Influence of Rayleigh Number in Turbulent and Laminar Region in Parallel-Plate Vertical Channels

1Hayder A. Abdul Bari and 2Rana Thabet Abd Al-Rubai
1Faculty of Chemical and Natural Resources Engineering, University Malaysia Pahang, Kuantan, Pahang, Malaysia
2Department of Chemical Engineering, College of Engineering, University of Baghdad, Iraq

Abstract: The main objectives of this research is to study the influence of Rayleigh number in turbulent and laminar region in parallel-plate vertical plate so we will comparison the different between laminar and turbulence in Rayleigh number and validate it with existing experimental data available in literature. The channel was formed by two vertical parallel plates. Velocity measurements were carried out for natural convection flow in a symmetrically heated vertical channel. The plates taken were 12.5 mm thick the modified Rayleigh number (Ram) range 50 to $1 \times 10^7$ has been covered for symmetrically heated isothermal vertical surfaces. The aspect ratio of the channel was kept constant ($L/b = 12$) and width 200 mm. One plate was kept at fixed temperature above and the other at fixed temperature below the ambient temperature and local velocity and temperature profile at three different sections of channel were reported along with the local heat transfer coefficients and temperature distribution on the heated wall.

Keywords: Vertical parallel plate channel, Rayleigh number, velocity profile, natural convection flow

INTRODUCTION

In a smooth vertical parallel plate channel that are open to the ambient at top and bottom ends, natural convection occurs when at least one of the two plates forming the channel is heated or cooled. The resulting buoyancy-drive flow can be laminar or turbulent depending on the channel geometry, fluid properties and temperature difference between the plates and ambient. The Rayleigh number at which flow becomes turbulent in vertical channels is different from that of flow over a vertical flat plate. Surface thermal conditions may be idealized as being isothermal or isoflux and symmetrical or asymmetrical. For small aspect ratio (length to inter-plate spacing), independent boundary layer develops at each surface and a condition similar to that of a vertical plate in an infinite quiescent medium takes place. For large aspect ratio, however, boundary layers developed on opposing surfaces eventually merge to yield a fully developed condition. Due to modern application of cooling of electronic equipments such as printed circuit boards, there has been resurgence of interest in studying natural convection in vertical channels. Understanding the flow pattern in this equipment may significantly improve their design and consequently their operational performance (Incropera and DeWitt, 1996).

Corresponding Author: Hayder A. Abdul Bari, Faculty of Chemical and Natural Resources Engineering, University Malaysia Pahang, Kuantan, 25000, Pahang, Malaysia
Tel: 0600123456789 Fax: 0600987654321

265
During the last two decades, a number of studies involving experimental measurements and numerical solution of heat transfer in laminar free convection flows between two vertical plates were reported. Sparrow and Azavedo (1985) conducted experimental and numerical studies on the effect of inter-plate spacing on natural convection heat transfer characteristics of a one-sided heated vertical channel.

The first numerical solution for developing natural convection flow in an isothermal channel was carried out by Bodia and Osterle (1962) using the boundary-layer approximation. The authors assumed uniform velocity and temperature profiles and ambient pressure at the channel inlet while assuming fully-developed flow at the channel exit. The solution methodology developed by Bodia and Osterle (1962) as widely used to solve free convective channel flow problems for various boundary conditions as in the studies by Fedorov and Viskanta (1997), Badr et al. (2006), Miyamoto et al. (1986), Bessaih and Kadja (2000), Staatman et al. (1994), Omur and Aktas (1998), Baskaya et al. (1999) and Bianco et al. (2000).

Badr et al. (2006) discussed the problem of buoyancy driven turbulent flow in parallel-plate channels. Their investigation was limited to vertical channels of uniform cross-section with different modes of heating. The details of the flow and thermal fields are obtained from the solution of the conservation equations of mass, momentum and energy in addition to equations of the low Reynolds number turbulence model. Their study covers Rayleigh number ranging from 105 to 107 and focuses on the effect of channel geometry on the characteristic of the flow and thermal fields as well as the local and average Nusselt number variation. A Nusselt number correlation has been developed in terms of a modified Rayleigh number and channel aspect ratio for the cases of symmetrically heated isothermal and isoflux conditions.

Naylor et al. (1991) conducted a numerical study on developing free convection flow between isothermal vertical plates with aspect ratios between 10 and 24. The Navier-Stokes and energy equations were solved numerically assuming a special inletflow boundary conditions in the range of Grashof number $50 < Gr < 5 \times 10^5$.

On the other hand, Contrary to the large volume of research published on the problem of laminar natural convection flow in vertical channels, only few investigations were reported on the turbulent flow case.

