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Analysis of Availability and Reliability for Repairable Parallel Systems with Different Failure Rates

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Abstract: This study presents Markov models for analysis of availability and reliability for parallel repairable system subject to three types of failure rates (e.g., human, hardware and software) and common cause failure rate. The problem addressed is how applying a Continuous-Time Markov Chain (CTMC) to evaluate availability, reliability and Mean Time to System Failure (MTTFs) for parallel system with repair. In this study, assumed that the working time and the repair time of each component are both arbitrary distributed (e.g., Weibull, exponentially distributed). The Markov method is used to develop generalized expressions for system state probabilities, system availability, system reliability and system mean time to failure. A numerical example is presented in order to illustrate the performance of the model.

Key words: Repairable parallel system, arbitrary distribution for failure/repair rates, Markov models, availability, reliability, component

INTRODUCTION

Markov chain is a stochastic process that have a finite states at time t under consideration that the chain runs only through a continuous time, the basic assumption of Markov chain is the transition from the current state of the system is determined only by the present state and not by the previous state or the time at which it reached the present state.

Parallel can be used to increase the reliability of a system without any change in the reliability of the individual components that form the system. The probability of failure or unreliability for a system with n statistically-independent parallel components is the probability that component 1 fails and component 2 fails and all of the other components in the system fail. Therefore, in a parallel system, all n components must fail for the system to fail (Xie *et al*., 2004). The problem of evaluating the availability and reliability of the parallel system has been the subject of many studies throughout the literature (Pan and Nonaka, 1995; Ebeling, 2000; Kolowrocki, 1994; Kwiatuszewska-Sarnecka, 2001).

System reliability depends not only on the reliabilities of components in the system but also on their interactions, viz., the dependencies among them. Generally, in a system, not only statistically-independent failures but also statistically-dependent failures among components can occur thus there are many studies (Jack, 1986; Dorre, 1992; Lydersen, 1992; Rausand and Hoyland, 1994) where the statistically-dependent among components are taken into account in system reliability and availability analysis but in which the failure and repair rates assumed constant. Whereas, from a practical viewpoint, the constant failure rate assumption for components has been and is repeatedly challenged by knowledgeable reliability practitioners. Therefore, there are other studies which handled the problem of time-varying failure rates, among which all concerned repairable system did not involve statistically-dependent failures (Hassett *et al*., 1995; Amiri *et al*., 2009). In most cases, however, to combine statistically-dependent failures and time-varying failure and repair rates in system reliability and availability analysis is the most appropriate for real system (Zhang and Horigome, 2001).

This study shows a CTMC for performing availability, reliability and Mean Time to system Failure (MTTFs) analysis of n-identical component multiple repairable system with considering time varying (constant) three types of failure rates and common-cause failure rate. A parallel system of three component for an example is given to show the performance of the model.

ASSUMPTIONS

- The system is composed of n identical and independent components which are connected in parallel
- Each component is failed with failure rate λ_i (t), i = 1, 2, 3
- The failed component is repaired with repair rate μ_i (t), I = 1, 2, 3
- Common-cause failure rate for fully operating system λ_c (t) and μ (t) repair rate of the system when it failed due to a common-cause failure rate

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- The repaired component/system is as good as new
- At time t = 0, all component are up and the system failed when all components are down

THE BASIC MODEL

State probabilities: The state probabilities for the system can be viewed as a result of solving a the set of first order liner differential equations given by the following identity (Hoyland and Rausand, 2004):

$$\frac{dP_t(j)}{dt} = -[\sum_{i=0}^{j-1} b_{ji} + \sum_{i=j+1}^{n} a_{ji}] P_t(j) + \sum_{i=0}^{j-1} a_{ij} P_t(i) + \sum_{i=j+1}^{n} b_{ij} P_t(i) \quad (1)$$

where probability of state j at time t, α_{ji} failure rate of a component will be failed through transition from state j to state l, repair rate of a component will be good through transition from state j to state l.b_{ji}

Let (m_1, m_2, m_3)be the state of the system, where m_1, m_2 and m_3 represent the number of failed components due to failure of type I, type II and type III, respectively. Let P (m_1, m_2, m_3) be the probability of being in state (m_1, m_2, m_3) at time t and P_t(c) be the probability of being in the critical case at time t.

