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Mathematical Model Selection of a Haulage Mechanism with Static Equilibrium

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Abstract: Haulage installations are subjected to many efforts making their equilibrium difficult in spite, the use of equilibrium devices. In this study a mathematical model of this type of installation is developed and studied taking in consideration the factor of traction cable elasticity (dumping factor). If the dumping factor (or attenuation) of oscillations goes beyond a certain limit, this will provoke a fast and a considerable propagation of elastic deformations along the cables (on all their length) which unbalances the installation operation.

Key words: Haulage installations, equilibrium, elasticity, deformations, operation, traction

INTRODUCTION

Traction electromechanical systems (haulage systems) are one of the most research subject studied at the moment (Tolba *et al*., 2007; Tolba and Saad, 2003). Their regulation and control (Tchermalekh *et al*., 1996) become complicated especially when different elements constituting the mechanism and transient states to which are subjected are taken in consideration. The presence of such elements has made the mechanism movements followed up with additional dynamic efforts during the transient states (braking, accelerations and starting). In these conditions the load is at its maximal values and sometimes these values are superior to those corresponding to the opposite efforts introduced by the system.

Differential equations resolution governing the system dynamic, give the possibility to define not only the efforts maximum values during braking and starting states but also the possible different movements of different elements related by elastic cables.

From verification, solidity and motor robustness point of view, it is sufficient to know the load maximum value. In these conditions, dynamic loads evaluation is determined from dynamic coefficient (Walker, 1988; Davidof, 2001).

Because of masses of different elements and transmissions transient states cable deformations are appeared on cables, this will provoke the displacement of gravity center of elements in question and produce an increase in potential energy and will lead to an increase to the corresponding total force.

MATHEMATICAL MODEL DEVELOPMENT

In the machine studied, the presence of elements having a great deformability (cables), allow to consider other elements such as: drum, transmission element and motor as rigid elements. The most appropriate scheme of this study is a haulage system with loads static equilibrium where main and equilibrium cables are found Fig. 1a.

The dynamic of the extracting machine with static equilibrium and taking in consideration main cable bits and also those of equilibrium is shown in Fig. 1b.

All masses are drawn to the drum shaft of rolling up cable, it can also be found:

F_m	:	Motrice forces
F_r	:	Effort resulting from of all static resistant efforts;
m_1	:	Reduced mass of rotating elements;
m_2, m_3	:	Are respectively upward and downward load masses;
m_{c1}, m_{c2}	:	Masses of principal cable bits;
l_1, l_2, l_3, l_4	:	The length corresponding to cable bits;

Assuming :

$$\alpha_{11} = \frac{m_{c1}}{m_1}; \alpha_{12} = \frac{m_{c1}}{m_2}; \alpha_{21} = \frac{m_{c2}}{m_1}; \alpha_{23} = \frac{m_{c2}}{m_3}$$

$W_1(p)$, $W_2(p)$, $W_3(p)$, $W_4(p)$: Transfer functions of cable bits corresponding to dynamic efforts.

F'_{1y}, F''_{1y}, F'_{2y}, F''_{2y}, F'_{3y}, F''_{3y}, appearing at contact points between bits and concentrated masses (in two schemes all forces have the corresponding masses as origin), thus, transfer functions are:

$$W_1(p) = \frac{\pi^2}{8}\left(p^2 + \mu_c\beta_{c1}^2 p + \beta_{c1}^2\right)^{-1} \quad (1)$$

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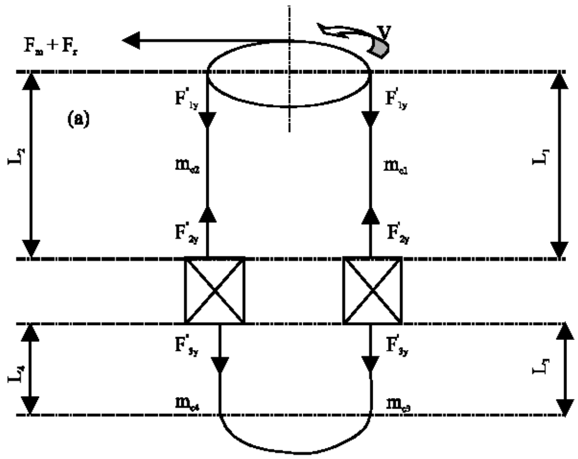