The main objectives of this research is to study the influence of Rayleigh number in turbulent and laminar region in parallel-plate vertical plate so we will comparison the different between laminar and turbulence in Rayleigh number and validate it with existing experimental data available in literature.

**MATERIALS AND METHODS**

The present study considers steady-state, turbulent, incompressible, two-dimensional natural convection flow of air in the vertical parallel-plate channel. All the thermo physical properties are assumed to be constants, except for the density in the buoyancy force term which can be adequately modeled by Boussinesq approximation (Jaltun, 1980).

The channel was formed by two vertical parallel plates. Velocity measurements were carried out for natural convection flow in a symmetrically heated vertical channel. The plates taken were 12.5 mm thick the modified Rayleigh number (Ran) range 50 to $1 \times 10^7$ has been covered for symmetrically heated isothermal vertical surfaces. The aspect ratio of the channel was kept constant ($L/b = 12$) and width 200 mm. One plate was kept at fixed temperature above and the other at fixed temperature below the ambient temperature and local velocity and temperature profile at three different sections of channel were reported along with the local heat transfer coefficients and temperature distribution on the heated wall.
RESULTS AND DISCUSSION

The heat transfer and fluid flow characteristics of natural convection in a Vertical Parallel-plate channel are studied for both laminar and turbulent regimes.

Influence of Rayleigh Number on the Laminar Flow Regimes

In the present investigation, the modified Rayleigh number (Ram) range 50 to $3 \times 10^4$ has been covered for symmetrically heated isothermal vertical surfaces. The aspect ratio of the channel was kept constant ($L/b = 12$). Figure 1 shows the variation of the average Nusselt number versus the modified Rayleigh for vertical parallel-plate channel. It can be noted that the average Nusselt number increases with the increase of Rayleigh number and that makes the present results closer to the experimental measurements carried out by Wirtz and Haag (1985) as indicated.

To study the influence of Rayleigh number on the flow field and heat transfer characteristics, the vertical velocity and temperature profiles as well as the isotherms are presented for four different Rayleigh numbers.

The variation of the mean vertical velocity across the channel at different channel cross-sections ($y/L = 0, 0.5, 1.0$) are plotted for a specific value of the modified Rayleigh number (Ram) and shown in Fig. 2 to 5. The vertical velocity profile in Fig. 2 shows fully developed flow at the channel exit section when Ram = 80. However, as Rayleigh number increases, two velocity peaks occur at the same section as shown in Fig. 3 to 5. These vertical velocity peaks become sharp and move towards the channel walls with the increase of the modified Rayleigh number (Ram).

The vertical velocity distributions across the channel are plotted in Fig. 6 at section $y/L = 0.5$ and in Fig. 7 at section $y/L = 1.0$ for different values of Ram. It can be noted that the dimensional vertical velocity increases as the Rayleigh number increases as expected. Accordingly, one would expect the mass flow rate to increase with the increase of Ram.

Figure 8 to 11 shows the dimensionless temperature distribution across the channel for four different modified Rayleigh numbers (Ram = 80, 3040, 7865 and 15732). These are plotted in Fig. 12 at channel mid-plane ($y/L = 0.5$) and in Fig. 13 at channel exit section ($y/L = 1.0$). It can be noted that the dimensionless temperature decreases in the core region as the Rayleigh number increases.

![Graph](image)

Fig. 1: The variation of average Nusselt number versus the modified Rayleigh number in laminar regime for vertical parallel-plate channel
Fig. 2: Mean vertical velocity distribution at different cross-section in the channel for $Ra_0(b/L) = 80$

Fig. 3: Mean vertical velocity distribution at different cross-section in the channel for $Ra_0(b/L) = 3940$

Figure 14 to 17 show the isotherms (°) for modified Rayleigh numbers 80, 3040, 7865 and 15732. These plots show that at low Rayleigh number the flow in Fig. 14 and 15 gets heated in the core region from inlet section of channel while at moderate Rayleigh numbers (Fig. 16, 17), the flow is heated in a layer close to the walls. It can be seen that the heated layer thickness decreases as the modified Rayleigh number increases.
Fig. 4: Mean vertical velocity distribution at different cross-section in the channel for \(Ra_0(b/L) = 7885\)

Fig. 5: Mean vertical velocity distribution at different cross-section in the channel for \(Ra_0(b/L) = 15732\) for \(Ra_0(b/L) = 3040\)

