Analysis of Improved Switching Constant False Alarm Rate Processor (IS-CFAR) for Different Swerling Radar Target Cases

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Abstract: The distinction of the undesired radar targets from the desired is one of the many important radar tasks and fixing the rate of the false alarms caused by these undesired targets is another important task which is performed by special processors within radar systems known as Constant False Alarm Rate (CFAR) processors. In this study, the Improved Switching-CFAR (IS-CFAR) processor is used and the performance detection equations have been achieved for Swerling cases (II-IV) radar targets in homogenous (Thermal noise only) environment and non-homogenous (Multiple interfering targets) environment. The existence probability equations of a thermal noise sample at homogenous environment and the existence probability equations of a reflected sample from interfering target at non-homogenous environment have been derived for each case of swerling radar targets cases (II-IV) for a single reflected pulse. The simulation results shows that the IS-CFAR processor has better performance in non-homogenous environment rather than in homogenous environment, comparing it with other CFAR processors.

Key words: IS-CFAR, CA-CFAR, OS-CFAR, multiple targets, swerling, Iraq

INTRODUCTION

Radar suffers many problems when detecting targets, one of these problems is the presence of other (undesired) targets in the region of the main (desired) target under detection. These (undesired) targets or interfering targets cause many false alarms when detecting the main target so in order to maintain the rate of false alarms fixed, the Constant False Alarm Rate (CFAR) processors were used (Barton and Leonov, 1998). In general, CFAR processors based on the existence or non-existence of a target in the Cell Under Test (CUT) of a reference window by comparing the contents of the CUT with the estimated noise from the rest cells surrounding the CUT. The estimation methods of the background noise differ from one CFAR to another depending on the environment requirements it operates in. The simple, well known Cell-Averaging (CA-CFAR) processor produced by Gandhi and Kassam (1988) was used with good performance in the homogenous environment but it suffers degradation in its performance when the reference window size get larger since multiple interfering targets will appear and affect on its performance. Later many CFAR processors were developed to operate in hard situation environments or non-homogenous environments such as the Ordered-Statistics (OS-CFAR) processor which operates with good performance in non-homogenous environments (Rohlting, 1983). Recently, the Switching CFAR (S-CFAR) processor was produced by Van Cao (2004) with good performance detection in the non-homogenous environment. Later, Erfanian and Vakili (2008) improved the S-CFAR to another CFAR known as the Improved Switching CFAR (IS-CFAR) processor with better detection performance in non-homogenous environment.

All the CFAR processors that were proposed to solve the problem of interfering targets assumed the Swerling I radar target model to describe the detection performance of these CFAR processors.

In this study, the rest of Swerling radar target models (II-IV) were used to evaluate the detection performance for the IS-CFAR processor in homogenous and non-homogenous environments. A comparison between the IS-CFAR processor and other CFAR processors is presented to explain its detection performance.

SWERLING RADAR TARGET MODELS

Radar Cross Section (RCS) most used statistical model is the chi-square ($\chi^2$) distribution. The probability density function of the chi-square ($\chi^2$) distribution is (Skolnik, 2001):

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290
\[ p(\sigma) = \frac{m}{(m-1)\sigma_w} \left( \frac{m0}{\sigma_w} \right)^{m-1} \exp \left( -\frac{m0}{\sigma} \right), \sigma \geq 0 \] (1)

Where:
- \( \sigma \) = The value of RCS
- \( \sigma_w \) = The average value of RCS
- \( m \) = Any real number decides target type

In Eq. 1, when \( m = 1 \) then it reduces to SWI and SWII radar targets and when \( m = 2 \) it reduces to SWIII and SWIV radar targets.

