Determination of Coefficient of Discharge for Piano Key Weirs of Varying Geometry

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ABSTRACT
A Piano Key (PK) weir is a type of nonlinear (labyrinth-type) weir developed specifically for free-surface flow control structures with relatively small spillway footprints. There are many geometrical parameters which influence the discharge capacity of the PK weir. The discharge capacity is a functional characteristic of geometrical parameters. To have a better understanding of the effects of PK weir geometry on discharge capacity, three PK weirs of varying geometrical parameters were physically tested in the laboratory. For each design the discharge was measured by keeping constant upstream depth and noting down the downstream depths at various sections for respective upstream depths. From the experimental results, $C_d$ and $Q_d$ values were found to be different for all configurations. The values $C_d$ and $Q_d$ were found to be dependent on various parameters viz., inlet width ($W_i$), outlet width ($W_o$), floor slope and parapet wall. The Weir height ($P$) and the width of the Weir ($w$) were kept constant for all the three Weirs. Also the profile of nappe in the downstream length was plotted for each model for different upstream depths and comparisons were made.

Key words: Piano key weir, spillway, coefficient of discharge ($C_d$), labyrinth-type

INTRODUCTION
The discharge capacity of the dam spillways can be increased by revising the design flood value of existing dam. In recent years, many labyrinth spillway weirs have been constructed to maximize the unit discharge per available width. Taylor (1968), did an extensive investigation of labyrinth spillway and developed a capacity ratio by comparison with sharp-crested linear weirs of same width. The design procedure for estimating the discharge over triangular and trapezoidal labyrinth weirs was proposed by Hay and Taylor (1970). The outcome of their work indicates that trapezoidal layout of labyrinth weir, is optimal. Based on the experimental results of model studies for the Wonorora and Avon weirs in Australia, a family of curves for designing labyrinth weir was established by Darvas (1971). The discharge coefficient values for labyrinth angles between $6^\circ$ and $35^\circ$ for labyrinth spillways was presented with the design procedure by Tullis et al. (1995). The equations developed by Tullis et al. (1995) are valid for a range of widths between $t$ and $2t$, where $t$ is the wall thickness and $H/P<0.9$ with $H$ as the total upstream head and $P$ is the height of the wall. Falvey (2003) proposed a spreadsheet for labyrinth designs using the equation developed by Tullis et al. (1995). Lamperiere et al. (2003) observed that traditional labyrinth spillways require 1 or 2 m$^3$ of reinforced concrete to increase the flow by 1 m$^3$ sec$^{-1}$. The construction cost of Labyrinth
weir is relatively high. More over the primary demerit of labyrinth weir is that it cannot be easily placed on the crest of common concrete gravity dams as it requires a flat area to support the vertical walls and flat bottom of the weirs.

PIANO KEY WEIR (PK WEIR)

A new type of spillway called Piano Key weir (PK weir) was developed to fulfill the spillway requirements. The PK weir is a flood control structure which is most efficient for low heads. The PK weir was originally developed to improve the performance of labyrinth-type weirs installed on smaller footprints. The PK weir has a simple rectangular crest layout with inclined inlet and outlet key floors. The PK weirs can facilitate a significant amount of weir length relative to their footprint size. Lamperiere and Ouamane (2003) described PK weir as an innovative alternative of labyrinth weirs. The special feature of PK weir over traditional labyrinth weir is that the bottom of the keys of PK weir is inclined to enhance the stability of the structure. Based on experiments performed in two types of PK weirs at the University of Briska, Algeria, Lamperiere and Ouamane (2003) presented rough design criteria.

In the hydraulic aspect, the behavior of PK weir is quite different from the traditional labyrinth weir. The flow in the PK weir is divided into two parts. One part of flow comes from the inlet key of the PK weir and another from outlet key. The water is overflowing as a thin sheet from the inlet key and a jet towards the bottom of the outlet key (Ouamane and Lamperiere, 2003; Leite Ribeiro et al., 2007). The design of PK weir is different from the design procedures proposed for labyrinth weir with trapezoidal or triangular shaped structure. In France, the upgrading of spillway capacity has been taken in accordance with the revised design flood. Bieri et al. (2009) mentioned that about 160 dams with an average age of 50 years, owned by Electricité de France (EDF) upgraded the spillway capacity. In the year 2006, the first PK weir spillway was constructed in EDF Goulours dam and successfully tested in December 2006. Laugier (2007) presented issues related to the design and construction of PK weir. Since PK weir is a new structure, the number of geometrical parameters influencing the capacity of a PK weir, the complexity of the flow and the downstream energy-dissipation system of PK weir are to be studied in details by using scaled laboratory models. The objective of this study is to determine the coefficient of discharge \( C_d \) for PK weirs of varying geometry as the \( C_d \) value and the discharge \( Q \) values depend upon geometrical parameters viz., inlet width \( W_i \), outlet width \( W_o \), floor slope and parapet wall. Hence these parameters can be altered to obtain various values of \( C_d \) and \( Q \).