The state probabilities $0 \le m_1+m_2+m_3 \le$) where P (m_1, m_2, m_3) can be viewed as result of solving the set of first order liner differential equations associated with Eq. 1 is as follows:

$$\frac{dP_t(0,0,0)}{dt} = -[n\sum_{i=1}^{3}\lambda_i(t) + \lambda_c(t)]P_t(0,0,0) + \mu_1(t)P_t(1,0,0) + \mu_2(t)P_t(0,1,0) + \mu_3(t)P_t(0,0,1) + \mu_c(t)P_t(c), \quad (2a)$$

$$\begin{aligned}\frac{dP_t(m_1,m_2,m_3)}{dt} = & -[(n-\sum_{i=1}^{3}m_i)\sum_{i=1}^{3}\lambda_i\ (t) + \sum_{i=1}^{3}m_i\mu_i(t)] \\ & P_t(m_1,m_2,m_3) + (n-\sum_{i=1}^{3}m_i+1)[\delta(m_1) \\ & \lambda_1(t)P_t(m_1-1,m_2,m_3) + \delta(m_2)\lambda_2(t) \\ & P_t(m_1,m_2-1,m_3) + \delta(m_3)\lambda_3(t) \\ & P_t(m_1,m_2,m_3-1)] + (m_1+1)\delta(m_1+1) \\ & \mu_1(t)P_t(m_1+1,m_2,m_3) + (m_2+1)\,\delta(m_2+1) \\ & \mu_2(t)P_t(m_1,m_2+1,m_3) + (m_3+1)\delta(m_3+1) \\ & \mu_3(t)P_t(m_1,m_2,m_3+1),\end{aligned} \quad (2b)$$

and:

$$\frac{dP_t(c)}{dt} = -\mu_c(t)P_t(c) + \lambda_c(t)P_t(0,0,0) \quad (2c)$$

Where:

$$\delta(m_i) = \begin{cases}1 & \text{for } 1 \le m_i \le n \\ 0 & \text{for } m_i = 0\end{cases} \quad \text{for } i = 1, 2, 3$$

$$\delta(m_i+1) = \begin{cases}1 & \text{for } 1 \le m_1+m_2+m_3+1 \le n \\ 0 & \text{for } m_1+m_2+m_3+1 \succ n\end{cases} \quad \text{for } i = 1, 2, 3$$

To evaluate the system availability:

$$\delta(m_i+1) = \begin{cases}1 & \text{for } 1 \le m_1+m_2+m_3+1 \le n \\ 0 & \text{for } m_1+m_2+m_3+1 \succ n\end{cases} \quad \text{for } i = 1,2,3 \quad (3)$$

To evaluate the system reliability:

$$\delta(m_i+1) = \begin{cases}1 & \text{for } 1 \le m_1+m_2+m_3+1 \prec n \\ 0 & \text{for } m_1+m_2+m_3+1 \ge n\end{cases} \quad \text{for } i = 1, 2, 3 \quad (4)$$

At time t = 0 Eq. 2a-c with the following initial conditions:

$$\begin{cases}P_0(0,0,0) = 1, \\ P_0(m_1,m_2,m_3) = 0 \quad \text{for } 1 \le m_1+m_2+m_3 \le n \\ \text{and } P_0(c) = 0\end{cases} \quad (5)$$

SYSTEM AVAILABILITY

By definition, the general form solution of parallel system availability at time t is given by:

$$A(t) = \sum_{m=0}^{n-1} P_t(m_1,m_2,m_3), \quad m = m_1 + m_2 + m_3 \quad (6)$$

Here, according to the value of n we use the numerical method based on the Runge Kutta method to find the solution time-varying (constant) failure and repair rates in system availability A (t) of Eq. 6 with the initial condition Eq. 5.

The steady state availability is one of the more important factors in specifications and choice of hardware The steady state availability of the system is the limit of the instantaneous availability function as time approaches infinity. Therefore:

$$A(\infty) = \lim_{t\to\infty} A(t) \quad (7)$$

However, in this case we have:

$$\frac{dP_t(m_1,m_2,m_3)}{dt} = 0,\ 0 \le m_1+m_2+m_3 \le n, \frac{dP_t(c)}{dt} = 0$$

$$P_t(m_1,m_2,m_3) \to P(m_1,m_2,m_3) \text{ and } P_t(c) \to P(c)$$

The parallel system steady state availabilityA (∞) becomes:

$$A(\infty) = \sum_{m=0}^{n-1} P(m_1, m_2, m_3), \quad m = m_1 + m_2 + m_3 \tag{8}$$