Figure 18 shows the variation of local Nusselt number along the channel for 4 different Rayleigh numbers. It can be seen that as the modified Rayleigh number increases, the local Nusselt number also increases over the entire length of the channel. This is expected since the increase in Rayleigh number results in an increase of the flow velocity which tends to a reduction in the thermal layer thickness. Comparison of present calculation with the experimental local Nusselt number distribution measured in air by Wirtz
Fig. 6: Vertical velocity distribution at cross section \( y/L = 0.5 \) for four different modified Rayleigh numbers in laminar regime

and Haag (1985) and the numerical results by Naylor et al. (1991) is shown in Fig. 18. The present calculations of the local Nusselt number are closer to the experimental values than the numerical values obtained by Naylor et al. (1991). The experiments by Wirtz and Haag (1985) were performed in air as the working medium with channel aspect ratio, \( L/b = 26.25 \). The numerical study by Naylor et al. (1991) was done with the channel aspect ratio, \( L/b = 24 \). To verify the accuracy of the present
Fig. 8: Dimensionless temperature distribution at different cross-section in the channel for $\text{Ra}_b(b/L) = 80$

Fig. 9: Dimensionless temperature distribution at different cross-section in the channel for $\text{Ra}_b(b/L) = 3040$

Experimental work, the work are carried out for a channel with aspect ratio, $L/b = 12$. The results of the present study show a better agreement with the experimental results of Wirtz and Haag (1985) than the numerical results of Naylor et al. (1991). The difference with the experimental results might be due to the difference in the channel aspect ratio.
Fig. 10: Dimensionless temperature distribution at different cross-section in the channel for $Ra_b(b/L) = 7865$

Fig. 11: Dimensionless temperature distribution at different cross-section in the channel for $Ra_b(b/L) = 15732$

**Influence of Rayleigh Number on the Turbulent Flow Regime**

This study covers the range of modified Rayleigh number from $1 \times 10^5$ to $1 \times 10^7$ for symmetrically heated isothermal vertical surfaces. The aspect ratio of the channel was kept constant ($L/b = 12$). Figure 19 shows the variation of the average Nusselt number versus Rayleigh number for vertical parallel-plate channel. It can be noted that the average Nusselt number increases continuously with the increase of Rayleigh number.
Fig. 12: Dimensionless temperature distribution for different modified Rayleigh number in laminar regime at channel cross section $y/L = 0.5$

Fig. 13: Dimensionless temperature distribution for different modified Rayleigh number in laminar regime at channel cross section $y/L = 1.0$

To study the influence of Rayleigh number on the flow field and heat transfer characteristics the vertical velocity and temperature profiles as well as isotherms are presented for 4 different Rayleigh numbers ($1.9 \times 10^5$, $9.04 \times 10^5$, $1.84 \times 10^6$ and $7.06 \times 10^6$). The plots of dimensionless mean vertical velocity versus the dimensionless distance across the channel at different channel cross-sections
Fig. 14: Isotherm for a modified Rayleigh number 80

Fig. 15: Isotherm for a modified Rayleigh number 3040

(y/L = 0, 0.5, 1.0) are plotted for a specific value of the Rayleigh as shown in Fig. 20 to 23. From these figures, it can be seen that the vertical velocity profiles have two velocity peaks in the entire range of Rayleigh number considered. These velocity peaks become sharp and move towards the channel walls with the increase of modified Rayleigh number. The vertical velocity profiles across the channel for four values of Rayleigh are plotted at section y/L = 0.5 and y/L = 1.0 and are shown in Fig. 24 and 25, respectively. It can be seen that for turbulent flows the dimensional vertical velocity increases as the Rayleigh increases. Accordingly, the mass flow rate through the channel increases with increase of Rayleigh as...
Fig. 16: Isotherm for a modified Rayleigh number 7865

Fig. 17: Isotherm for a modified Rayleigh number 15732

expected. Because of buoyant fluid motion arising from heat transfer through the vertical walls, the greatest effect of buoyancy occurs very close to the wall where viscous effects are appreciable and the results are compared with the experimental results of Miyamoto et al. (1986) and show that will exhibit a good agreement.

Figure 26 to 29 show the dimensionless temperature distribution across the channel for four different Rayleigh numbers $1.9 \times 10^5$, $9.0 \times 10^5$, $1.84 \times 10^6$ and $7.06 \times 10^6$ which are all in the turbulent flow regime. At all these Rayleigh numbers the temperature at outlet section is higher than mid-section.
Fig. 18: Local Nusselt number distributions along the channel for different modified Rayleigh numbers

Fig. 19: The variation of the average Nusselt number versus the modified Rayleigh number in turbulent regime for vertical parallel-plate channel

The fluid temperature increases along the flow direction as expected. Figure 30 and 31 show the dimensionless temperature distribution across the channel at section $y/L = 0.5$ and 1.0 for the abovementioned Rayleigh numbers. It can be seen that as the Rayleigh number increases the temperature at both sections decreases.
Fig. 20: Mean vertical velocity distribution at different cross-section in the channel for $Ra_b(L/L) = 1.9 \times 10^5$

