence aperiodic autocorrelation functions is {90, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 20, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}.

Construction 3 of TZCS: Let $(F_1^0, F_2^0, F_3^0, \cdots, F_{2P}^0)$ be an arbitrary TZCS with 2P ternary sequences, each of length N_0, zero correlation zone Z_0, a new TZCS with 2P ternary sequences, each of length $N_n = 2^nN_0$, zero correlation zone $Z_n = 2^nZ_0$, can be constructed using Eq. 9 and 10:

$$F^n_{2k-1} = F^{n-1}_{2k-1} \lozenge F^{n-1}_{2k}, n=1, 2, \cdots, k=1, 2, \cdots, P \quad (9)$$

$$F^n_{2k} = F^{n-1}_{2k-1} \lozenge (-F^{n-1}_{2k}), n=1, 2, \cdots, k=1, 2, \cdots, P \quad (10)$$

where, $-F^{n-1}_{2k}$ is formed by negating the ternary sequence $-F^{n-1}_{2k}$ and $F^{n-1}_{2k-1} \lozenge F^{n-1}_{2k}$ denotes the bit-interleaved operation between ternary sequences F^{n-1}_{2k-1} and F^{n-1}_{2k}.

Example 3: Let $\{F_1^0, F_2^0, F_3^0, F_4^0\}$ be a TZCS with Z = 6:

Where:
F_1^0 = {1, 1, 1, 1, -1, 1, -1}
F_2^0 = {1, 1, -1, -1, 1, 0, 0}
F_3^0 = {1, 1, 1, 1, -1, 0, 1}
F_4^0 = {1, 0, -1, -1, 1, -1, 1}

The sum of their aperiodic autocorrelation functions is {24, 0, 0, 0, 0, 0, 1}. Then, a new TZCS is obtained with Z = 12, which is denoted by $\{F_1^1, F_2^1, F_3^1, F_4^1\}$ using Eq. 9 and 10:

Where:
F_1^1 = {1, 1, 1, 1, 1, -1, 1, -1, -1, 1, 1, 0, -1, 0}
F_2^1 = {1, -1, 1, -1, 1, 1, 1, 1, -1, -1, 1, 0, -1, 0}
F_3^1 = {1, 1, 1, 0, 1, -1, 1, -1, -1, 1, 0, -1, 1, 1}
F_4^1 = {1, -1, 1, 0, 1, 1, 1, 1, -1, -1, 0, 1, 1, -1}

The sum of the new sequence autocorrelation functions is {48, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0}.

Constructions of TZCS Mates: Three recursive constructing methods of quadriphase Z-complementary mates similarly are applicable to TZCS mates (Li *et al*., 2010). Three recursive methods of constructing TZCS mates are given in this section and they are obtained by modifying the original methods of quadriphase Z-complementary mates (Li *et al*., 2010).

Let $TZCS_{P,N}^{M,Z} = \{(A_i^1, A_i^2, \cdots, A_i^M)\}$ be a class of M ternary Z-complementary mates, each set with p ternary sequences of the same length N and zero relation zones Z, denoted by Eq. 11 the matrix form as follow:

$$TZCS_{P,N}^{M,Z} = \begin{bmatrix} A_1^1 & A_1^2 & & A_1^M \\ A_2^1 & A_2^2 & & A_2^M \\ \cdots & \cdots & \cdots & \cdots \\ A_P^1 & A_P^2 & & A_P^M \end{bmatrix} \quad (11)$$