**Swerling case one (SWI):** This model describes targets with many equal small scattering centers. The reflected echo pulses from this target have the same amplitude during the entire scan but they fluctuate from scan to scan. The PDF of the RCS (\( \sigma \)) is:

\[ p(\sigma) = \frac{1}{\sigma_w} e^{-\frac{\sigma}{\sigma_w}}, \sigma \geq 0 \] (2)

**Swerling case two (SWII):** This model describes targets with many equal small scattering centers. The reflected echo pulses fluctuate rapidly from pulse to pulse during the entire scan. The PDF of the RCS (\( \sigma \)) is the same as Eq. 2.

**Swerling case three (SWIII):** This model describes targets with dominant scattering center and many other small scattering centers. The reflected echo pulses fluctuate as in SWI radar target. The PDF of the RCS (\( \sigma \)) is:

\[ p(\sigma) = \frac{4\sigma}{\sigma_w^2} e^{-\frac{\sigma}{\sigma_w^2}}, \sigma \geq 0 \] (3)

**Swerling case four (SWIV):** This model describes targets that have same specifications as in SWIII. The reflected echo pulses fluctuate from pulse to pulse during the entire scan. The PDF of the RCS (\( \sigma \)) is the same as Eq. 3.

**IS-CFAR PROCESSOR ALGORITHM**

The IS-CFAR processor is an improved version of the Switching CFAR (S-CFAR) processor and it consists of the following two stages (Erfanian and Vakili, 2008):

**Stage one:** Samples in the reference window are partitioned into two sub-windows \((S_0)\) and \((S_1)\) according to the following conditions:

If \( (x_0 > n \alpha x_0) \) then \( x_i \in S_1 \)

Else if \( (x_0 < n \alpha x_0) \) then \( x_i \in S_0 \)

Where:
- \( x_i \) = The amplitude of the reference sample (cell)
- \( x_i \) = The amplitude of the sample in the CUT
- \( \alpha \) = The scale factor which is used to scale the content of the CUT
- \( \alpha x_0 \) = The threshold that is used to split the reference window into sub-windows

**Stage two:** Count the length of the sub window \((S_0)\) and denote this length as \((n_0)\), then depending on the value of the parameter integer threshold \((N_r)\), a target is declared available in the CUT as follows, If \( n_0 > N_r \) then:

- Use the sub window \((S_0)\) to extract the background noise or clutter power
- Multiply the result from (1) by a constant \((\beta_0)\) to get a threshold level

Now if \( (x_0 > \text{results from (2)}) \) then a target is declared in CUT

If \( n_0 \leq N_r \) then:

- Use the sub window \((S_1)\) to extract the background noise or clutter power
- Multiply the results from (1) by a constant \((\beta_1)\) to get a threshold level

Now if \( (x_0 > \text{results from (2)}) \) then a target is declared in CUT

In Fig. 1 block diagram shows the algorithm of the IS-CFAR processor.

![Fig. 1: Block diagram of IS-CFAR processor](image-url)
MATHEMATICAL ANALYSIS

The analysis of the IS-CFAR processor is performed in both homogenous and non-homogenous environments for Swerling radar targets (SWII-SWIV) and by considering Gaussian noise background. For both homogenous and non-homogenous environments, it is assumed that the CUT \((X_0)\) contains a reflected echo samples from main target to be detected (Erfanian and Vakili, 2008).

**Homogenous environment:** It is assumed here that all reference cells \((X_i)\)'s contain thermal noise samples only and their PDF is distributed exponentially as:

\[
p_n(x_i) = \frac{1}{\mu} e^{-\frac{x_i}{\mu}}
\]

(4)

Where:

\(\mu = \) The total background clutter-plus-thermal noise power

\(x_i = \) The reference cells holding thermal noise samples only

For thermal noise only it is assumed that \((\mu = \mu_t)\), then Eq. 4 became as:

\[
p_n(x_i) = \frac{1}{\mu_t} e^{-\frac{x_i}{\mu_t}}
\]