Where:

$$\begin{cases} P(m_1, m_2, m_3) = [\frac{m!}{m_1!\, m_2!\, m_3!}] \binom{n}{m} \\ [(\lambda_1^{m_1}\lambda_2^{m_2}\lambda_3^{m_3})/(\mu_1^{m_1}\mu_2^{m_2}\mu_3^{m_3})]\; P(0,0,0) \\ P(c) = [\frac{\lambda_c}{\mu_c}]\, P(0,0,0) \\ \sum_{m=0}^{n} P(m_1, m_2, m_3) + P(c) = 1, \\ m = m_1 + m_2 + m_3 \end{cases} \tag{9}$$

SYSTEM RELIABILITY

To obtain the reliability of the system, we consider that the set of failed states are absorbing states.
Now let:

$$P_t(m_1, m_2, m_3) \rightarrow \tilde{P}_t(m_1, m_2, m_3)$$

The general form solution of parallel system reliability at time t is obtained as:

$$R(t) = \sum_{m=0}^{n-1} \tilde{P}_t(m_1, m_2, m_3), \quad m = m_1 + m_2 + m_3 \tag{10}$$

Here, according to the value of n we use the numerical method based on the Runge Kutta method to find the solution time-varying failure and repair rates in system reliability R (t) of Eq. 10 with the initial condition Eq. 5.

The method of Laplace was used transform for Eq. 2a-c according to Eq. 4 when failure and repair rates assumed constant by combining the initial condition Eq. 5, the numerical solution for R (t) can be obtained through inverse Laplace transform of state probabilities:

$$\tilde{P}_s(m_1, m_2, m_3) = \int_0^\infty e^{-st}\tilde{P}_t(m_1, m_2, m_3)\,dt$$

Where:

$$s\tilde{P}_s(m_1, m_2, m_3) - \tilde{P}_{t=0}(m_1, m_2, m_3) = \int_0^\infty e^{-st}(d\tilde{P}_t(m_1, m_2, m_3)/dt)\,dt$$

Therefore:

$$R(t) = L^{-1}(R(s)) = \sum_{m=0}^{n-1} L^{-1}(\tilde{P}_s(m_1, m_2, m_3)), \quad m = m_1 + m_2 + m_3 \tag{11}$$

The MTTFs is defined as the mean time elapsed in successful states before entering a failed absorbing states:

$$\begin{aligned} \text{MTTFs} &= \int_0^\infty R(t)\,dt = \lim_{s\to 0}\int_0^\infty e^{-st}\,R(t)\,dt \\ &= \lim_{s\to 0}\sum_{m=0}^{n-1} \tilde{P}_s(m_1, m_2, m_3) \\ &= \sum_{m=0}^{n-1} \tilde{P}_{s=0}(m_1, m_2, m_3), \\ m &= m_1 + m_2 + m_3 \end{aligned} \tag{12}$$

AN ILLUSTRATIVE EXAMPLE

In the Table 1 the above procedure is applied to availability, reliability and mean time to failure for parallel system with repair when n = 3.

And state (c) represent failed state for three components due to common-cause failure.

Table 1: Event space of states for the system of failure

$m = m_1+m_2+m_3$	(m_1, m_2, m_3)
0	(0, 0, 0)
1	(1, 0, 0), (0, 1, 0), (0, 0, 1)
2	(2, 0, 0), (0, 2, 0), (0, 0, 2), (1, 1, 0), (1,0, 1), (0, 1, 1)
3	(3, 0, 0), (0, 3, 0), (0, 0, 3), (2, 1, 0), (1, 2, 0), (2, 0, 1), (1, 0, 2), (0, 2, 1), (0, 1, 2), (1, 1, 1)

SYSTEM AVAILABILITY

According to Eq. 2a-c the set of differential equations associated with the system availability state are:

$$\frac{dP_t(0,0,0)}{dt} = -[3\sum_{i=1}^{3}\lambda_i(t) + \lambda_c(t)]P_t(0,0,0) + \mu_1(t)P_t(1,0,0) + \mu_2(t)P_t(0,1,0) + \mu_3(t)P_t(0,0,1) + \mu_c(t)P_t(c)$$

$$\frac{dP_t(1,0,0)}{dt} = -[2\sum_{i=1}^{3}\lambda_i(t) + \mu_1(t)]P_t(1,0,0) + 2\mu_1(t)P_t(2,0,0) + \mu_2(t)P_t(1,1,0) + \mu_3(t)P_t(1,0,1) + 3\lambda_1(t)P_t(0,0,0)$$

