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The Analysis of Frequency Response for Pyramidal Directional Filter Banks

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Abstract: Aiming at the frequency aliasing issue existing in Contourlet Transform, through analyzing Laplacian Pyramidal transform, making sure that neither of lowpass filter banks in Laplace pyramid transform satisfies Nyquist sampling theorem and cutoff frequency of stopband was over $\pi/2$ was obviously the basic reason to the frequency aliasing in Contourlet Transform. Based on this, a lowpass filter which satisfies Nyquist sampling theorem was designed and an Non-aliasing Contourlet Transform was proposed, namely N-Contourlet Transform. Experimental Results of hard thresholding denosing indicated that N-Contourlet Transform denoising not only give a 2 dB ($\sigma = 30$) higher PSNR than that of Contourlet Transform, but also effectively suppressed scratch after the latter denoised, providing a better visual effect.

Key words: Contourlet transform, frequency aliasing, poly-phase representation

INTRODUCTION

Recently, wavelet theory has had a significant development in the application of signal processing and its position has been also increasingly important, which benefits from the fact that wavelet can analyze 1-D piece-wise continuous signal effectively, meanwhile it is the optimal basis to represent these functions with 1-D point singularity. Nevertheless, 2-D wavelet on commonly used is the tensor product of 1-D wavelet and it only has limited directions (strictly speaking, only horizontal, vertical and diagonal 3 directions), but the singularity of 2-D image mainly reflects as edge and contour, 2-D tensor wavelet could not make full use of direction information in image, so it would not represent image optimally or sparsely. Based on this, scholars presented a series of multi-scale geometrical analysis approaches including Ridgelet transform (Candès, 1998), Curvelet transform (Candès, 1999) and Contourlet transform (Do and Vetterli, 2005) etc. to solve the representation issue of high dimensional singularity. Ridglet transform was proposed by Candes (1998), whose kernel idea was to transform line singularity into point singularity by using Radon transform, then point singularity was captured by wavelet transform. Therefore, Ridgelet transform was the optimal basis to represent image with line singularity, but it wasn't suitable to curve singularity. However, emergence of Curvelet transform solved this problem preferably. Curvelet transform was the generalization of ridgelet transformation and it realized the multiscale and multidirection decomposition on image by block ridgelet transform and subband decomposition algorithm, was the optimal basis to represent smoothing curve edge image with bi-differential and having the good time-frequency localization and nonlinear approximation. But its disadvantage was that it had a lot of redundancy (16J+1, J denotes decomposition scale), which was difficult to cope with in practical processing. In order to overcome this defect of Curvelet transform, do presented a new image decomposition method with low redundancy, namely Contourlet transform. Definitely, it implemented multiscale and multidirection decomposition on image by the integration of Laplacian Pyramidal transform (Burt and Adelson, 1983) and Directional Filter Banks (Bamberger *et al.*, 1992) and its basis function accorded with every anistropy scale relation, was undoubtedly able to represent high-dimensional singularity in image including contour and edge by extraordinarily approaching to optimal way, so it is an excellent representation for 2-D image in a real sense (Eslami and Radha, 2006; Cunha *et al.*, 2006).

However, regularity of Contourlet basis function was not high enough and the localization of its spatial domain and frequent domain was not ideal either (Lu and Do, 2003; Po and Do, 2006), obviously frequency aliasing existed, which greatly affected the application of Contourlet transform in the practical image processing. Accordingly, this study starts with the basic concept of 2D multirate decimation system (Vaidyanathan, 1993), analyzing aliasing source and influence from Directional

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filter banks and Laplacian Pyramidal transform 2 aspects. On this basis, it proposes corresponding solution, studying and realizing an Non-aliasing Contourlet transform.

CONTOURLET TRANSFORM AND ITS POLYPHASE REPRESENTATION

Contourlet transform consists of Laplacian Pyramidal decomposition and Directional filter banks, Laplacian Pyramidal transform performs multiscale decomposition on image to acquire point singularity and produce approximation subband and detail subband, where approximation subband is gained by 2-D lowpass filtering and downsampling on original image. By upsampling and lowpass filtering, approximation subband produces low frequency component whose scale is the same as original image and finally to subtract low frequency component from original image so as to obtain detail subband. Afterwards, perform the application of Directional Filter Banks in detail subband to capture direction information and integrate singular points distributing in the same direction into a coefficient. This procedure can be performed by iteration in approximation subband, normally, it can realize multiscale and multidirection decomposition of image, so this is the filter banks which have dual-iterative structure. Figure 1a denotes the principle diagram of Contourlet transform.