(5)

where, \(\mu_t\) is thermal noise sample. The existence probability of a sample with thermal noise in the \((S_0)\) group is (Erfanian and Vakili, 2008):

\[
p_0 = p(X_i < \alpha_{nr}) = \int_{x_a}^{\alpha_{nr}} \int_{x_a}^{\alpha_{nr}} p_n(x_i) d_{x_i} d_{x_0}
\]

(6)

where, \(P_{0x}(X_0)\) is the PDF of the sample in the CUT and it is depending on the Swerling radar target type. From stage 2 in the IS-CFAR algorithm, the probability of detection is (Erfanian and Vakili, 2008):

\[
p_d(N_\tau, \alpha, \beta, \theta) = \sum_{n=0}^{2^{2H}} C_{2n}^{2H} P_{0}^{n} (1-P_{0})^{2^{2H}-n} \left(1 + \frac{\beta_n}{1+\delta_i} \right)^{(2^{2H}+n_0)\theta_i}(2n_0)
\]

(7)

\[
+ \sum_{n=0}^{2^{2H}} C_{2n}^{2H} P_{0}^{n} (1-P_{0})^{2^{2H}-n} \left(1 + \frac{\beta_n}{1+\delta_i} \right)^{n_0}\theta_i
\]

The probability of false alarm equation is obtained from Eq. 7 by setting \((\delta_i = 0)\) as (Erfanian and Vakili, 2008):

\[
p_0(N_\tau, \alpha, \beta, \theta) = \sum_{n=0}^{2^{2H}} C_{2n}^{2H} P_{0}^{n} (1-P_{0})^{2^{2H}-n} \left(1 + \frac{\beta_n}{2n_0n_0} \right)^{(2^{2H}+n_0)\theta_i}(2n_0)
\]

(8)

For each Swerling case (II-IV), the existence probability of a thermal noise sample in the \((S_0)\) group is obtained in the following study.

**Swerling case two radar target:** The CUT containing samples from SWII main target to be detected an assumption was made that \(\mu = \mu_t = \mu_{\delta} + (1+\delta_i)\) (Erfanian and Vakili, 2008) then \(P_{0x}(X_0)\) which is the PDF of the CUT is:

\[
p_0(X_0) = \frac{1}{\mu_t} e^{-\frac{x_i}{\mu_t}}
\]

(9)

Substituting Eq. 5 and 9 into Eq. 6, researchers get Eq. 10:

\[
p_0 = p(X_i < \alpha_{nr}) = \int_{x_a}^{\alpha_{nr}} \int_{x_a}^{\alpha_{nr}} \frac{1}{\mu_t} e^{-\frac{x_i}{\mu_t}}
\]

(10)

Integrating Eq. 10 researchers obtain Eq. 11 which is the existence probability of a thermal noise sample in the \((S_0)\) group given that SWII radar target is available in the CUT:

\[
p_s = \frac{a(1+\delta_i)}{1+a(1+\delta_i)}
\]

(11)

**Swerling case three and Swerling case four radar targets:** The CUT containing samples from SWIII or SWIV (since we are dealing with returns of samples with SNR per single pulse) target to be detected, same assumption was made that \(\mu = \mu_t = \mu_{\delta} + (1+\delta_i)\) then \(P_{0x}(X_0)\) is:

\[
p_0(X_0) = \frac{4X_0}{(\mu_t + (1+\delta_i))} \frac{e^{-\frac{X_0}{\mu_t}}}{(1+\delta_i)^{\theta_i}}
\]

(12)

Substituting Eq. 5 and 12 into Eq. 6, researchers get Eq. 13:

\[
p_0 = p(X_i < \alpha_{nr}) = \int_{x_a}^{\alpha_{nr}} \int_{x_a}^{\alpha_{nr}} \frac{1}{\mu_t} e^{-\frac{x_i}{\mu_t}}
\]