Fig. 1a: Main and equilibrium cables diagram

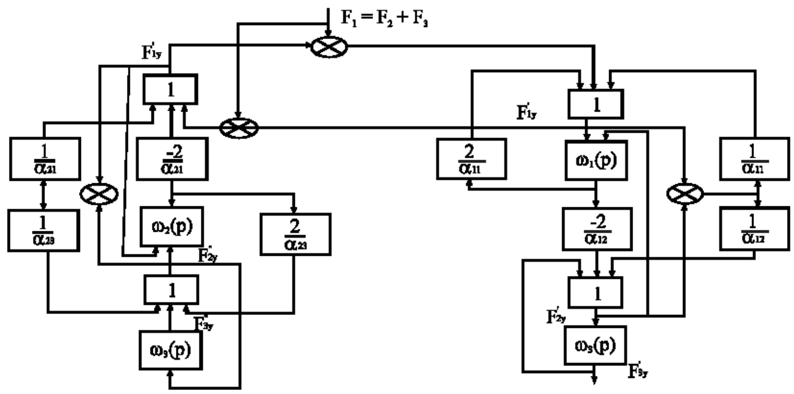

Fig. 1b: System diagram presented by transfer function

$$W_2(p) = \frac{\pi^2}{8}\left(p^2 + \mu_c\beta_{c2}^2 p + \beta_{c2}^2\right)^{-1} \tag{2}$$

$$W_3(p) = \frac{\alpha_{32}\left(p^2 + \mu_c\beta_{c3}^2 p + \beta_{c3}^2\right)}{\left(1+\alpha_{32}+\frac{\pi^2}{4}\right)p^2 + \left(1+\alpha_{43}\right)\left(\mu_c p + 1\right)\beta_{c3}^2} \tag{3}$$

$$W_4(p) = \frac{\alpha_{43}\left(p^2 + \mu_c\beta_{c4}^2 p + \beta_{c4}^2\right)}{\left(1+\alpha_{43}+\frac{\pi^2}{4}\right)p^2 + \left(1+\alpha_{43}\right)\left(\mu_c p + 1\right)\beta_{c4}^2} \tag{4}$$

Where,

$$\alpha_{32} = \frac{m_{c3}}{m_2} \text{ et } \alpha_{43} = \frac{m_{c4}}{m_3}$$

oscillating dumping factors in main cable bits and equilibrium cables;

β_{c1}, β_{c2}, β_3, β_4: oscillating frequency in the bits;

$\beta_{ci} = \frac{\pi.a_c}{l_i}$:α_c Propagation speed of elastic deformation along the cable, usually α_c = 4000m/s and $\mu_c \approx$ 0,005....0, 01s.

In order to neglect the elasticity of one or many cable bits, in their corresponding transfer function, assuming that:

$$\mu_c = \beta_{ci} = \infty$$

If in the system, the equilibrium cable does not exist, thus:

$$\alpha_{32} = \alpha_{43} = 0$$

This will lead to establish that the expressions (3) and (4) are equal to zero.

In order to obtain the mathematical model of the extracting machine, composed from masses m_1, m_2 and one cable bit mass m_c from which m_2 is obtained having a length of l_1, we must take in consideration:

$\beta_{c2} = \mu_c = \infty$ and $\alpha_{21} = \alpha_{23} = 0$, donc $F''_{1y} = 0$ car

$$\frac{F''_{1y}(p)}{F_{02}(p)} = \frac{\alpha_{21}}{1+\alpha_{22}} = 0$$

As a result, the functional scheme of Fig. 2a is obtained. If a force is applied to m_1 then:

$$F_1 = F_m + F_r$$

Therefore, the dynamic forces F'_{1y} and F'_{2y} are determined by the following expressions under operational form:

$$F'_{1y}(p) = F_1(p)\frac{\alpha_{11}\left[\left(1+\alpha_{12}+\frac{\pi^2}{4}\right)p^2 + (1+\alpha_{12})(\mu_c p+1)\beta_{c1}^2\right]}{\Delta_{c1}\left(p^2+\mu_c\omega_{c1}^2+\omega_{c1}^2\right)}; \quad (5)$$

$$F'_{2y}(p) = F_1(p)\frac{\alpha_{11}\left[\left(1-\frac{\pi^2}{4}\right)p^2 + (\mu_c p+1)\beta_{c1}^2\right]}{\Delta_{c1}\left(p^2+\mu_c\omega_{c1}^2+\omega_{c1}^2\right)}, \quad (6)$$

with:

$$\Delta_{c1} = \alpha_{11}+\alpha_{12}+\alpha_{11}.\alpha_{12}+\frac{\pi^2}{4}(4+\alpha_{11}+\alpha_{12}).$$

Using expressions (5) and (6), for any law of force variation F_1(in time), It will be easy to determine the dynamic forces F'_{1y} (t)et F'_{2y}(t).