In order to analyze frequency aliasing characteristics of Contourlet transform further, we define Contourlet transform as the form of equivalent filter banks. For L level decomposition, every level has d_i Contourlet transforms of directional subband. The definition is as follow: for DFB in the ith level decomposition, we define its directional filter as , output directional subband as y_{ij} (n): $\{1 \le i \le L, 1 \le j \le d_i\}$, i denotes this DFB is in the ith level decomposition, j shows the directional filter banks (directional subband) correspond to the jth direction. Similarly, when x (n) and y_{ij} (n): $\{1 \le j \le d_i\}$ are regarded as input and output of a directional filter banks, $\hat{H}_{ij}$ is the definition of its equivalent directional filter. 1-level decomposition, 4-channel Contourlet transform is taken as an example, such as given in Fig. 1 (b), dashed block diagram denotes 1-level Lapalcian pyramidal decomposition, x (n) is its input, y_{10} (n) and d_1 (n) are output, which represent approximation subband and detail subband, respectively, G (z) and F (z) are 2 half-band lowpass filter. Lowpass filter G (z) and F (z) are, respectively proposed as type-I polyphase representation and type-II polyphase representation (Vaidyanathan, 1993):

$$\begin{aligned} G(z) &= G^{(0)}(z^{D_2}) + z_1^{-1}G^{(1)}(z^{D_2}) + z_2^{-1}G^{(1)}(z^{D_2}) \\ &\quad + z_1^{-1}z_2^{-1}G^{(3)}(z^{D_2}) = g^T(z^{D_2})e(z) \\ F(z) &= F^{(0)}(z^{D_2}) + z_1F^{(1)}(z^{D_2}) + z_2F^{(2)}(z^{D_2}) \\ &\quad + z_1z_2F^{(3)}(z^{D_2}) = f^T(z^{D_2})e(z^{-1}) \end{aligned} \quad (1)$$

Where,

$g(z) = [G^{(0)}(z), G^{(1)}(z), G^{(2)}(z), G^{(3)}(z)]^T$,

$f(z) = [F^{(0)}(z), F^{(1)}(z), F^{(2)}(z), F^{(3)}(z)]^T$

$e(z) = [1, z_1^{-1}, z_2^{-1}, z_1^{-1}z_2^{-1}]^T$

$e(z^{-1}) = [1, z_1, z_2, z_1z_2]^T$

If LP transform between x (n) and y_{1j} (n): $\{1 \le j \le 4\}$ and DFB are together equivalently an filter bank, its corresponding polyphase matrix is:

$$\hat{E}(z) = E(z)\left[I - f(z)g^T(z)\right] \quad (3)$$

and there is a relation as:

$$\hat{E}(z^{D_2})e(z) = \left[\hat{H}_{11}(z), \hat{H}_{12}(z), \hat{H}_{13}(z), \hat{H}_{14}(z)\right]^T \quad (4)$$

According to the relation mentioned Fig. 1a shows Contourlet transform is also equivalently Fig. 1b represents a 5-channel filter banks, in Fig. 1b G is a lowpass filter and $\hat{H}_{ij}$ is an equivalent directional filter banks. In the ideal condition, frequency support region of $\hat{H}_{ij}$ should be 2 trapezoids that are symmetric along diagonal direction as shown in Fig. 1b, other region frequency responses are

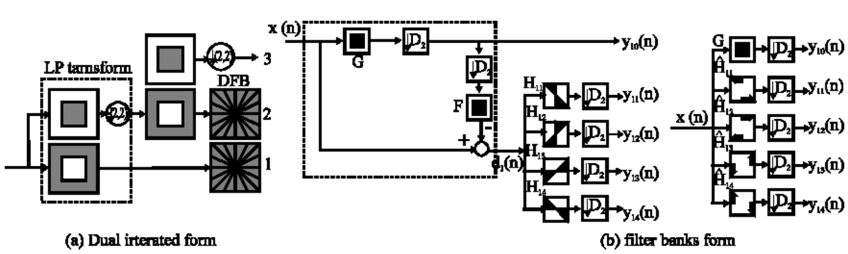

Fig. 1: Contourlet transform

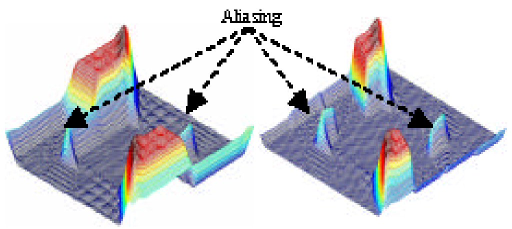

Fig. 2: The frequency spectrum of equivalent directional filter

approximately 0. Figure 2 (a) provides frequency response of Contourlet transform equivalent direction filter $\hat{H}_{11}$ which is shown in Fig. 2. We can see that outside trapezoid region there is an obvious salient, which is also frequency aliasing component. Figure 2b shows requency response of 8-channel Contourlet transform, Similarly, frequency aliasing also exists. Existence of frequency aliasing results in low frequency domain localization of Contourlet transform, which directly affects the application of Contourlet transform in fields including image denoise and compress. Then we will analyze the cause of frequency aliasing from Laplacian pyramidal transform and give out the corresponding solution.