$$\frac{dP_t(0,1,0)}{dt} = -[2\sum_{i=1}^{3}\lambda_i(t) + \mu_2(t)]P_t(0,1,0) + \mu_1(t)P_t(1,1,0) + 2\mu_2(t)P_t(0,2,0) + \mu_3(t)P_t(0,1,1) + 3\lambda_2(t)P_t(0,0,0)$$

$$\frac{dP_t(0,0,1)}{dt} = -[2\sum_{i=1}^{3}\lambda_i(t) + \mu_3(t)]P_t(0,0,1) + \mu_1(t)P_t(1,0,1) + \mu_2(t)P_t(0,1,1) + 2\mu_3(t)P_t(0,0,2) + 3\lambda_3(t)P_t(0,0,0)$$

$$\frac{dP_t(2,0,0)}{dt} = -[\sum_{i=1}^{3} \lambda_i(t) + 2\mu_1(t)]P_t(2,0,0) + 3\mu_1(t)P_t(3,0,0) + \mu_2(t)P_t(2,1,0) + \mu_3(t)P_t(2,0,1) + 2\lambda_1(t)P_t(1,0,0)$$

$$\frac{dP_t(0,2,0)}{dt} = -[\sum_{i=1}^{3} \lambda_i(t) + 2\mu_2(t)]P_t(0,2,0) + \mu_1(t)P_t(1,2,0) + 3\mu_2(t)P_t(0,3,0) + \mu_3(t)P_t(0,2,1) + 2\lambda_2(t)P_t(0,1,0)$$

$$\frac{dP_t(0,0,2)}{dt} = -[\sum_{i=1}^{3} \lambda_i(t) + 2\mu_3(t)]P_t(0,0,2) + \mu_1(t)P_t(1,0,2) + \mu_2(t)P_t(0,1,2) + 3\mu_3(t)P_t(0,0,3) + 2\lambda_3(t)P_t(0,0,1)$$

$$\frac{dP_t(1,1,0)}{dt} = -[\sum_{i=1}^{3} \lambda_i(t) + \mu_1(t) + \mu_2(t)]P_t(1,1,0) + 2\mu_1(t) P_t(2,1,0) + 2\mu_2(t)P_t(1,2,0) + \mu_3(t)P_t(1,1,1) + 2\lambda_1(t)P_t(0,1,0) + 2\lambda_2(t)P_t(1,0,0)$$

$$\frac{dP_t(1,0,1)}{dt} = -[\sum_{i=1}^{3} \lambda_i(t) + \mu_1(t) + \mu_3(t)]P_t(1,0,1) + 2\mu_1(t) P_t(2,0,1) + \mu_2(t)P_t(1,1,1) + 2\mu_3(t)P_t(1,0,2) + 2\lambda_1(t)P_t(0,0,1) + 2\lambda_3(t)P_t(1,0,0)$$

$$\frac{dP_t(0,1,1)}{dt} = -[\sum_{i=1}^{3} \lambda_i(t) + \mu_2(t) + \mu_3(t)]P_t(0,1,1) + \mu_1(t) P_t(1,1,1) + 2\mu_2(t)P_t(0,2,1) + 2\mu_3(t)P_t(0,1,2) + 2\lambda_2(t)P_t(0,0,1) + 2\lambda_3(t)P_t(0,1,0)$$

$$\frac{dP_t(3,0,0)}{dt} = -3\mu_1(t)P_t(3,0,0) + \lambda_1(t)P_t(2,0,0)$$

$$\frac{dP_t(0,3,0)}{dt} = -3\mu_2(t)P_t(0,3,0) + \lambda_2(t)P_t(0,2,0)$$

$$\frac{dP_t(0,0,3)}{dt} = -3\mu_3(t)P_t(0,0,3) + \lambda_3(t)P_t(0,0,2)$$

$$\frac{dP_t(2,1,0)}{dt} = -[2\mu_1(t) + \mu_2(t)]P_t(2,1,0) + \lambda_1(t) P_t(1,1,0) + \lambda_2(t)P_t(2,0,0)$$

$$\frac{dP_t(1,2,0)}{dt} = -[\mu_1(t) + 2\mu_2(t)]P_t(1,2,0) + \lambda_1(t) P_t(0,2,0) + \lambda_2(t)P_t(1,1,0)$$

