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Decomposition-coordination Fuzzy Methods for Optimal Control of Large Scale Systems

Chokri Rekik, Mohamed Djemel and Nabil Derbel
Research unit on Intelligent Control, Design and Optimisation of Complex Systems
University of Sfax, ENIS, BP. W, 3038, Sfax, Tunisia

Abstract: The object of this study concerns the determination of an optimal control of a nonlinear complex process. The proposed idea is based on the decomposition of the system into interconnected subsystems which are linear with respect to their state and control vectors. The nonlinearities are located in the interconnection terms. We have used the mixed method with two levels iterative computation structure. A fuzzy logic approach has been used in order to identify the optimal gains. The considered applications treat the optimal control of a rotary crane.

Key words: Optimal control, fuzzy method, large system, decomposition-coordination

INTRODUCTION

The evolution of decentralized and hierarchical computational and control techniques have known recent advances. Recently there has been an increasing amount of study in the development of efficient decomposition-coordination or multi-level techniques for solving complex optimization problems.

This approach, first introduced by Dantzing and Wolfe to solve large linear programming problems, was further developed by Mesarovic and co-workers[1-3], for large-scale optimal control problems, where the system is mainly composed of an interconnected sub-system. The methodology of these techniques consists of decomposing a large-scale system into interconnected subsystems of smaller dimensions, which become therefore easier to study. But, when the interactions between subsystems are strong, conflicts could arise between the controllers if none of subsystems have priority of action[4-5]. To resolve these conflicts, is considered a second level of control which takes into account the interactions, modifies if necessary the controllers and coordinates between different subsystems. Several study have been presented in the literature following this approach in the case of linear systems[6-9].

In this study, the proposed method consists in the application of the hierarchical control for a complex system by determining optimal control laws, which minimize a quadratic cost function. Such optimization has been successfully achieved using a fuzzy logic controller yielding the optimal gains at each step time.

PROBLEM FORMULATION

Let us consider a nonlinear system which can be represented by interconnected subsystems. We consider the case where subsystems are linear with respect to their state and control vectors. We assume that subsystems can be described by the following differential equation:

$$\dot{X}_i = A_i(Z_i)X_i + B_i(Z_i)U_i \tag{1}$$

Moreover, we assume that subsystems are controllable. The interconnection between subsystems is represented by:

$$Z_i = \sum_{j \neq i} L_{ij}X_j \tag{2}$$

where X_i is an n_i dimensional state vector, U_i is an m_i dimensional control vector and Z_i is an r_i dimensional vector. The optimization problem deals with choosing the controls $U_1, U_2, \ldots U_M$ in order to minimize the following quadratic criterion:

$$J = \sum_{i=1}^{M} \frac{1}{2} \int_0^{+\infty} (X_i^T Q_i X_i + U_i^T R_i U_i)\,dt \tag{3}$$

with Q_i and R_i the ponderation matrices. Q_i is a symmetric positive semi-definite matrix ($Q_i \geq 0$) and R_i is a symmetric positive definite matrix ($R_i > 0$), for $i=1,2,\ldots,M$. It is clear that the principle behind hierarchical control is to decompose the global problem into subproblems. In general, there are many interactions between subproblems. For this reason, it is necessary to introduce an interaction vector or coordination parameters in order

Corresponding Author: Chokri Rekik, Research unit on Intelligent Control, design and Optimisation of complex Systems (ICOS) University of Sfax, ENIS, BP. W, 3038, Sfax, Tunisia. Tel : 2164 274088, Fax: 2164 275595

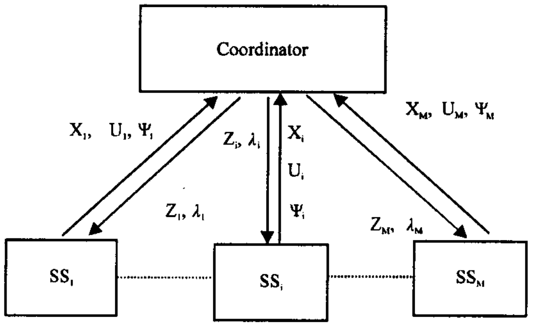

Fig. 1: Information transfer for the mixed method

To coordinate between different subproblems. Hierarchical optimization involves the choice of the coordination parameters from some initial values and iterating to the final values which solve the coordination problem in hierarchical control[5]. In this study , we use the mixed method with two levels iterative computation structure (Fig.1). In this method, Lagrange multiplier vectors and interaction input vector Z are determined by the second level and are used at the first level as given vectors[6].

