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Low Complexity Quantum Assisted Linear Multiuser Detection for DS-CDMA System

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Abstract: In this study, we consider the performance of the Direct Sequence Code Division Multiple Access (DS-CDMA) system. This study investigated, the Bit Error Rate (BER) performance of a synchronous DS-CDMA system over Additive White Gaussian Noise and Rayleigh channel which is inflated by the different number of users as well as different number of Qubits. We present a quantum detection system based on the circuit model of quantum constraints. In this model, Quantum algorithms are represented by sequential and combinational circuits which comprise quantum gates and quantum registers. The well known Grover algorithm is described and it is compared with classical detection methods along with Improved Quantum Weighted Sum Algorithm (IQWSA). The simulation results demonstrate that the proposed data detection algorithm for MIMO system has a very close to Maximum Likelihood based data detection in BER performance but it has lower complexity than ML-MUD based data detection algorithm.

Key words: Qubit, bit error rate, inter channel interference, orthogonality, cost function, entanglement, teleporation

INTRODUCTION

CDMA is the multiple channel access technique preferred for the 3G mobile communication systems and it has the important role in the research beyond 3G systems. The CDMA systems over wireless channels have to manage with fading multipath propagation which makes the channel estimation a significant issue in CDMA receivers. In spite of a precise literature on CDMA receivers, there are still open problems about the coefficient estimation in hostile environment and the design of low complexity DSP based channel estimators for CDMA applications. Theoretical measures of performance in CDMA detection in the presence of fading multipath channels have mainly derived for ideal channel estimators. Developing such analytical models in presence of channel estimation error significantly decrease the computational time when analyzing the performance of different algorithms.

The next generation wireless systems will communicate simultaneously and the subscriber share the same frequency band. Multicarrier communication based on Multi carrier Code Division Multiplexing Access (MC-CDMA) principles are increasingly being deployed in broadband wireless communication standards such IEEE 802.11 (Wi-Fi) and IEEE 802.16 (WiMax) (Bigham, 2000). In recent years, Orthogonal Frequency Division Multiplexing (OFDM) (Patidar and Parikh, 2011) where the subscriber's signal is transmitted via a group of orthogonal frequencies, providing Inter Channel Interference (ICI) exemption and CDMA systems (Fa *et al*., 2009) have gained considerable attention due to their use in high speed wireless communication.

Both OFDM and CDMA have unique features, for example the former is good in bandwidth utilization and the later has multi-user potential. In 3G mobile systems, Direct Sequence-Code Division Multiple Access (DS CDMA) is applied due to its high capacity and in-built resistance to interference. However, in case of CDMA communication due to the frequency selective property of the channel the orthogonality property between two different user codes at the receiver is lost which indicate degradation in the performance. Single-User detectors were overloaded and showed quite poor performance even in multi-path environment. To overcome this problem, in recent years Multi-User Detection (MUD) (Verdu, 1998) has received considerable attention and become one of the most important signal processing task in wireless. The third-generation wideband-CDMA systems for example the single-carrier Direct-Sequence Code Division Multiple Access (DS-CDMA) are capable of increasing the throughput linearly with the transmit power. For each of the K users we assign multiple superimposed spreading codes in the system.

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In the same way, the Multi-Carrier (MC) MC-CDMA (Hanzo *et al*., 2003; Xu *et al*., 2008) system supports a diversity of users by assigning unique, user-specific spreading codes to them, referred to as user signatures. The throughput of MC-CDMA may also be improved linearly with the transmit power, since we can create superimposed parallel streams in both the time-domain and frequency-domain. Similarly to MC-CDMA, all the above-mentioned concepts are also pertinent to the fourth-generation Multiple-Input Multiple Output (MIMO) assisted Orthogonal Frequency Division Multiplexing (OFDM) systems (Li *et al*., 2002; Jiang and Hanzo, 2007) where the users convey their information to and from the Base Station (BS) over multiple subcarriers.

However, so far we have only stated to the multiple streams generated by multiple users. The severe performance limitation of wireless systems is created by the MUI imposed by the adjacent cells, because this can only be mitigated with the aid of Cooperative Multi-cell Processing (COMP) (Liu *et al*., 2011). More explicitly, the basic philosophy of COMP is that the base-stations are linked with the aid of either optical fibre or by a point-to-point microwave link and this way they exchange all their information including all the uplink and downlink data of all the users. Nonlinear sub-optimal solutions (Kim and Choi, 2011) afford quite good performance, however only asymptotically. Quantum computation based algorithms seem to be able to fill this long-felt gap. Next to the classical explanation, researchers in the early 20th century hoist the idea of quantum theory become remarkable in information theory communication and for signal processing.

In simple physically tangible terms, we may argue that provided all the K users' signals in the above-mentioned holistically optimized system arrive at the base-station synchronously and they transmit M-ary signals then the optimum full-search-based receiver has to tentatively check all the M^K symbol combinations, in order to reliably detect each of the K user's symbols. More specifically, this is achieved by identifying the most likely transmitted M-ary symbol of all the K users of the complete multi-cell system by estimating Cost Function (CF) which may be the Mean Squared Error (MSE) or the Bit Error Ratio (BER), etc. (Chen *et al*., 2006). The complexity of the above-mentioned processes is reduced by exploiting its inherent parallelism engaged by quantum constraints.