FREQUENCY ALIASING OF CONTOURLET TRANSFORM

The direction filter H_{11} (ω) is described in Fig. 1, which shows its equivalent structure in the following Fig. 3. We assume $FH_{11}(z) = F(z)H_{11}(z)$ and express FH_{11} (z) in the form of polynomial representation:
where,

$$FH_{11}^{(0)}(z^{D_1})$$

is gotten through subsampling in advance and then upper sampling with D_1 toward FH_{11} (z), whose frequency response is given by as follows:

$$FH_{11}(z) = FH_{11}^{(0)}(z^{D_1}) + z_1^{-1}FH_{11}^{(1)}(z^{D_1}) + z_2^{-1}FH_{11}^{(2)}(z^{D_1}) + z_1^{-1}z_2^{-1}FH_{11}^{(3)}(z^{D_1}) \quad (5)$$

$$FH_{11}^{(0)}(D_2^T\omega) = \frac{1}{|D_2|}\sum_{k\in\mathcal{N}(D_2^T)} FH_{11}(\omega - 2\pi D_2^T k) \quad (6)$$

$$\begin{aligned}\hat{H}_{11}(\omega) &= H_{11}(\omega) - \frac{1}{|D_2|}G(\omega)FH_{11}^{(0)}(D_2^T\omega) \\ &= H_{11}(\omega) - \frac{1}{|D_2|}G(\omega)\sum_{k\in\mathcal{N}(D_2^T)} F(\omega - 2pD_2^T k)H_{11}(\omega - 2\pi D_2^T k)\end{aligned} \quad (7)$$

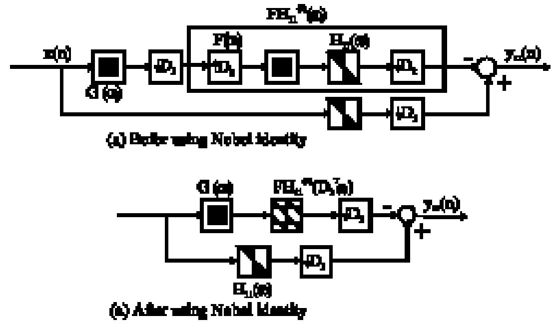

Fig. 3: The equivalent structure of directional filter $\hat{H}_{11}(\omega)$ in Fig. 2

$$\begin{aligned}\hat{H}_{11}(\omega) = {} & H_{11}(\omega)\left(1 - \frac{1}{|D_2|}G(\omega)F(\omega)\right) \\ & - \frac{1}{|D_2|}G(\omega)F(\omega-\omega_1)H_{11}(\omega-\omega_1) \\ & - \frac{1}{|D_2|}G(\omega)F(\omega-\omega_2)H_{11}(\omega-\omega_2) \\ & - \frac{1}{|D_2|}G(\omega)F(\omega-\omega_3)H_{11}(\omega-\omega_3)\end{aligned} \quad (8)$$

On the right of the equal mark in Formula (8), the second, the third and the forth items are the aliasing heaves in Fig. 2. Due to the half-band lowpasses G (ω) and F (ω), heaves are caused by the superposition of modulation direction filter H_{11} $(\omega-\omega_i)$'s pass-band and G (ω), F $(\omega-\omega_i)$'s. Therefore, to eliminate those heaves, one way is to try to reduce the band superposition among these filters, especially the transition-band superposition. In LP transform, G (ω) and F (ω) are 2-dimension separable lowpasses filter (Fig. 3). If they both satisfy the Nyquist sampling law, that means the ranges of pass-band and stop-band are both in $[-\pi/2, \pi/2]^2$, G (ω) and F $(\omega-\omega_i)$ won't superpose and their product is zero constantly through the whole range $[-\pi, \pi]^2$. With this method, the aliasing items could be eliminated.

Hence, G (ω) and F (ω) should satisfy the following conditions:

$$\begin{cases} G(\omega)\approx 0,\ F(\omega)\approx 0 & \omega_s \le |\omega| \le \pi \\ G(\omega) > 0,\ F(\omega) > 0 & |\omega| \le \omega_p \\ \omega_p < \omega_s < p/2 \end{cases} \quad (9)$$

Where,
ω_p = Denotes the pass-band cutoff frequency.
ω_s = Denotes the stop-band cutoff frequency.