To solve this problem, let us write the Hamiltonian of the problem as follows[6]:

$$H=\sum_{i=1}^{M}\frac{1}{2}(X_i^TQ_iX_i+U_i^TR_iU_i)+\psi_i^T(A_iX_i+B_iU_i) \quad (4)$$

$$+\lambda_i^T(Z_i-\sum_{j\neq i}L_{ij}X_j(t))$$

where Ψ_i is the adjoint vector of the state vector X_i and λ_i is the Lagrange multiplier vector associated to the coupling constraint. Using the mixed coordination method and for given vectors λ_i and Z_i the Hamiltonian is additively separable. It can be decomposed into M independent sub-Hamiltonians:

$$H=\sum_{i=1}^{M}H_i \quad (5)$$

with :

$$H_i=\frac{1}{2}(X_i^TQ_iX_i+U_i^TR_iU_i)+\psi_i^T(A_iX_i+B_iU_i)+\lambda_i^TZ_i \quad (6)$$

$$-(\sum_{j\neq i}\lambda_j^TL_{ji})X_i$$

The optimal conditions can be described by the following equations:

$$\frac{\partial H_i}{\partial X_i}=-\dot{\psi}_i \quad \frac{\partial H_i}{\partial \psi_i}=\dot{X}_i \quad \frac{\partial H_i}{\partial U_i}=0 \quad (7)$$

In this method, vectors λ and Z are determined by the second level and are used at the first level as given vectors, so the solution of each subsystem corresponds to the treatment of equations (7) for the first level and for the second level:

$$\frac{\partial H}{\partial Z_i}=0 \quad \frac{\partial H}{\partial \lambda_i}=0 \quad (8)$$

First level:
Equations (7) give:

$$\dot{\psi}_i=-Q_iX_i-A_i^T\psi_i+\sum_{j\neq i}^{M}L_{ji}^T\lambda_j \quad (9)$$

$$U_i=-R_i^{-1}B_i^T\psi_i \quad (10)$$

One can easily show that vector Ψ_i can be expressed in terms of vector X_i as:

$$\psi_i=P_iX_i+S_i \quad (11)$$

where matrix P_i and vector S_i will be determined below.

Differentiating Eq 11 with respect to time and using Eq 1 and 10 yields:

$$\dot{\psi}_i=(\dot{P}_i+P_iA_i-P_iB_iR_i^{-1}B_i^TP_i)X_i-P_iB_iR_i^{-1}B_i^TS_i+\dot{S}_i \quad (12)$$

Substituting Eq 11 into Eq. 9 gives:

$$\dot{\psi}_i=(-Q_i-A_i^TP_i)X_i-A_i^TS_i+\sum_{j\neq i}^{M}L_{ji}^T\lambda_j \quad (13)$$

Referring to Eq. 12 and 13, one can easily write:

$$\dot{P}_i+P_iA_i+A_i^TP_i-P_iB_iR_i^{-1}B_i^TP_i+Q_i=0 \quad (14)$$

$$\dot{S}_i+(A_i^T-P_iB_iR_i^{-1}B_i^T)S_i-\sum_{j\neq i}^{M}L_{ji}^T\lambda_j=0 \quad (15)$$

As an approximation, we can consider the stationary solutions of matrix P_i and vector S_i ($\dot{P}_i\approx 0$, $\dot{S}_i\approx 0$). The efficiency of this approximation can be proved below. Then, the control U_i is given by:

$$U_i=-R_i^{-1}B_i^TP_iX_i-R_i^{-1}B_i^TS_i \quad (16)$$

It is easy to verify that matrix P_i, which is the solution of the Riccati nonlinear equation, is a symmetric positive definite matrix.