$$\frac{dP_t(2,0,1)}{dt} = -[2\mu_1(t) + \mu_3(t)]P_t(2,0,1) + \lambda_1(t) P_t(1,0,1) + \lambda_3(t)P_t(2,0,0)$$

$$\frac{dP_t(1,0,2)}{dt} = -[\mu_1(t) + 2\mu_3(t)]P_t(1,0,2) + \lambda_1(t) P_t(0,0,2) + \lambda_3(t)P_t(1,0,1)$$

$$\frac{dP_t(0,2,1)}{dt} = -[2\mu_2(t) + \mu_3(t)]P_t(0,2,1) + \lambda_2(t) P_t(0,1,1) + \lambda_3(t)P_t(0,2,0)$$

$$\frac{dP_t(0,1,2)}{dt} = -[\mu_2(t) + 2\mu_3(t)]P_t(0,1,2) + \lambda_2(t)P_t(0,0,2) + \lambda_3(t)P_t(0,1,1)$$

$$\frac{dP_t(1,1,1)}{dt} = -\sum_{i=1}^{3}\mu_i(t)P_t(1,1,1) + \lambda_1(t)P_t(0,1,1) + \lambda_2(t)P_t(1,0,1) + \lambda_3(t)P_t(1,1,0)$$

$$\frac{dP_t(c)}{dt} = -\mu_c(t)P_t(c) + \lambda_c(t)P_t(0,0,0)$$

With the initial conditions $P_0(0, 0, 0) = 1$ and all other initial probabilities are equal to zero. The availability function which in this case is given by:

by: $$A(t) = P_t(0,0,0) + P_t(1,0,0) + P_t(0,1,0) + P_t(0,0,1) + P_t(2,0,0) + P_t(0,2,0) + P_t(0,0,2) + P_t(1,1,0) + P_t(1,0,1) + P_t(0,1,1)$$

The working time and the repair time of each component are both Weibull distributed as a particular case, let us assume that the working time and the repair time of each component are both Weibull distributed. We can then write:

$$\lambda_1(t) = 1.1\, t^{0.1} \qquad \mu_1(t) = 1.4\, t^{0.4}$$
$$\lambda_2(t) = 1.2\, t^{0.2} \qquad \mu_2(t) = 1.5\, t^{0.5}$$
$$\lambda_3(t) = 1.3\, t^{0.3} \qquad \mu_3(t) = 1.6\, t^{0.6}$$
$$\lambda_c(t) = 1.7\, t^{0.7} \qquad \mu_c(t) = 1.8\, t^{0.8}$$

Using Maple program, the system availability against time is shown in Fig. 1 with numerical solutions based on Runge-Kutta method.

The working time and the repair time of each component are both exponentially distributed as a particular case, let us assume that the working time and the repair time of each component are both exponentially distributed. We can then write:

$$\lambda_1(t) = \lambda_1 = 0.007 \qquad \mu_1(t) = \mu_1 = 0.7$$
$$\lambda_2(t) = \lambda_2 = 0.006 \qquad \mu_2(t) = \mu_2 = 0.6$$
$$\lambda_3(t) = \lambda_3 = 0.005 \qquad \mu_3(t) = \mu_3 = 0.5$$
$$\lambda_c(t) = \lambda_c = 0.008 \qquad \mu_c(t) = \mu_c = 0.8$$