The design of automatic control system and its dynamic analysis requires knowing, firstly the speed and displacement of moving masses. Therefore, it is necessary to know the forces acting on these masses and the movements produced by their displacements. In this case x_1 is provoked by the masses m_1, x_2, m_2 et $x_3$and m_3, for initial conditions equal to zero, the following relations are obtained:

$$X_1(p) = \frac{1}{m_1p^2}\left[F_1(p) - F'_{1y}(p) + F''_{1y}(p)\right] \quad (7)$$

$$X_2(p) = \frac{1}{m_2p^2}\left[F'_{2y}(p) - F'_{3y}(p)\right] \quad (8)$$

$$X_3(p) = \frac{1}{m_3p^2}\left[F''_{3y}(p) - F''_{2y}(p)\right]. \quad (9)$$

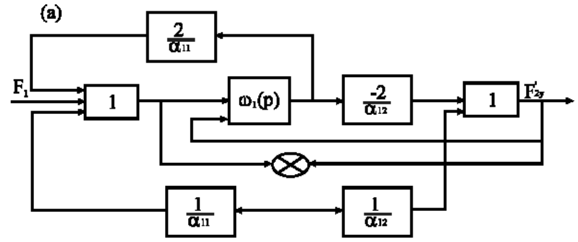

Fig. 2a: Bit represented by functional diagram

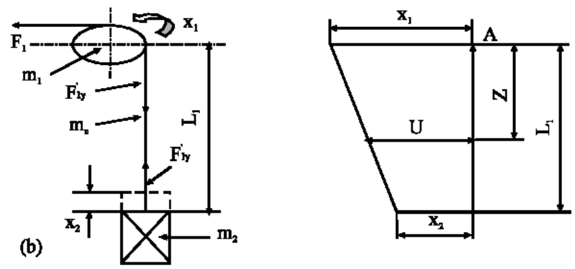

Fig. 2b: Machine with one cable bit

For a machine with one cable bit, illustrated in Fig. 2b and using the expressions (5) and (6), the equations below are obtained:

$$X_1(p) = \frac{1}{m_1p^2}\left[F_1(p) - F'_{2y}(p)\right] =$$

$$= \frac{F_1}{m_1p^2}\left[1-\alpha_{11}\frac{\left(1+\alpha_{12}+\frac{\pi^2}{4}\right)p^2+(1+\alpha_{22})(\mu_c p+1)\omega_{c1}^2}{\Delta_{c1}\left(p^2+\mu_c\omega_{c1}^2 p+\omega_{c1}^2\right)}\right]$$

If the oscillating dumping factor is neglected ($\mu_c = 0$), thus, the following expressions are obtained:

$$X_1(p) = \frac{F_1}{m_1p^2}\frac{\left[\alpha_{12}+\frac{\pi^2}{4}(4+\alpha_{12})\right]p^2+\alpha_{12}}{\Delta_{c1}\left(p^2+\omega_{c1}^2\right)} =$$

$$= \frac{F_1}{m_1\Delta_{c1}}\left[\frac{\alpha_{12}+\frac{\pi^2}{4}(4+\alpha_{12})}{p^2+\omega_{c1}^2}+\frac{\alpha_{12}}{\omega_{c1}^2}\left(\frac{1}{p^2}-\frac{1}{p^2+\omega_{c1}^2}\right)\right]$$

After some necessary transformations, the function below is found:

$$X_1(p) = \frac{F_1(p)}{m_0}\left[\frac{1}{p^2}+\frac{\pi^2}{4}.\frac{(2m_2+m_c)^2}{m_1.m_2.\Delta_{c1}\left(p^2+\omega_{c1}^2\right)}\right] \quad (10)$$

If it is considered that, the force ⌂ is applied gradually, therefore:

$$x_1(t) = \frac{F_1}{m_0}\left[\frac{t^2}{2} + \frac{\frac{\pi^2}{4}(2m_2 + m_c)^2}{m_1.m_2(\alpha_{11} + \alpha_{12} + \alpha_{11}.\alpha_{12})}(1 - \cos\omega_{c1}t)\right] = (11)$$

$$= \frac{F_1}{m_0}\left[\frac{t^2}{2} + \frac{2(m_2 + m_c)}{4.m_0.C_c}(1 - \cos\omega_{c1}t)\right]$$

Where:

$$C_c = \frac{\lambda_c.S_c}{l_1} = \frac{a_c^2.\gamma_c S_c}{l_1.g}$$

With, λ_c, S_c, γ_c, l_1: are respectively, elasticity factor, transversal section, metric weight and cable length $g = 9{,}81 m/s^2$.