Second level
In this level, we treat the two coordination equations:

$$\frac{\partial H}{\partial \lambda_i}=0 \quad \Rightarrow \quad Z_i=\sum_{j\neq i}^{M}L_{ij}X_j \quad (17)$$

$$\frac{\partial H}{\partial Z_i}=0 \;\Rightarrow\; \lambda_{ik}=-\psi_i^T\left(\frac{\partial A_i}{\partial Z_{ik}}X_i+\frac{\partial B_i}{\partial Z_{ik}}U_i\right) \qquad (18)$$

$$\lambda_{ik}=-\psi_i^T\,\eta_{ik}=-\eta_{ik}^T\,\psi_i \qquad (19)$$

with

$$\eta_{ik}=\frac{\partial A_i}{\partial Z_{ik}}X_i+\frac{\partial B_i}{\partial Z_{ik}}U_i \qquad (20)$$

$$\lambda_i=\begin{bmatrix}\lambda_{i1}\\ \vdots\\ \lambda_{ik}\\ \vdots\\ \lambda_{ir_i}\end{bmatrix},\; Z_i=\begin{bmatrix}Z_{i1}\\ \vdots\\ Z_{ik}\\ \vdots\\ Z_{ir_i}\end{bmatrix},\; \eta_i=\begin{bmatrix}\eta_{i1}\\ \vdots\\ \eta_{ik}\\ \vdots\\ \eta_{ir_i}\end{bmatrix} \qquad (21)$$

The information transfer between the two levels is still quite limited. The stationary solution of vector S_i can be expressed as the following:

$$S_i=\sum_{j\neq i}^{M}\upsilon_{ij}\,\lambda_j \qquad (22)$$

with:

$$\upsilon_{ij}=(A_i^T-P_iB_iR_i^{-1}B_i^T)^{-1}L_{ji}^T \qquad (23)$$

Substituting Eq .22 into Eq .11 gives:

$$\psi_i=P_iX_i+\sum_{j\neq i}^{M}\upsilon_{ij}\lambda_j$$

$$\psi_i+\sum_{i\neq j}\upsilon_{ij}\,\eta_j^T\,\psi_j=P_i\,X_i \qquad (24)$$

which represents a quadratic equation in terms of Ψ_i. We can notice here, that the variable η_i is a function of U_i and the control U_i is a function of Ψ_i. This yields that the analytic resolution becomes difficult. To overcome this problem, we can takes$\eta_i=f(U_i(t_i-\Delta t))$. To justify this approximation, Δt can be chosen smaller. In this case, one can easily write:

$$\begin{pmatrix} I & \upsilon_{12}\eta_2^T & \cdots & \upsilon_{1M}\eta_M^T\\ \upsilon_{11}\eta_1^T & I & & \vdots\\ \vdots & & \ddots & \vdots\\ \upsilon_{M}\eta_1^T & \cdots & \cdots & I\end{pmatrix}\begin{pmatrix}\psi_1\\ \vdots\\ \vdots\\ \psi_M\end{pmatrix}=\begin{pmatrix}P_1 & & \\ & \ddots\; 0 & \\ & 0\;\ddots & \\ & & P_M\end{pmatrix}\begin{pmatrix}X_1\\ \vdots\\ \vdots\\ X_M\end{pmatrix} \qquad (25)$$

$$\begin{pmatrix}\psi_1\\ \vdots\\ \vdots\\ \psi_i\end{pmatrix}=\begin{pmatrix} I & \upsilon_{12}\eta_2^T & \cdots & \upsilon_{1M}\eta_M^T\\ \upsilon_{21}\eta_1^T & I & & \vdots\\ \vdots & & \ddots & \vdots\\ \upsilon_{M1}\eta_1^T & \cdots & \cdots & I\end{pmatrix}^{-1}\begin{pmatrix}P_1 & & \\ & \ddots\; 0 & \\ & 0\;\ddots & \\ & & P_M\end{pmatrix}\begin{pmatrix}X_1\\ \vdots\\ \vdots\\ X_M\end{pmatrix} \qquad (26)$$

and $$U_i=-R_i^{-1}B_i^T\psi_i$$

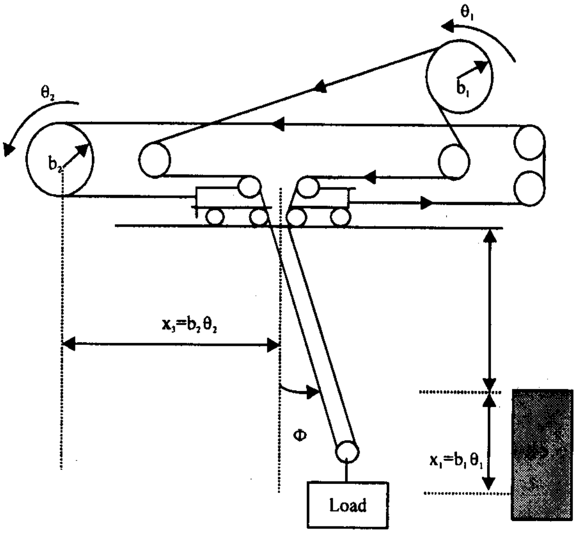

Fig. 2: Schematization of the rotary crane

We can notice here, that the terms η_i and P_i of the subsystem i are a function of the interconnexion terms of the subsystem j. So, the main idea is the construction a database, which for each Z_j, we compute the above terms. Then, a fuzzy logic controller is then identified.