METHODOLOGY

The preliminary general design criteria for PK weirs was obtained using data from a number of sources viz., data from the experimental results carried out in laboratory flumes at Ho Chi Minh City University of Technology, (HCMUT), Vietnam (Chi Hien et al., 2006), data from the Indian Institute of Technology (IIT), Roorkee, India (Sharma and Singhal, 2008) and data from the physical models studies on Saint-Marcs (St-Marcs) dam and Gloriettes dams at the Laboratoire de Constructions Hydrauliques (LHC), Switzerland (Laugier et al., 2009; Leite Ribeiro et al., 2009, 2012). The PK weir model was made up of 10 mm thick water proof wood. Three PK weir models viz., PK 1, PK 2 and PK 3 was made with constant width of the weir \( W \) and weir height \( P \). The other dimensions adopted viz., inlet cycle width \( W_i \) and outlet cycle width \( W_o \) are given in the Table 1. The influence of ratio of wall thickness to height of PK weir and the ratio of wall thickness to radius of vertical crest curvature is very much important for low heads.
Fig. 1: PK weir inside the flume

Table 1: Dimensions of PK 1, PK 2 and PK 3 models

<table>
<thead>
<tr>
<th>Description of the elements of the PK weir</th>
<th>PK (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of the weir (P)</td>
<td>15</td>
</tr>
<tr>
<td>Width of the weir (W)</td>
<td>14.8</td>
</tr>
<tr>
<td>Outlet key width (W_o)</td>
<td>2.6</td>
</tr>
<tr>
<td>Inlet key width (W_i)</td>
<td>2.6</td>
</tr>
<tr>
<td>Outer wall thickness (T_o)</td>
<td>1.4</td>
</tr>
<tr>
<td>Inner wall thickness (T_i)</td>
<td>1</td>
</tr>
<tr>
<td>Parapet wall height</td>
<td>3</td>
</tr>
<tr>
<td>Rib thickness</td>
<td>No rib</td>
</tr>
<tr>
<td>Rib height</td>
<td>No rib</td>
</tr>
<tr>
<td>Slope of the spillway</td>
<td>45°</td>
</tr>
</tbody>
</table>

(Machiels et al., 2009). In this preliminary investigation the above ratios are not considered. The slope portion of the inlet and outlet keys and the walls were connected with nails to avoid the leakage of water through small openings in the weir. Wooden sealant was also applied on all the joints in the weir. The first PK weir model (PK 1) and second PK weir model (PK 2) had parapet wall and third PK weir model (PK 3) had no parapet wall. Moreover in PK 3 model, step like ribs of 1 cm width and 1.3 cm thick were provided with uniform spacing of 2 cm across the slopes of inlet keys and outlet keys. For the three PK weir models, C_q and Q values are to be computed for different upstream depths and compared. Detailed drawings of PK 1, PK 2 and PK 3 were given in the appendix-I.

All tests were conducted in a rectangular tilting flume measuring 400 cm long, 15 cm wide and 30 cm deep. Water was allowed to enter the flume through the head box and the flow could be controlled with help of valve. The flume was equipped with a rolling point gauge instrumentation carriage, which was used to measure the depth of water surface and crest elevations at various locations (Fig. 1). The pressure taps were attached to the flume for the measurement of upstream depths. The PK weir model was placed at certain location in the flume.
**Test procedure:** Each weir was coated with varnish as a damp proof, the as-built weir dimensions were checked to ensure they agreed well with the design drawings. The weirs were then installed in the flume, the gap between the weir and the inner walls of the flume was sealed for water tightness and a leak test was performed to ensure all joints were water-tight. To avoid the variations in the computation of discharge and $C_d$, before taking the point gauge reading, the flow was allowed to stabilize for some time and then readings were taken at the place where there was no turbulence. The point gauge was installed on a rolling carriage above the flume, using which the elevation of the crest was measured two times at two different locations of the weir. The downstream depth was also measured at seven points at different locations where the flow found to be stable. Initial depth of water was observed by keeping the water level up to the brim of the weir crest by using the rolling carriage point gauge. The elevation difference between the water surface and the weir crest (i.e., the head over the weir $H_l$), as measured by the rolling carriage point gauge, was calculated. The manometer readings for respective upstream depths were noted down and flow rates were obtained by using Orifice meter by using the formula given:

$$ Q = 0.00235 \sqrt{\Delta h} $$

Where:

$Q_s = \text{Actual discharge through the flume}$

$\Delta h = \text{Difference in pressure head between upstream and downstream sides of Orifice meter}$

The profiles were plotted for various upstream depths and its respective downstream depths, along the length of the channel (Table 2). Generally PK weir tends to behave like a linear weir as the upstream head increases (Leite Ribeiro *et al.*, 2007). Based on this behaviour, the value of coefficient of discharge ($C_d$) is assumed as 0.42 (average $C_d$ value of sharp crested weir) in calculating the discharge enhancement ratio between PK weir discharge and rectangular sharp crested weir discharge (Hager and Schleiss, 2009). But in this investigation the coefficient of discharge ($C_d$) is to be calculated. The coefficient of discharge of PK weir ($C_d$) is the ratio between actual discharge ($Q_s$) and theoretical discharge ($Q_t$). A spreadsheet program was utilized to calculate $Q_s$, $H_l$ and $C_d$ using standard rectangular weir equation. The theoretical discharge of PK weir is calculated using the usual rectangular weir formula:

<table>
<thead>
<tr>
<th>Table 2: Upstream depth for each profile for the models</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Upstream depth (cm)</strong></td>
</tr>
<tr>
<td>-------------------------</td>
</tr>
<tr>
<td>d-p1</td>
</tr>
<tr>
<td>d-p2</td>
</tr>
<tr>
<td>d-p3</td>
</tr>
<tr>
<td>d-p4</td>
</tr>
<tr>
<td>d-p5</td>
</tr>
<tr>
<td>d-p6</td>
</tr>
<tr>
<td>d-p7</td>
</tr>
<tr>
<td>d-p8</td>
</tr>
</tbody>
</table>
\[ C_s = \frac{Q_s}{Q_i} \]

and:

\[ Q_i = \frac{2}{3} L_w \sqrt{2gH^2} \]

Where:

- \( L_w \) = Total developed crest length
- \( g \) = Acceleration due to gravity
- \( H \) = Total upstream hydraulic head

**RESULTS AND DISCUSSION**

This analysis was based on three PK weir configurations (PK 1, PK 2 and PK 3) only. The PK weir was very efficient in low heads. Under increased upstream head PK weir tends to behave like a linear weir. The outlet key of the PK weir evacuated the upstream side streamwise flow and the lateral overflow from the inlet key. In the merged profile of PK 1 weir (Fig. 2) the flow did not seem to be smooth in the downstream side with splash and jump at the tail of outlet keys in all the profiles. In the profile 2 with up stream depth of 16.5 cm, the flow in the downstream side was uniform after jump. The maximum head flow might have caused separation of nappe leading to an uneven flow in the downstream side. In the PK 1 and PK 2 weirs (Fig. 3) the nappe seemed to be steep in all the profiles leading to the development of negative pressure in the downstream side. This obviously increased the velocity of flow in the downstream side, resulting in enhanced scouring, which may cause structural problem to the weir. In the profile of PK 2 weir also, the flow was steep with more splash and deep jump at the tail of outlet keys in all the profiles. The flow after the jump in the downstream side was smooth in profile 2 and 7 (upstream depths 15.8 and 16.8 cm, respectively) compared to the other profiles. In the profile of PK 3 weir (Fig. 4) the fall of nappe in the downstream side was not steep as compared to the profiles of PK 1 and PK 2. All the profiles except profile 5 (Fig. 5) had jump in the downstream side tail of outer keys. The provision of keys (step like strip) along the slopes of outer key helped to reduce the steepness of the nappe profile and

![Fig. 2: Merged profile of PK 1 weir](image-url)
more over the downstream velocity. This reduction in velocity minimized the turbulence in the downstream side which resulted in a smooth flow. Due to this, scouring in the downstream was less. The apex overhang increases the discharge. The reason was the reduction in flow contraction with increased wetted perimeter. This resulted in decreased inlet velocity and inlet head loss.

The relation between total head ($H_t$) (piezometric head and velocity head) and the theoretical discharge ($Q_t$) was linear (Fig. 6 and 8). In PK 2 model, at the higher heads there was a reduction in discharge (Fig. 7). The efficiency of the PK weir was less with higher head in the upstream side. The increase in head effected flow drowning and lateral jet over crossing resulted in decrease in
Fig. 6: Head over the weir ($H_i$) and $Q_i$ of PK 1