Construction 1 of TZCS mates: Similar to the theorem 13 (Tseng and Liu, 1972), the following construction of ternary Z-complementary mates was obtained immediately. Let $TZCS_{P,N}^{M,Z}$ be a matrix of ternary sequences whose columns are ternary Z-complementary mates, then Eq. 12:

$$TZCS_{2P,2N}^{2M,Z} = \begin{bmatrix} (TZCS_{P,N}^{M,Z})\,(TZCS_{P,N}^{M,Z}) & (-TZCS_{P,N}^{M,Z})\,(TZCS_{P,N}^{M,Z}) \\ (-TZCS_{P,N}^{M,Z})\,(TZCS_{P,N}^{M,Z}) & (TZCS_{P,N}^{M,Z})\,(TZCS_{P,N}^{M,Z}) \end{bmatrix} \quad (12)$$

is also a matrix of ternary sequences, whose columns are new ternary Z-complementary mates, where, $(-TZCS_{P,N}^{M,Z})(TZCS_{P,N}^{M,Z})$ denotes the matrix whose mp-th entry is the concatenation of the mp-th entry of $-TZCS_{P,N}^{M,Z}$ and the mp-th entry of $TZCS_{P,N}^{M,Z}$. In general, $TZCS_{2^rP,2^rN}^{2^rM,Z}$ can be constructed by repeated applications of the Eq. 12, r times from $TZCS_{P,N}^{M,Z}$.

Construction 2 of TZCS mates: Given a set of ternary Z-complementary mates $TZCS_{2P,N}^{M,Z}$, a new set of ternary Z-complementary mates $TZCS_{2P,2^rN}^{M,2^rZ}$ was obtained by employing Eq. 9 and 10, r times.

Construction 3 of TZCS mates: Similar to the theorem 12 (Tseng and Liu, 1972), the following construction of ternary Z-complementary mates was obtained immediately. Let $TZCS_{P,N}^{M,Z}$ be a matrix of ternary sequences whose columns are mates, then Eq. 13:

$$TZCS_{2P,2N}^{2M,2Z} = \begin{bmatrix} (TZCS_{P,N}^{M,Z})\Diamond(TZCS_{P,N}^{M,Z}) & (-TZCS_{P,N}^{M,Z})\Diamond(TZCS_{P,N}^{M,Z}) \\ (-TZCS_{P,N}^{M,Z})\Diamond(TZCS_{P,N}^{M,Z}) & (TZCS_{P,N}^{M,Z})\Diamond(TZCS_{P,N}^{M,Z}) \end{bmatrix} \quad (13)$$

is also a matrix of ternary sequences whose columns are mates where, $(-TZCS_{P,N}^{M,Z})(TZCS_{P,N}^{M,Z})$ denotes the matrix whose mp-th entry is the interleaved sequence of the mp-th entry of $-TZCS_{P,N}^{M,Z}$ and the mp-th entry of $TZCS_{P,N}^{M,Z}$. In general, from $TZCS_{P,N}^{M,Z}$, $TZCS_{2^rP,2^rN}^{2^rM,2^rZ}$ can be constructed by using the Eq. 13, r times.

RESULTS AND DISCUSSION

Aperiodic ternary Z-complementary pairs are better than aperiodic ternary complementary ones of the same length in terms of the number of them. Constructing methods of ternary Z-complementary sets and their mates are given. Compared with Table 1 (Fan *et al*., 2007), Table 1 in this study, indicates that the maximum zero correlation zone of ternary Z-complementary pairs is generally much longer than that of binary Z-complementary ones. For the fixed length, the maximum zero correlation zone of ternary Z-complementary pairs is the same as that of quadriphase Z-complementary pairs (Li *et al*., 2010). However, the maximum zero correlation zone of ternary Z-complementary pairs of length 6 is longer than that of four-level Z-complementary ones (Li *et al*., 2010). Constructions of TZCS and their mates are obtained by modifying or improving the original methods (9, 10, 11, 15). Besides, other synthesizing methods of TZCS and their mates are presented here.

CONCLUSION

Aperiodic ternary Z-complementary sequences are investigated in this study. The notions of elementary transformations on ternary sequences and elementary operations on TZCS are put forward. The results show that aperiodic ternary Z-complementary pairs are better than aperiodic ternary complementary ones of the same length in terms of the number of them. Constructing methods of ternary Z-complementary sets and their mates are presented here. However, searching the set of nonequivalent generators of ternary Z-complementary pairs of longer length remains an open problem.

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