APPLICATION

To illustrate the above approach, we consider the optimal control of a rotary crane.

Problem formulation: The fundamental motion of a rotary crane is the rotation and the load hoisting[10-11]. We assume that the container load can be regarded as a material point and that frictional torques which may exist in torque-transfer mechanisms can be neglected. We use the following notations (Fig. 2):

θ_1 : the rotation angle of the trolley drive motor;
θ_2 : the rotation angle of the hoist motor
ϕ : the load swing angle;

The state variables are x_1, x_2, x_3, x_1, $x_2$and $x_3$where: $x_1=b_1\theta_1$,. $x_3=b_2\theta_2$, $x_5=\phi$

The process to be controlled can be described by a sixth-order nonlinear differential system such as:

$$\frac{dx_1}{dt}=x_2$$

$$\frac{dx_2}{dt}=u_1$$

$$\frac{dx_3}{dt}=x_4 \qquad (27)$$

$$\frac{dx_4}{dt} = u_2 + \delta_1 x_5$$

$$\frac{dx_5}{dt} = x_6$$

$$\frac{dx_6}{dt} = -\frac{1}{x_1 + h}[u_2 + \delta_2 x_5 + 2x_2 x_6]$$

Parameters $\delta_1$and δ_2 are given by δ_1=77 and δ_2=67.

The objective is to determine the optimal control strategies of such system, in order to transfer the rotary crane from any initial state to the desired state described by $X_d = [0\ 0\ 0\ 0\ 0\ 0]^T$.

We choose the following quadratic criterion defined as:

$$J = \frac{1}{2}\int_0^{\infty} (X^T Q\ X\ + U^T R\ U\ )dt \qquad (28)$$

where $X=[x_1\ x_2\ x_3\ x_4\ x_5\ x_6]^T$, the state vector, $U=[u_1\ u_2]^T$, the control vector and Q and R are the ponderation matrices, which are chosen as: Q=diag([1 10 1 40 20000 20000]), R=diag([20 4]).

Decomposition of the system: Consider the rotary crane described above by system (27) which can be divided into two interconnected subsystems defined as follow:

- The subsystem 1 is defined by:

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = u_1 \qquad (29)$$

where:

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and } X_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},\ Z_1 = [\ ]$$

- The subsystem 2 is described by:

$$\frac{dx_3}{dt} = x_4$$

$$\frac{dx_4}{dt} = u_2 + \delta_1 x_5$$

$$\frac{dx_5}{dt} = x_6$$

$$\frac{dx_6}{dt} = -\frac{1}{x_1 + h}[u_2 + \delta_2 x_5 + 2x_2 x_6] \qquad (30)$$

where:

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \delta_1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\alpha\delta_2 & -\alpha\beta \end{bmatrix} B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -\alpha \end{bmatrix} X_2 = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

with

$$\alpha = \frac{1}{x_1 + 17}\ ,\ \beta = 2x_2$$

It's clear that subsystem 2, which depends on the state vector of subsystem 1, is linear with respect to its state vector and its control vector. However subsystem 1, which is also linear, is independent from subsystem 2. Equation (26) gives:

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} I & \upsilon_{12}\eta_2^T \\ \upsilon_{21}\eta_1^T & I \end{pmatrix}^{-1} \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \qquad (31)$$

with:

$$\upsilon_{12} = (A_1^T - P_1 B_1 R_1^{-1} B_1^T)^{-1} L_{21}^T \qquad (32)$$

$$\upsilon_{21} = (A_2^T - P_2 B_2 R_2^{-1} B_2^T)^{-1} L_{12}^T \qquad (33)$$

and:

$$L_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ L_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad (34)$$