(13)
Integrating Eq. 13 researchers obtain Eq. 14 which is the existence probability of a thermal noise sample in the \((S_b)\) group given that SWII or SWIV radar target is available in the CUT:

\[
p_b = 1 - \frac{4}{(2 + \alpha(1 + \delta))}
\]  

(14)

**Non-homogenous environment**: It is assumed here that all reference cells \((x_i)\) contain thermal noise samples and interfering target samples. For interfering radar target sample it assumed that \(\{\mu = \mu_0, (1 + \delta_i)\}\) then Eq. 4 became as:

\[
p_n(x_i) = \frac{1}{\mu_0(1 + \delta_i)} e^{-\frac{x_i}{\mu_0(1 + \delta_i)}}
\]  

(15)

From stage 2 in the IS-CFAR algorithm, the probability of detection considering the presence of interfering targets is (Erfanian and Vakili, 2008):

\[
p_d(N_T, \alpha, \beta_s, \beta, \delta) = \sum_{n=M}^{N} \sum_{n=1}^{M} c^{n+M} \sum_{n=1}^{M} c_{n-1}^{n+M} \left[ \frac{1}{(2N - n)} \frac{1}{1 + \delta_i} \right]^{(n-M) - (2N-n)} \left[ 1 + \frac{\beta_s}{n} \right]^{-n} \left[ 1 + \frac{\beta}{n} \right] \left[ 1 + \frac{\beta}{n} \right]^{-n_0}
\]

\[
(1 - p_f)^{n-1} \cdot \left[ 1 + \frac{\beta_s}{n} \right]^{-n_0} \cdot \left[ 1 + \frac{\beta}{n} \right] \left[ 1 + \frac{\beta}{n} \right]^{-n_0}
\]

where, \(p_f\) is the existence probability of a sample with interference noise in the \((S_b)\) group and it is also determined using Eq. 6. The probability of false alarm is obtained by setting \((\delta_i = 0, \delta = 0, M = 0\) and \(m = 0\)) in Eq. 16, then the equation of probability of false alarm is the same as Eq. 8. For each Swerling case (II-IV), the existence probability of a sample with thermal noise in the \((S_b)\) group \((p_n)\) is the same as in the homogenous environment but the existence probability of an interfering target sample in the \((S_b)\) group \((p_n)\) at the non-homogenous environment is obtained.

**Swerling case two radar target**: The CUT containing samples from SWII main target to be detected, same assumption was made that \(\{\mu = \mu_0, (1 + \delta_i)\}\) and \(p_n(x_i)\) is the same as Eq. 12. Substituting Eq. 12 and 15 into Eq. 6, researchers get Eq. 19:

\[
p_f = p(x_i < \alpha x_i) = \int_{-\infty}^{x_i} \left[ 1 - \frac{x_i}{\mu_i} \right]^{-\alpha/\mu_i} \cdot e^{-\frac{x_i}{\mu_i}} \cdot d_{x_i} \cdot d_{x_i}
\]

(17)

Integrating Eq. 19 researchers obtain Eq. 20 which is the existence probability of an interfering target sample in the \((S_b)\) group given that SWII or SWIV radar target is available in the CUT:

\[
p_f = 1 - \frac{4(1 + \delta)^2}{(2(1 + \delta) + \alpha(1 + \delta))}
\]

(20)

**IS-CFAR PERFORMANCE**

The performance of the IS-CFAR processor is studied in both homogenous and non-homogenous environments to investigate the detection performance of this processor when detecting Swerling radar target cases (SWII-SWIV). In homogenous environment the existence probability of a thermal noise sample in the \((S_b)\) group is considered and in non-homogenous environment the existence probability of a thermal noise sample with existence probability of an interfering target sample in the \((S_b)\) group are considered. The performance of the IS-CFAR processor algorithm is a function of \(\beta, \beta_s, N_T\) and \(\alpha\). By plotting the \(P_{\text{fa}}\) curve using Eq. 8 for reference window size \((2N = 24)\) \(\beta_s, \beta, \beta_s, \alpha\) and \(N_T = 12\). Figure 2 shows the \(P_{\text{fa}}\) curves for SWII radar target.