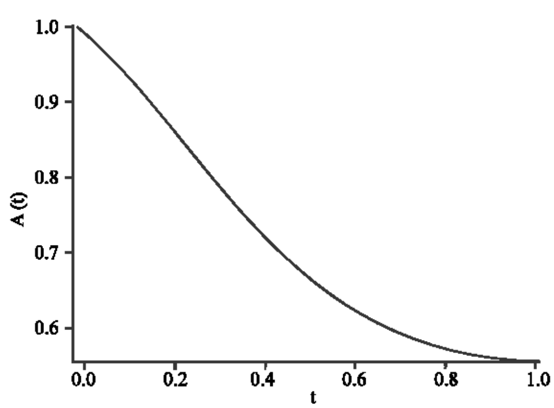

Fig. 1: The availability function A(t) versus the time t

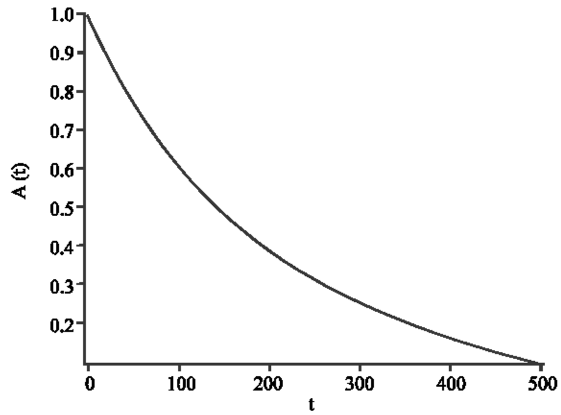

Fig. 2: The availability function A(t) versus the time t

Using Maple program, the system availability against time is shown in Fig. 2 with numerical solutions based on Runge-Kutta method.

STEAD STATE AVAILABILITY

By using the expressions of Eq. 8, 9, the parallel system steady state availability A (∞) is calculate:

$$A(\infty) = \sum_{m=0}^{2} P(m_1, m_2, m_3),\ m = m_1 + m_2 + m_3$$

Where:

$$P(1,0,0) = \frac{3\lambda_1}{\mu_1} P(0,0,0),\quad P(0,1,0) = \frac{3\lambda_2}{\mu_2} P(0,0,0),\quad P(0,0,1) = \frac{3\lambda_3}{\mu_3} P(0,0,0),$$

$$P(2,0,0) = \frac{3\lambda_1^2}{\mu_1^2} P(0,0,0),\ P(0,2,0) = \frac{3\lambda_2^2}{\mu_2^2} P(0,0,0),$$

$$P(0,0,2) = \frac{3\lambda_3^2}{\mu_3^2} P(0,0,0),$$

$$P(1,1,0) = \frac{6\lambda_1\lambda_2}{\mu_1\mu_2} P(0,0,0),\quad P(1,0,1) = \frac{6\lambda_1\lambda_3}{\mu_1\mu_3} P(0,0,0),\quad P(0,1,1) = \frac{6\lambda_2\lambda_3}{\mu_2\mu_3} P(0,0,0),$$

$$P(3,0,0) = \frac{\lambda_3^3}{\mu_1^3} P(0,0,0),\quad P(0,3,0) = \frac{\lambda_2^3}{\mu_2^3} P(0,0,0),$$

$$P(0,0,3) = \frac{\lambda_3^3}{\mu_3^3} P(0,0,0),$$

$$P(2,1,0) = \frac{3\lambda_1^2\lambda_2}{\mu_1^2\mu_2} P(0,0,0),\quad P(2,0,1) = \frac{3\lambda_1^2\lambda_3}{\mu_1^2\mu_3} P(0,0,0),\quad P(1,2,0) = \frac{3\lambda_1\lambda_2^2}{\mu_1\mu_2^2} P(0,0,0),$$

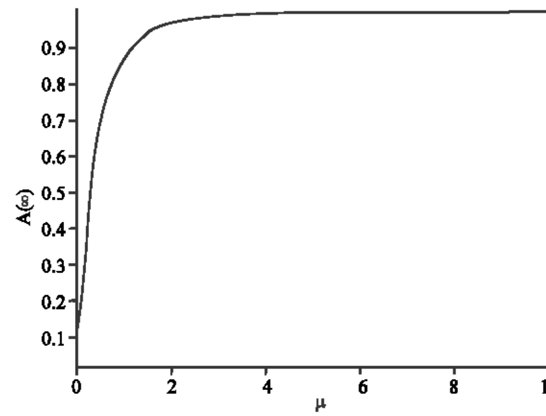

Fig. 3: Steady state availability variation with u (utilization factor)

$$P(1,0,2) = \frac{3\lambda_1\lambda_3^2}{\mu_1\mu_3^2} P(0,0,0),\quad P(0,2,1) = \frac{3\lambda_2^2\lambda_3}{\mu_2^2\mu_3} P(0,0,0),\quad P(0,1,2) = \frac{3\lambda_2\lambda_3^2}{\mu_2\mu_3^2} P(0,0,0),$$

$$P(1,1,1) = \frac{6\lambda_1\lambda_2\lambda_3}{\mu_1\mu_2\mu_3} P(0,0,0),\ P(c) = \frac{\lambda_c}{\mu_c} P(0,0,0),$$

$$\sum_{m=0}^{3} P(m_1, m_2, m_3) + P(c) = 1$$

Which is $u = \mu_1/\lambda_1$. The steady state availability is strongly effected by the ratio varis with A (∞) which is commonly termed utilization factor. Figure 3 Shows how A (∞) varies with utilization factor, when $\mu_2/\lambda_2 = \mu_3/\lambda_3 = \mu_c/\lambda_c = 100$.