To verify the validity of this mathematical model illustrated by the functional schemes of Fig. 1b and Fig. 2a, the displacement x_1 of the mass m_1 is determined by Relé method and using Lagrange equation for all system comprise haulage cable. From this fact, the relation U (z) must be used, where cable bit static deformation is taken in consideration.

$$U = x_2 + \frac{x_1 - x_2}{l_1}(l_1 - z) \quad (12)$$

Where, U: A relative displacement of cable section during localized elastic deformations to a distance z from point A.

Therefore, the kinetic energy of the system is determined as follows:

$$E_{CT} = \frac{m_1.\dot{x}_1^2}{2} + \frac{m_2.\dot{x}_2^2}{2} + E_{CC} \quad (13)$$

The derived of the Eq. 12 gives:

$$\dot{U} = \dot{X}_2 + \frac{\dot{X}_1 - \dot{X}_2}{l_1}(l_1 - Z)$$

In this case, the expression of cable bit kinetic energy is given by the following formula:

$$E_{cc} = \frac{m_c}{2l_1}\int_0^{l_1} \dot{U}^2.dz = \frac{m_c}{2l_1}\int_0^{l_1}\left[\dot{X}_2 + \frac{\dot{X}_1 - \dot{X}_2}{l_1}(l_1 - Z)\right]^2 \quad (14)$$

$$dz = E_{cc} = \frac{m_c}{3} \times \frac{\dot{X}_1^2 + \dot{X}_1.\dot{X}_2 + \dot{X}_2^2}{2}$$

Thus, the system total kinetic energy is:

$$E_{CT} = \frac{m_1.\dot{X}_1^2}{2} + \frac{m_2.\dot{X}_2^2}{2} + \frac{m_c}{3}.\frac{\dot{X}_1^2 + \dot{X}_1.\dot{X}_2 + \dot{X}_2^2}{2} \quad (15)$$

The potential energy variation of the considered cable bit according to displacements and deformations is given as follows:

$$E_P = \frac{C_c(x_1 - x_2 + S_{CT})^2}{2} - \frac{C_c S_{CT}}{2}$$

Where, S_{CT}: Static deformations before transient state.

Having the Kinetic and potential energies expressions, it is possible to determine the Lagrange equation parameters having the following form:

$$\frac{d}{dt}\left(\frac{\partial E_{CT}}{\partial \dot{X}_i}\right) - \frac{\partial E_{CT}}{\partial X_i} + \frac{\partial E_P}{\partial X_i} = F_i$$

For each coordinate, x_1 and x_2, the system equation of the following form is obtained:

$$\begin{cases} \left(m_1 + \frac{1}{3}m_c\right)\ddot{X}_1 + \frac{1}{6}m_c\ddot{X}_2 + C_c(X_1 - X_2) = F_1; \\ \left(m_2 + \frac{1}{3}m_c\right)\ddot{X}_2 + \frac{1}{6}m_c\ddot{X}_1 + C_c(X_2 - X_1) = 0 \end{cases} \quad (16)$$

If the system initial conditions are considered to be null, the expressions (16), under the operational form can be written as follows:

$$\left(m_1 + \frac{1}{3}m_c\right)p^2X_1(p) + \frac{1}{6}m_c p^2 X_2(p) + C_c[X_1(p) - X_2(p)] = F_1(p);$$

$$\left(m_2 + \frac{1}{3}m_c\right)p^2X_2(p) + \frac{1}{6}m_c p^2 X_1(p) + C_c[X_2(p) - X_1(p)] = 0.$$

where:

$$X_1(p) = \frac{F_1 + X_2(p)\left(C_c - \frac{m_c p^2}{6}\right)}{m_1 p^2 + C_c + \frac{m_c p^2}{3}};$$

$$X_2(p) = \frac{X_1(p)\left(C_c - \frac{m_c p^2}{6}\right)}{m_2 p^2 + C_c + \frac{m_c p^2}{3}}.$$

$$\text{if } m_1^1 = m_1 + \frac{1}{3} m_c \text{ and } m_2^1 = m_2 + \frac{1}{3} m_c$$

therefore, the equation below is obtained:

$$X_1(p) = \frac{F_1\left(m_2^1 p^2 + C_c\right)}{p^2\left(m_1 m_2 - \frac{1}{36} m_c^2\right)\left(p^2 + \omega_{c2}^2\right)} \quad (17)$$

$$X_2(p) = \frac{m_1^2 p^2 + C_c}{p^2\left(m_1 m_2 - \frac{1}{36} m_c^2\right)\left(p^2 + \omega_{c1}^2\right)} \quad (18)$$