From Fig. 2 and 3, one can observe that by decreasing the scale factor \((\alpha)\) then the probability of false alarm decreases for the same value of the constant \((\beta)\) where the \((\alpha)\) parameter affects the CUT amplitude and it has a large effect on the performance of the
Fig. 2: False alarm probability of the IS-CFAR processor for SWII at different scale factors

Fig. 3: False alarm probability of the IS-CFAR processor for SWIII at different scale factors and \( P_n \) curves for SWIII or SWIV radar target

IS-CFAR processor, since this CFAR processor unlike other CFAR processors, uses a scaled CUT amplitude in processing the cells of the reference window.

The detection performance of Swerling radar targets (SWII-SWIV) and for both homogenous and non-homogenous (Multiple targets) environments are obtained for \( P_n = 10^{-6}, 2N = 24, N_r = 12 \) and \( p_n = 10^{-6}, 2N = 24, N_r = 12 \). Figure 4 which is plotted by using Eq. 7, 11, 16 and 18 shows the detection performance curves the IS-CFAR processor detecting SWII radar target in homogenous \( (M = 0) \) and non-homogenous \( (M = 1 \) and \( 2) \) environments.

Figure 5 which is plotted by using Eq. 7, 14, 16 and 20 shows the detection performance curves of SWIII or SWIV radar targets. From Fig. 4 and 5, it is obvious that the IS-CFAR processor is affected by the increase in the number of the interfering targets where the probability of detection is decreased by increasing the number of interfering targets.

Fig. 4: Comparison of detection probability of SWII in non-homogenous environment (Number of interference targets \( M = 0, 1, 2 \)) at \( P_n = 10^{-6} \)

Fig. 5: Comparison of detection probability of SWIII in non-homogenous environment (Number of interference targets \( M = 0, 1, 2 \)) at \( P_n = 10^{-6} \)

**IS-CFAR PERFORMANCE COMPARISON**

A comparison of the IS-CFAR performance, detecting SWII radar target with other CFAR processors in homogenous environment at \( P_n = 10^{-6} \) is shown in Fig. 6 for IS-CFAR \( (2N = 24, N_r = 12, \alpha = 0.5, \beta = 22.5) \), S-CFAR \( (2N = 24, N_r = 12, \alpha = 0.03, \beta = 24.16) \) and for C-CFAR \( (2N = 24) \).

It is obvious from Fig. 6 that the IS-CFAR processor curve lies below the Optimum and CA-CFAR curves while there is a little improvement over the S-CFAR processor. Actually, CFAR-loss (Detection loss) is a good performance comparison measure of CFAR processors and exact CFAR-loss is measured when comparing the specified CFAR processor with the Optimum CFAR processor. So from Fig. 6 at \( (P_d = 0.5) \), the SNR of SWII
The S-CFAR, especially for the targets with SNR values from 10-30 dB but over (SNR = 17.5 dB) the IS-CFAR performance degrades under the OS-CFAR performance.

Table 1 shows the difference between these processors by the values of the SNR (16, 20, 24 and 28 dB), it can be seen that up to SNR = 16 dB the IS-CFAR has better performance than all the other processors. Over SNR = 20 dB, the IS-CFAR processor still has better performance than (S-CFAR and CA-CFAR) processors but its performance, comparing with the OS-CFAR processor, degrades with some percentages.

**CONCLUSION**

The IS-CFAR has better detection performance in non-homogenous environment rather than in homogenous environment. The scale factor parameter effects obviously in the performance of the IS-CFAR especially when controlling its probability of false alarm rate. In homogenous environment, there is a small improvement in the performance of the IS-CFAR over the S-CFAR while in non-homogenous environment there is an obvious improvement in its performance over the S-CFAR.

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