Fig. 7: Head over the weir ($H_i$) and $Q_i$ of PK 2

Fig. 8: Head over the weir ($H_i$) and $Q_i$ of PK 3

efficiency. The relation between ($H_i/P$) and $C_d$ for PK 1 were shown in Fig. 9, indicated the increase in $C_d$ value with the increase of ($H_i/P$) ratio. The value of $C_d = 0.47$ when $H_i$ is about 0.144 times $P$ and $C_d$ value is maximum (0.58) at $H_i/P$ ratio was 0.215. For PK 2 (Fig. 10), the value of $C_d = 0.73$ when $H_i$ was about 4.3% of $P$ and it was 0.27 when $H_i$ was 16% of $P$. For the PK 3 weir, the value of $C_d = 0.26$ when $H_i/P = 0.16$ and it was 0.37 when $H_i$ was about 10.5% of $P$. Further from the Fig. 10 and 11, it was observed that the value of $C_d$ was getting decreased with the increase of ratio ($H_i/P$). This indicated that the ratio of certain geometric parameters viz., ratio of total head to the weir height ($H_i/P$), ratio between inlet and outlet key widths ($W_i/W_o$), ratio of the total developed crest length to the weir width ($L_w/W$) etc., are responsible for enhancing the value of $C_d$, discharge and efficiency.
Fig. 9: \((H/P)\) vs. \(C_d\) of PK 1

Fig. 10: \((H/P)\) vs. \(C_d\) of PK 2

Fig. 11: \((H/P)\) vs. \(C_d\) of PK 3

<table>
<thead>
<tr>
<th>Configuration of PK weir</th>
<th>Range of coefficient of discharge ((C_d))</th>
<th>Average value of (C_d)</th>
<th>Range of actual discharge ((Q_a)(10^{-3} \text{ m}^3 \text{ sec}^{-1}))</th>
<th>Average of actual discharge ((Q_a)(10^{-3} \text{ m}^3 \text{ sec}^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>PK 1</td>
<td>0.47-0.58</td>
<td>0.525</td>
<td>1.20-2.10</td>
<td>1.650</td>
</tr>
<tr>
<td>PK 2</td>
<td>0.27-0.78</td>
<td>0.500</td>
<td>1.06-1.80</td>
<td>1.430</td>
</tr>
<tr>
<td>PK 3</td>
<td>0.26-0.87</td>
<td>0.315</td>
<td>1.15-1.38</td>
<td>1.295</td>
</tr>
</tbody>
</table>

The results shown in the Table 3 indicated that: (1) Average \(C_d\) value of PK 1 was the highest (0.525) and the PK 2 was the second with mean \(C_d\) value of 0.50. (2) Average actual discharge \((Q_a)\) for PK 1 had a maximum value when compared with other two (PK 2 and PK 3) and, PK 3 had a
minimum mean discharge value. In PK 3, there was no parapet wall above the apex overhang to guide the flow as in PK 1 and PK 2 and hence the actual discharge of PK 3 was less. (3) PK 3 was very efficient and suitable in storing flood water in the reservoir and for in-stream storage in canal/channel.

CONCLUSION

The purpose of this study was to ascertain the $C_d$ value of the PK weir with varying geometry. The PK 1 model was more efficient in discharging the water with high $C_d$ value. The model had inlet and outlet slope angle of 45°. The PK 2 model was moderate in discharging the water with almost same $C_d$ value of PK 1. The slope angle was 60°. The discharge of PK 3 model is less because there is no guidance of nappe as there is no parapet well. But PK 3 weir was very efficient in flood controlling and very useful in in-stream storage in canal/channel.

Appendix-I: Detailed drawing of PK 1, PK 2 and PK 3 weir. All dimensions are measured in mm
LIST OF ABBREVIATIONS AND SYMBOLS

The following symbols were used:

- \( W_i \) = Inlet cycle width
- \( W_o \) = Outlet cycle width
- \( C_d \) = Discharge coefficient
- \( d \) = Depth of water
- \( d-p_i \) = Profile for \( i \)th upstream depth
- \( g \) = Acceleration of gravity
- \( H \) = Piezometric head
- \( H_t \) = Total head (piezometric head plus velocity head)
- \( L_w \) = Total crest length of weir
- \( L \) = Length of the channel
- \( N \) = Weir cycles
- \( P \) = Weir height
- \( PK \) weir = Weir Piano key weir
- \( PK \) 1 = Piano key weir 1
- \( PK \) 2 = Piano key weir 2
- \( PK \) 3 = Piano key weir 3
- \( P_m \) = Distance between intersection of the slope of the inlet and outlet key to the top of the weir
- \( Q_s \) = Discharge (actual)
- \( Q_{th} \) = Discharge (theoretical) or \( Q \) theoretical discharge
- \( S_i \) = Slope of inlet cycle or key floor
- \( S_o \) = Slope of outlet cycle or key floor
- \( T_o \) = Outer wall thickness
- \( T_i \) = Inner wall thickness
- \( V \) = Velocity
- \( W \) = Width of weir
- \( w \) = Cycle width

REFERENCES