The equation below is deduced:

$$\omega_{c2} = \sqrt{\frac{C_c}{m_c} \cdot \frac{1 + \alpha_{11}^{-1} + \alpha_{12}^{-1}}{\left(\alpha_{11}^{-1} + \frac{1}{3}\right)\left(\alpha_{12}^{-1} + \frac{1}{3}\right) - \frac{1}{36}}} \quad (19)$$

From Eq. (17) the equation below is obtained:

$$x(t) = \frac{F_1}{m_0}\left[\frac{t^2}{2} + \frac{(2m_2 + m_c)^2}{4 m_0 C_c} \times (1 - \cos\omega_{c2} t)\right] \quad (20)$$

Recalling that:

$$\frac{1}{p^2} \subset \frac{t^2}{2} \text{ and } \frac{1}{p^2 + \omega^2} \subset \frac{1}{\omega^2}(1 - \cos\omega t)$$

Taking in consideration:

$\frac{C_c}{m_c} = \frac{a_c^2}{l_1} = \frac{\beta_{c1}^2}{\pi^2}$; Therefore, the expression (19) becomes:

Table 1: The obtained values of Eq. 23

α_{11}	0,25	1,0	2	4	16	∞
γ_c	0.996	0.985	0.975	0.96	0.93	0.906

$$\omega_{c2} = \frac{\beta_{c1}}{\pi}\left(1 + \alpha_{11}^{-1} + \alpha_{12}^{-1}\right)^{0,5} \cdot \left[\alpha_{11}^{-1}\alpha_{12}^{-1} + \frac{1}{3}\left(\alpha_{11}^{-1} + \alpha_{12}^{-1}\right) + \frac{1}{12}\right]^{0,5} \quad (21)$$

On the other hand:

$$\omega_{c1} = \beta_{c1}\left(\alpha_{11} + \alpha_{12} + \alpha_{11}\alpha_{12}\right)^{0,5} . \left(\Delta_{c1}\right)^{0,5} \quad (22)$$

The ratio of the two frequencies ω_{c1} and ω_{c2} is:

$$\gamma_c = \frac{\omega_{c1}}{\omega_{c2}} = \left(\pi\left[1 + \frac{1}{3}(\alpha_{11} + \alpha_{12}) + \frac{\alpha_{11}.\alpha_{12}}{12}\right]\right)^{0,5} \quad (23)$$

For $\alpha_{11} = \alpha_{12}$ = 0,25.... ∞, the obtained values are illustrated in the Table 1 below:

E_{TP} : Power transformer electromotive force;
I_A : Current in the induced circuit;
ω_m, E_m : Rotational speed and electromotive force;
R_A : Equivalent resistance of rotor circuit;
T_M : Motor electromagnetic time constant;
R : Drum ray of rolling up cable;
i : Speed reducing gear transmission ratio;
J : Inertia of rotating masses des masses reduced to the motor shaft;
MC : Mechanical part of the system

RESULTS AND DISCUSSION

The obtained results presented in the above Table will lead to the method giving the more accurate results. In the case where $\alpha_{11} = \alpha_{12} = \infty$ for any value of the mass m_c, this will correspond to the absence of m_1 and m_2 masses, which means that $m_1 = m_2 = 0$ consequently, cable bit has to be considered as a simple free elastic rod to its extremities. For such system the main frequency is equal to

$$\beta_c = \frac{\pi a_c}{l_1}$$

corresponding to ω_1 frequency, obtained by the first method. The value of ω_2, with the same conditions, is determined by $2\beta_c\sqrt{3}/\pi = 1{,}103\ \omega_c$, that is ω_2 10% superior to ω_1. This rise will condition an increase in the difference between forces followed by an increase in the factors α_{11} et α_{12}.

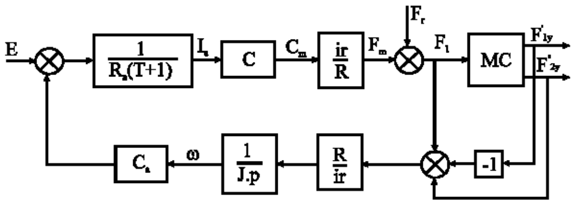

Fig. 3: Motor functional and power diagram

CONCLUSION

It can be concluded that, the mathematical model obtained from the limited values method (first method) is more accurate in describing the evolution of real processes in the system having elastic property.

This method is well adapted to the analysis of processes of electromechanical systems with complicated structures and containing elements having certain elasticity. This will favors vibration dispersions and from this, the deformations acting on the installation will disappear.

Functional diagram of the power part of driving motor a) with elastic elements and b) absolutely rigid are illustrated in Fig. 3.

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