Formula (9) indicates that in order to ensure the anti-aliasing, the range of lowpass's pass-band and transition-band have to be in $[-\pi/2, \pi/2]^2$ and the frequency response should approximate zero within the stop-band region.

THE DESIGN OF THE NON-ALIASING CONTOURLET TRANSFORM

In LP transform, if lowpasses filter G (ω) and F (ω) don't satisfy Nyquist sampling law, the filter's response will still exist beyond $[-\pi/2, \pi/2]^2$. According to the former section, we could affirm that is the primary reason leading to the frequency aliasing of Contourlet transform. Thus, to design the Non-aliasing Contourlet transform (N-Contourlet), we must ensure that G (ω) and F (ω) satisfy Formula (9), i.e., G (ω) and F (ω)'s stop-band cutoff frequencies should be less than π/2 and their frequency response should approximate zero in stop-band region. We design the N-Contourlet transform with the similar method of designing the Contourlet transform. Multi-scale decomposition and direction decomposition are carrying out separately. In the multi-scale decomposition, we define lowpass H_0 (ω) and highpass G_0 (ω) directly to implement the acquisition of appoximation sub-band and detail sub-band, assuming the H_0 (ω)'s pass-band cutoff frequency is π/4, its stop-band cutoff frequency is π/2 and the transition region is [π/4, π/2]. Supposing the frequency aliasing caused by lower-sampling could be eliminated absolutely, to guarantee the entire re-construction, H_0 (ω) and G_0 (ω) must satisfy the equation: $|H_0(\omega)|^2 + |G_0(\omega)|^2 = 1$. Direction decomposition is implemented through DFB, whose conformation method is consistent with the one in Contourlet transform. Concretely speaking, it uses the exaltation-structure based PKVA fan filter banks as the basic modules and cascade them in a appropriate way. Defining highpass directly could avoid the sampling operation in the process of generating detail sub-band in LP transform. Image directly cascades DFB after passing through the highpass. So far as the highpass satisfies the sampling law, the frequency aliasing caused by multi-scale decomposition won't be taken into those direction sub-bands, so that the aliasing could be restrained in a further step.

Figure 4 is the frequency support of a N-Contourlet transform direction filter. Compared with Fig. 2, we can find that N-Contourlet transform won't cause obvious frequency aliasing. Furthermore, the stop-band region is quite flat and the heaves in Contourlet transform are observably restrained. That indicates the localization of N-Contourlet transform's frequency region is also superior to the Contourlet transform's, with better direction selectivity.

Fig. 4: The frequency spectrum of equivalent directional filter for N-contourlet transform

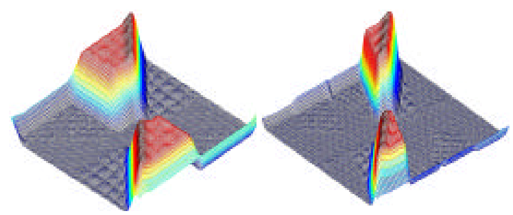

Fig. 5: The zoomed denoised lena with different transform (σ =40)

EXPERIMENTS

For simulation, we choose image Lena as the testing image. Gaussian white noise at different levels is added into the image previously. We use 9/7 wavelet, Contourlet transform and N-Contourlet transform to act on the image, respectively for the hard threshold denoising experiment. All these 4 transforms contain 4-layer decompositions. For the later 2 transforms, the number of the direction sub-bands in every layer's decomposition is [16, 16, 8, 8]. When the niose's variance is beyond 40, compared to 9/7 wavelet and Contourlet transform, the re-construction image's PSNR value could be enhance 3.32dB and 2.46 dB by using N-Contourlet transform denoising. We can also conclude from the figure that with the method of N-Contourlet transform denoising, the detail and texture of the image are well kept and the nisks caused by Contourlet transform denoising are well restrained. Its vision impression is obvious superior to the ones of Contourlet transform and 9/7 wavelet (Fig. 5).

CONCLUSION

Aiming at the frequency aliasing issue existing in Contourlet Transform, this study starts from multirate filter banks, through analysis of frequency aliasing in Laplacian pyramidal transform, indicates the basic reason to frequency aliasing in Contourlet Transform is neither of lowpass filters in LP transform satisfies Nyquist sampling theorem and on this basis it defines an Non-aliasing Contourlet transform which is suitable to Nyquist sampling theorem, namely N-Contourlet transform. N-Contourlet transform can effectively suppress the frequency aliasing in Contourlet transform, absolutely its basis function has a higher regularity and a better directional selectivity. The hard thresholding denoise experiment on standard image shows that N-Contourlet transform has a better performance than Contourlet transform, especially, for images have rich texture and detail, the effect of N-Contourlet transform is greatly better.

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