$$\eta_1 = \begin{bmatrix} \eta_{11} \\ \eta_{12} \end{bmatrix},\ \eta_2 = \begin{bmatrix} \eta_{21} \\ \eta_{22} \end{bmatrix} \qquad (35)$$

$$\eta_{11} = \frac{\partial A_1}{\partial Z_{11}} X_1 + \frac{\partial B_1}{\partial Z_{11}} u_1 = 0,\ \eta_{12} = \frac{\partial A_1}{\partial Z_{12}} X_1 + \frac{\partial B_1}{\partial Z_{12}} u_1 = 0 \qquad (36)$$

so:

$$\eta_1 = [\ ] \qquad (37)$$

and:

$$\eta_{21} = \frac{\partial A_2}{\partial Z_{21}} X_2 + \frac{\partial B_2}{\partial Z_{21}} u_2,\ \eta_{22} = \frac{\partial A_2}{\partial Z_{22}} X_2 + \frac{\partial B_2}{\partial Z_{22}} u_2 \qquad (38)$$

with:

$$\eta_{21} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ x_5\delta_2\alpha^2 + x_6\beta\alpha^2 + u_2\alpha^2 \end{bmatrix},\quad \eta_{22} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -2x_6\alpha \end{bmatrix} \qquad (39)$$

so:

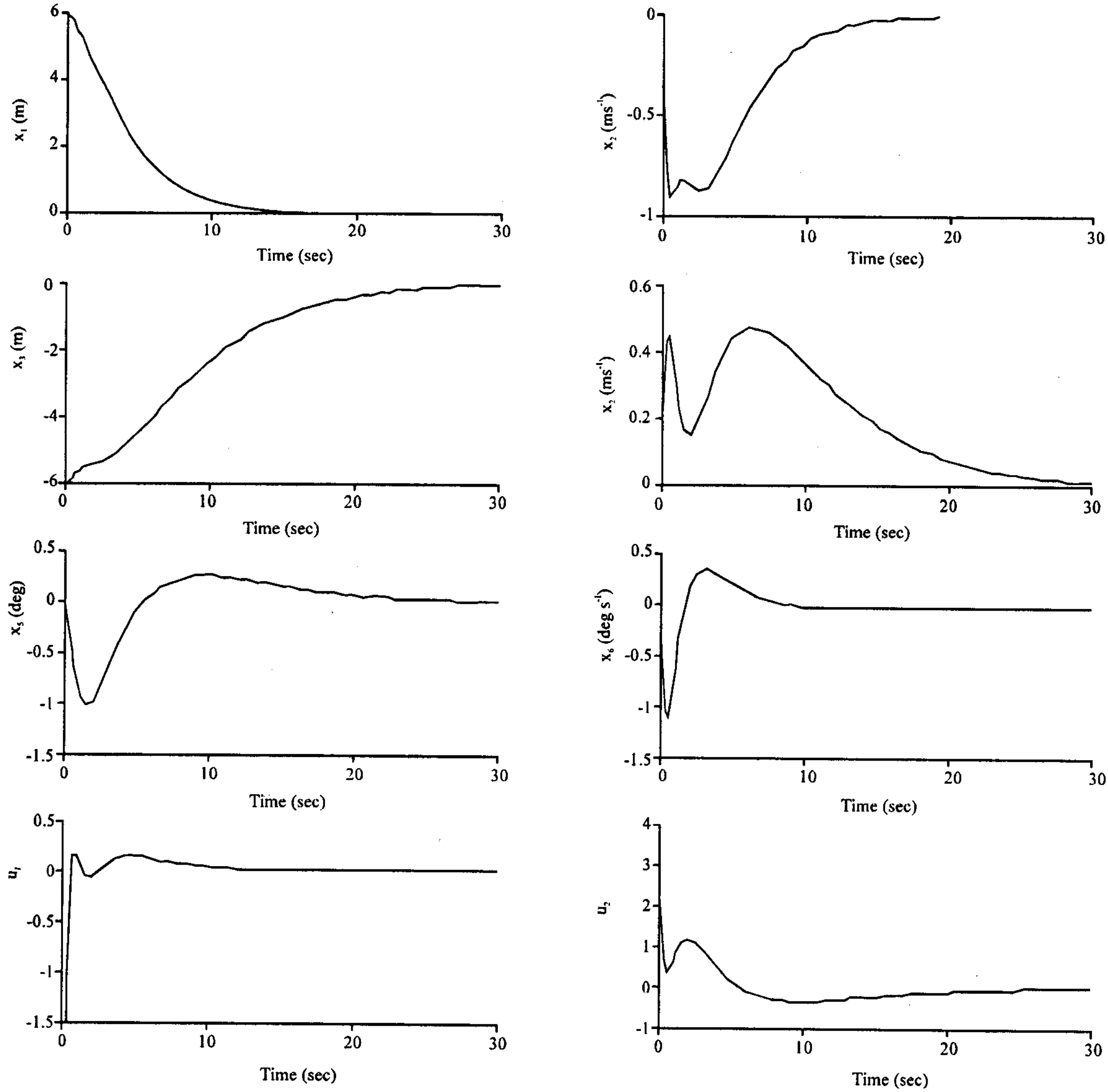

Fig. 3: Dynamic optimal control behaviour of rotary crane

$$\eta_2^T = \begin{bmatrix} 0 & 0 & 0 & x_5\delta_2\alpha^2 + x_6\beta\alpha^2 + u_2\alpha^2 \\ 0 & 0 & 0 & -2x_6\alpha \end{bmatrix} \quad (40)$$

so, Equation (31) gives:

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} I & \upsilon_{12}\eta_2^T \\ 0 & I \end{pmatrix}^{-1} \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \quad (41)$$

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} I & -\upsilon_{12}\eta_2^T \\ 0 & I \end{pmatrix} \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \quad (42)$$

so, the follow system:

$$\begin{cases} \psi_1 = P_1X_1 - \vartheta_{12}\eta_2^T P_2 X_2 \\ \psi_2 = P_2 X_2 \end{cases} \quad (43)$$

thus, the control laws:

$$\begin{cases} u_1 = -K_{11}X_1 - K_{12}X_2 \\ u_2 = -K_{22}X_2 \end{cases} \quad (44)$$

with:

$$K_{11} = R_1^{-1}B_1^TP_1,\ K_{12} = R_1^{-1}B_1^T\upsilon_{12}\eta_2^TP_2,\ K_{22} = R_2^{-1}B_2^TP_2 \quad (45)$$

finally, we can write:

$$u_1 = -0.2236x_1 - 0.9732x_2 - \frac{\alpha^2(x_5\delta_2 + \beta x_6 + u_2)\ (p_{14}x_3 + p_{24}x_4 + p_{34}x_5 + p_{44}x_6)}{4.4720} \quad (46)$$

$$u_2 = -k_3x_3 - k_4x_4 - k_5x_5 - k_6x_6 \quad (47)$$

To determine the two control laws u_1, two fuzzy logic controllers are considered. The proposed controller employs the symmetrical gaussian functions to represent the membership functions to represent the membership function of the antecedent parts of fuzzy rules.

The first one is considered to determine the coefficients of the solution of the Riccati equation. The rule has the following form:

If ($x_1$is F_1) and (x_2 is F_2) Then u_2 [P_{14}, P_{24}, P_{34}, P_{44}].

The second one is considered to determine the optimal gain k_i (I=1...4). The rule have the following form:

If (x_1 is F_1) and (x_2 is F_2) Then u_2. (K_3 x_3+ k_4 x_4+ k_5 x_5 k_6x^6)

RESULTS

Simulation results are presented in Fig. 3. The rotary crane starts from the initial state X_i =[6 0 –6 0 0 0]^T. It reaches the desired terminal state X_d=[0 0 0 0]^T after 30 sec. Observing the state variations presented in Fig. 3, we can notice some interesting comments.

The state vector (x_1 and x_2) of subsystem 1 has rapid variation compared to the variations of the state vector of subsystem 2 (x_3 to x_6). This can be explained by the fact that the vertical movement of the load (subsystem 1) is practically independent from the hoisting (variables x_5 and x^6). However, the horizontal movement of the load (subsystem2) is the principal factor generating the hoisting. Thus, the control vector of subsystem 2 is the principal factor responsible in order to limited the hoisting. This is why subsystem 1 is more rapid than subsystem 2.

CONCLUSIONS

The interest of this study was concerned by the determining of optimal control strategies of nonlinear systems. Firstly, the global system was divided into interconnected linear subsystems. Nonlinearities are located in the interconnection terms. Secondly, different optimal gains are computed in terms of interconnection vectors. This was done by the use of the fuzzy mixed coordination method.

As an illustration, we considered the optimal control problem of a rotary crane.

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