SYSTEM RELIABILITY

In order to obtain the reliability function associated with the model Eq. 2a-c and Eq. 4, therefore define states (m_1, m_2, m_3) for $(m_1+m_2+m_3) = 3$ to be an absorbing states and set all departure rates from these states equal to zero. Then the reliability of the parallel system of three components is defined by:

$$R(t) = \sum_{m=0}^{2} \tilde{P}_t(m_1, m_2, m_3),$$

$$m = m_1 + m_2 + m_3$$

The working time and the repair time of each component are both Weibull distributed as a particular case, let us assume that the working time and the repair time of each component are both Weibull distributed and given the same data in availability.

Using Maple program, the system reliability against time is shown in Fig. 4 with numerical solutions based on

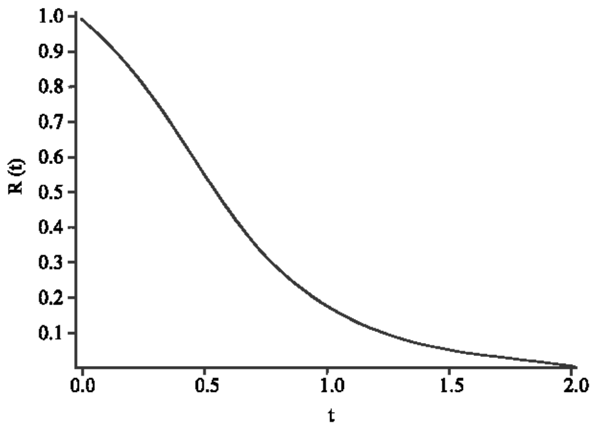

Fig. 4: The reliability function R(t) versus the time t

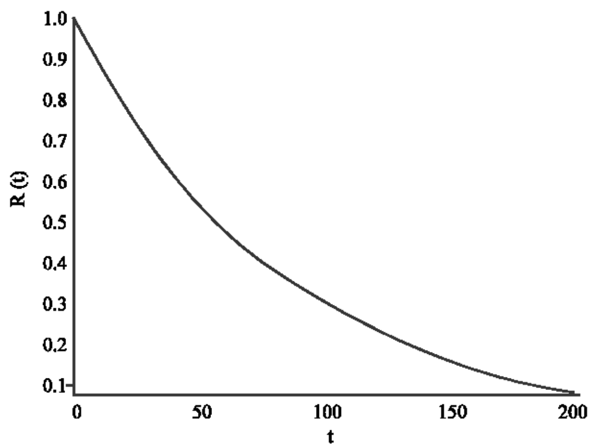

Fig. 5: The reliability function R(t) versus the time t

Runge-Kutta method.The working time and the repair time of each component are both exponentially distributed as a particular case, let us assume that the working time and the repair time of each component are both exponentially distributed and given the same data in availability.

Using Maple program, the system reliability against time is shown in Fig. 5 with numerical solutions based on Laplace transform according to Eq. 11.

MTTF OF THE PARALLEL SYSTEM

The mean time to system failure can be obtained by using Eq. 2a-2c , 4 and 12 when n = 3 and the working time and the repair time of each component are both exponentially distributed:

$$-1 = -[3\sum_{i=1}^{3}\lambda_i + \lambda_c]\tilde{P}_{s=0}(0,0,0) + \mu_1\tilde{P}_{s=0}(1,0,0) + \mu_2\tilde{P}_{s=0}(0,1,0) + \mu_3\tilde{P}_{s=0}(0,0,1)$$