Fig. 21: Mean vertical velocity distribution at different cross-section in the channel for $Ra_b(L/L) = 9.04 \times 10^5$

In case when the channel width is considerable, the free convection flow near each plate is expected to be similar to that of free convection from a single flat plate. In a single flat vertical heated surface, fluid at a far distance is stagnant. The fluid at the surface is also stationary due to the no-slip condition. The temperature decreases continuously from surface temperature to ambient temperature.
Fig. 22: Mean vertical velocity distribution at different cross-section in the channel for $Ra_0(b/L) = 1.84 \times 10^6$

Fig. 23: Mean vertical velocity distribution at different cross-section in the channel for $Ra_0(b/L) = 7.06 \times 10^6$

Therefore, the maximum velocity occurs at some distance away from the vertical surface. Similarly, in a vertical channel there is a no-slip condition at both the walls and maximum velocity (or velocity hump) occurs close to the heated wall. The exact location and magnitude of max V to be determined through analysis. However, as the flow proceeds vertically, the flow gets more and more disordered.

278
Fig. 24: Vertical velocity distribution at cross section \( y/L = 0.5 \) for four different modified Rayleigh numbers in laminar regime.

Fig. 25: Vertical velocity distribution at cross section \( y/L = 1.0 \) for four different modified Rayleigh numbers in laminar regime.

and disturbed, eventually becoming turbulent. The flow region between the laminar and the turbulent flow regions is called transition region. The location and spread of which is a function of several variables such as surface temperature, the nature and magnitude of external disturbances.
Fig. 26: Dimensionless temperature distribution at different cross-section in the channel for $Ra_b(b/L) = 1.9 \times 10^5$

Fig. 27: Dimensionless temperature distribution at different cross-section in the channel for $Ra_b(b/L) = 9.04 \times 10^3$

Figure 32 to 35 show isotherms for the above modified Rayleigh numbers. In the considered Rayleigh number range there is no thermally fully developed flow visible. In the turbulent flow regime, as Ram increases the thickness of heated layer decreases. Figure 36 shows the variation of local Nusselt number along the channel for the four different values of Ram. It can be seen that as
Fig. 28: Dimensionless temperature distribution at different cross-section in the channel for $\text{Ra}_b(b/L) = 1.84\times10^6$

Fig. 29: Dimensionless temperature distribution at different cross-section in the channel for $\text{Ra}_b(b/L) = 7.06\times10^6$

Ram increases the local Nusselt number also increases due to decrease of the thermal layer thickness.

In the turbulent flow regime, it can be concluded that the average Nusselt number increases with the increase of the modified Rayleigh number as observed earlier in the laminar flow regime. This
Fig. 30: Dimensionless temperature distribution for different modified Rayleigh number in turbulent regime at channel cross-section $y/L = 0.5$

Fig. 31: Dimensionless temperature distribution for different modified Rayleigh number in turbulent regime at channel cross-section $y/L = 1.0$

is due to increase in flow rate and decrease in the heated layer thickness. It is clear that the difference between $Nu$ obtained and the experimental values reported by La Pica et al. (1993), is very small up to $Ra_i(b/L) = 5 \times 10^5$. However, that difference increased as the modified Rayleigh number increased to $6 \times 10^6$. The maximum difference of $Nu$ in the entire range of the modified Rayleigh number did not exceed 7%.
Fig. 32: Isotherm for a modified Rayleigh number $1.9 \times 10^5$

Fig. 33: Isotherm for a modified Rayleigh number $9.05 \times 10^5$

Fig. 34: Isotherm for a modified Rayleigh number $7.06 \times 10^6$
CONCLUSION

The research described here constitutes a comprehensive study of the turbulent and laminar heat transfer and fluid flow characteristic of flow through buoyancy driven natural convection in vertical parallel plate channel.

Different Rayleigh numbers in the range of 80 to $10^5$ have been used to study the flow of the turbulent and laminar heat transfer. During the investigation, various parameters such as Rayleigh number, channel aspect ratio, temperature difference between walls and ambient and different wall heat flux were systematically varied. The investigation yielded patterns of fluid flow and heat transfer presented in the form of streamlines, isotherms, velocity and temperature profiles, in addition to the local and average Nusselt number distribution.
The conclusions derived from the present study can be summarized as follows:

- Laminar natural convection in vertical parallel-plate channel for uniform pressure inlet condition is in good agreement with experimental results of Witz and Haag (1985).
- For both laminar and turbulent flow regimes, the average Nusselt number increases with increasing modified Rayleigh number.

REFERENCES


