

## Zhang *et al.*'s Partially Blind Signature Revisited

<sup>1</sup>Wei Gao, <sup>2</sup>Xueli Wang and <sup>3</sup>Dongqing Xie

<sup>1</sup>College of Mathematics, Hunan University, Changsha,

<sup>2</sup>College of Mathematics, Guangzhou University, Guangzhou,

<sup>3</sup>College of Software, Hunan University, Changsha, Peoples Republic of China

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**Abstract:** Partially blind signature is an important cryptographic primitive in privacy oriented e-commerce applications. In INDOCRYPT 2003, Zhang *et al.* proposed a new partially blind signature scheme more efficient than previous schemes. It was later showed linkable by Chow *et al.* and then respectively improved by both Chow *et al.* and Zhang *et al.* at the cost of poorer efficiency. However, in this short study we show that the cryptanalysis of Chow *et al.* is wrong. So the original partially blind signature scheme remains secure and the corresponding improvements are needless.

**Key words:** Zhang *et al.*, partially blind signature

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### INTRODUCTION

Blind signatures were first introduced by Chaum<sup>[1]</sup> and play a central role in cryptographic protocols such as e-cash or e-voting that require user anonymity. Partially blind signatures were introduced by Abe and Fujisaki<sup>[2]</sup> to allow the signer to explicitly include some agreed information in the blind signature. The partially blind signature is a regular digital signature, but it needs to satisfy two additional requirements (partial blindness):

- The signer assures that an issued signature contains the information that it desires and none can remove the embedded information from the signature.
- For the same embedded information, the signer cannot link a signature to the instance of the signing protocol that produces the corresponding blind signature.

In INDOCRYPT 2003, Zhang *et al.*<sup>[3]</sup> proposed a partially blind signature which is very efficient in terms of computation and communication and can provide batch verification. Unfortunately, it was later showed linkable by Chow *et al.*<sup>[4]</sup> and improved. And the original authors themselves, Zhang *et al.* also accepted this disadvantage and revised their original scheme. Of course, both improved schemes are less efficient than the original scheme. Especially, Chow *et al.*'s scheme lost a lot of efficiency since they applied the conventional paradigm mentioned in<sup>[2]</sup>, i.e., converting a identity-based blind signature to a partially blind signature scheme.

### REVIEW OF ZHANG *et al.*'s PARTIALLY BLIND SIGNATURE SCHEME

First, let us recall the corresponding mathematical tools used in the scheme. Let  $G_1$  be a cyclic additive group generated by  $P$  whose order is a prime and  $G_2$  be a cyclic multiplicative group with the same order. Let  $e: G_1 \times G_1 \rightarrow G_2$  be a bilinear pairing with the following properties:

- Bilinearity:  $e(aP, bQ) = e(P, Q)^{ab}$  for all  $P, Q \in G_1$  and  $a, b \in \mathbb{Z}_q^*$
- Non-degeneracy: There exists  $P, Q \in G_1$  such that  $e(P, Q) \in G_2$ .
- Computability: There is an efficient algorithm to compute  $e(P, Q)$  for all  $P, Q \in G_1$ .

Zhang *et al.*'s scheme<sup>[3]</sup> is given as follows. The system parameters are  $\text{params} = (G_1, G_2, e, q, P, H, H_0)$ , where  $G_1 = \langle P \rangle$ ,  $G_2, e, q$  are defined as above and  $H: \{0,1\}^* \rightarrow \mathbb{Z}_q, H: \{0,1\}^* \rightarrow G_1$  are hash functions.

**KeyGen:** The signers picks random  $x \in \mathbb{Z}_q^*$  and compute  $P_{\text{pub}} = xP$ . The public key is  $P_{\text{pub}}$  and the secret key is  $x$ .

**Issue:** Suppose the requester now wants to get the signature of message  $m$  and the requester has already negotiated with the signer on the agreed information  $c$  to be attached to the message. The interaction between the requester and the signer is as follows:

**Blinding:** The user randomly chooses a number  $r \in \mathbb{Z}_q^*$ ,  $U = rH_0(m||c)$  computes and sends  $U$  to the signer.

**Signing:** The signer sends back  $V$ , where  $V = (H(c)+x)^{-1}U$ .

**Unblinding:** The user computes  $S = r^{-1}V$ . Then  $(S,m,c)$  is the partially blind signature of the message  $m$  and public information  $c$ .

**Verification:** A verifier can accept this partially blind signature if and only if.

$$e(H(c)P, P_{pub}, S) = e(P, H_0(m || c))$$

**REVIEW OF CHOW *et al.*'S  
CRYPTANALYSIS AND OUR DISCUSSION**

The cryptanalysis given by Chow *et al* is very simple as follows. Given a valid signature  $(S,m,c)$  and the view  $(U,V)$ , any party can accept the partially blind signature  $(S,m,c)$  as the one produced by the instance of the Issue protocol  $(U,V)$  if and only if  $e(S,U) = e(V,H_0(m || c))$ . However, the following theorem is enough to show their conclusion is wrong.

**Theorem 1.** Given any view  $(U,V)$  and any valid signature  $(S',m',c)$ , the above mentioned equation  $e(S',U) = e(V,H_0(m' || c))$  always holds.

**Proof.** Let the signature and message corresponding to  $(U,V)$  is  $(S',m',c)$ , then there exists  $r \in Z_q^*$  such that:

$$U = rH_0(m || c), V = \frac{1}{x + H(c)}U, S = \frac{1}{x + H(c)}H_0(m || c)$$

So, for some valid signature and message  $(S',m',c)$  satisfying

$$S' = \frac{1}{x + H(c)}H_0(m' || c), \text{ we have:}$$

and

$$\begin{aligned} e(V, H_0(m' || c)) &= e\left(\frac{1}{x + H(c)}U, H_0(m' || c)\right) \\ &= e\left(\frac{1}{x + H(c)}rH_0(m || c), H_0(m' || c)\right) \\ &= e(H_0(m' || c), H_0(m || c))^{\frac{r}{x + H(c)}} \end{aligned}$$

Now we have:

$$e(S',U) = e(V, H_0(m' || c))$$

So, for any given valid signature  $(S',m',c)$  and the view of the execution of the issuing protocol  $(U,V)$  corresponding to any signature pair  $(S',m',c)$ , the following equation always holds:  $e(S',U) = e(V, H_0(m || c))$ . So we can not determine whether a signature  $(S,m,c)$  is issued by the view  $(U,V)$  through checking the above equation.

**CONCLUSION**

We point out the error in the cryptanalysis of the art-and-state partially blind signature based pairing in<sup>[4]</sup>. So we conclude that the original scheme remains secure and of course much more efficient than the so-called improved schemes in<sup>[4,3]</sup>.

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