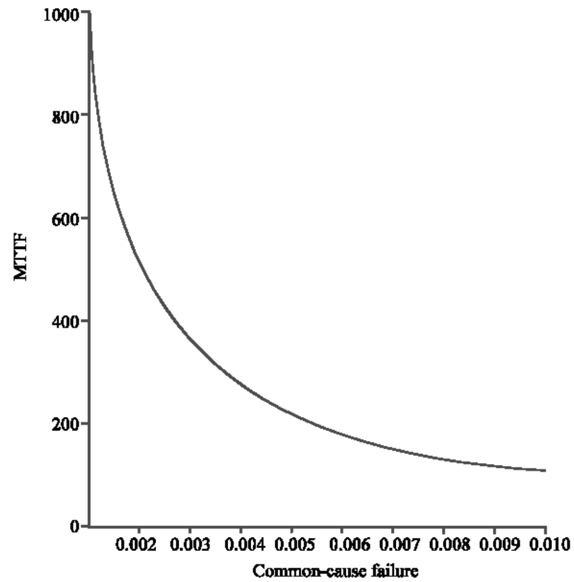

Fig. 6: MTTF of the parallel system against λ_c

$$0 = -[2\sum_{i=1}^{3}\lambda_i + \mu_1]\tilde{P}_{s=0}(1,0,0) + 2\mu_1\tilde{P}_{s=0}(2,0,0) + \mu_2\tilde{P}_{s=0}(1,1,0) + \mu_3\tilde{P}_{s=0}(1,0,1) + 3\lambda_1\tilde{P}_{s=0}(0,0,0)$$

$$0 = -[2\sum_{i=1}^{3}\lambda_i + \mu_2]\tilde{P}_{s=0}(0,1,0) + \mu_1\tilde{P}_{s=0}(1,1,0) + 2\mu_2\tilde{P}_{s=0}(0,2,0) + \mu_3\tilde{P}_{s=0}(0,1,1) + 3\lambda_2\tilde{P}_{s=0}(0,0,0)$$

$$0 = -[2\sum_{i=1}^{3}\lambda_i + \mu_3]\tilde{P}_{s=0}(0,0,1) + \mu_1\tilde{P}_{s=0}(1,0,1) + \mu_2\tilde{P}_{s=0}(0,1,1) + 2\mu_3\tilde{P}_{s=0}(0,0,2) + 3\lambda_3\tilde{P}_{s=0}(0,0,0)$$

$$0 = -[\sum_{i=1}^{3}\lambda_i + 2\mu_1]\tilde{P}_{s=0}(2,0,0) + 2\lambda_1\tilde{P}_{s=0}(1,0,0)$$

$$0 = -[\sum_{i=1}^{3}\lambda_i + 2\mu_2]\tilde{P}_{s=0}(0,2,0) + 2\lambda_2\tilde{P}_{s=0}(0,1,0)$$

$$0 = -[\sum_{i=1}^{3}\lambda_i + 2\mu_3]\tilde{P}_{s=0}(0,0,2) + 2\lambda_3)\tilde{P}_{s=0}(0,0,1)$$

$$0 = -[\sum_{i=1}^{3}\lambda_i + \mu_1 + \mu_2]\tilde{P}_{s=0}(1,1,0) + 2\lambda_1\tilde{P}_{s=0}(0,1,0) + 2\lambda_2\tilde{P}_{s=0}(1,0,0)$$

$$0 = -[\sum_{i=1}^{3}\lambda_i + \mu_1 + \mu_3]\tilde{P}_{s=0}(1,0,1) + 2\lambda_1\tilde{P}_{s=0}(0,0,1) + 2\lambda_3\tilde{P}_{s=0}(1,0,0)$$

$$0 = -[\sum_{i=1}^{3}\lambda_i + \mu_2 + \mu_3]\tilde{P}_{s=0}(0,1,1) + 2\lambda_2\tilde{P}_{s=0}(0,0,1) + 2\lambda_3\tilde{P}_{s=0}(0,1,0)$$

Where:

$$\lambda_1 = 0.007,\ \lambda_2 = 0.006,\ \lambda_3 = 0.005,$$
$$\mu_1 = 0.7,\ \mu_2 = 0.6,\ \text{and}\ \mu_3 = 0.5$$

Using Maple program, the MTTFs:

$$= \sum_{m=0}^{2} \tilde{P}_{s=0}(m_1, m_2, m_3),\ m = m_1 + m_2 + m_3$$

The MTTF of the parallel system against λ_c is shown in Fig. 6 with numerical solutions for equations given above.

CONCLUSION

The main objective for this study was to offer a methodology for analyzing repairable parallel system availability, reliability and MTTF with n identical components and different failure rates, Markov models and time varying failure and repair rates concepts have been employed to develop the methodology for the availability and reliability of such systems. The problem of evaluating the availability and reliability depending on size of the parallel system was formulated in set of first order liner differential equations form, which seems convenient for computation with software packages like Maple. Numerical solutions based on Runge-Kutta and Laplace transform methods was used in this model to evaluate the state probabilities from set of first order liner differential equations. Tractable solution were found for the parallel system of 3-component and 21-state.

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