In addition, quantum mechanics provides the narrative description of the physical phenomena and there are scenarios where a quantum model fits best as in deep space communications where the received signal is really weak or in a satellite network where we are interested in strongly reducing the power of transmitted signals, possibly without sacrificing performance significantly. By using the principles of quantum mechanics in order to improve intelligent computational systems, the field of quantum computing emerged. In 1981, Feynman introduced the concept of a quantum computer which would be able to accurately simulate the evolution of a quantum system (Feyman, 1982). The structural element of a quantum computer is a quantum bit or qubit that in comparison to the classic bit has values that are not limited to 0 and 1. Quite the contrary, it can have any of these two values like a spinning coin in a box which will only assume the value of "Heads" or "Tails" upon observing it when it stopped. This phenomenon is also often referred to as being in a superposition of the two orthogonal states, 0 and 1 (Nielsen and Chuang, 2000). The reason for this superposition of states being seemingly absent in the macroscopic world is relate to the observation of the qubit. When a qubit is observed or "measured", any superposition of states that it might have assumed "collapses" to the classic states of 0 or 1, stated by Bohr and Heisenberg in 1924 (Hanzo *et al*., 2012).

Quantum computing exploits a range of astonishing, non-intuitive characteristics of quantum mechanics such as quantum parallelism, a term coined by Deutsch (1985) and entanglement (Bell, 1966) to accomplish computational tasks of stunningly high complexity which would be deemed excessive in the classic computing world. Following an approach where the aim is to directly minimize the system's BER, the Minimum BER (MBER) detector was conceived (Tan *et al*., 2008). A surplus of both Multi-User Detection (MUD) and multi stream detections techniques has been proposed in the literature. Our goal is that of seeking algorithms which are more efficient than their best classic counterparts but are only available in the quantum world. The corresponding software-related efforts are in the realms of quantum computing has been explained by Imre and Balazs (2005). In order to understand how quantum computing and communication might improve the performance of our classic wireless systems, let us summarize the four basic rules (called postulates) of quantum mechanics from a telecommunications engineering point of view (Nielsen and Chang, 2000).

In order to pave the way further, we will invoke the well-known DS-CDMA Maximum Likelihood (ML-MUD) and Minimum Mean Square Error (MMSE) multi user detection as a bridge between the classic wireless and the quantum world. However, to find the optimum is a NP-hard problem as the number of users grows. Many authors proposed sub-optimal linear and nonlinear solutions such as Decorrelating Detector, MMSE (Minimum Mean Square Error) detector, Multistage Detector, Hoppfield neural network or stochastic hoppfield neural network (AlJerjawi and Hamouda, 2008). Nonlinear sub-optimal solutions provide quite good performance, however, only asymptotically (Durisi *et al.*, 2010). The first postulate declares how to describe the state of any physical system. Considering our ML MUD problem, similar to the classic vectors b or y, a quantum register consisting of K qubits stores. The 2^K legitimate states at any instant but the quantum register may assume all these states simultaneously, i.e., in parallel which is formulated as (Zhao *et al.*, 2006). This implies from our MUD perspective that a single quantum register is capable of storing all the legitimate b_m hypotheses.

The parallel processing capability of the quantum-search originating from the second postulate allows us to find the discrete PDF given by the relative frequencies of those transmitted signal vectors b_m that lead to a certain received signal vector y which are then combined by weighting with the corresponding a priori probabilities. Finally, in possession of the function f(y|x), we may opt for using quantum-search for finding the most likely transmitted signal vector b_m, given a specific received signal vector y. The third postulate connects the nano as well as the classic macroscopic world and it is referred to as the measurement.

From a ML MUD's perspective the received signal vector y is entered into the quantum detector which also prepares an additional register containing all legitimate hypotheses b_m associated with uniformly distributed coefficients according to the 1st postulate. Hence, if we performed a measurement on this register, we would find that any of the hypotheses m has a probability of $1/2^k$. The second postulate enables us to modify these coefficients in such a way that the most likely hypothesis will have the largest coefficient. An appropriately constructed measurement will deliver this particular hypothesis with a probability that is proportional to the absolute squared value of the largest coefficient. In order to increase this probability towards unity, we have to conceive a sophisticated Quantum algorithm and prepare an appropriate measurement. The fourth postulate defines the technique of combining individual quantum systems. The contributions in this study are: we have proposed an Improved Quantum assisted Multiuser Detection (IQMUD) where all the combinations of the users transmitted symbols are taken into consideration at the receiver.

We have investigated the Bit Error Rate (BER) performance of a synchronous DS-CDMA system over AWGN Rayleigh channel which is affected by the different number of users as well as different types spreading codes.

Improved Quantum Weighted Sum Algorithm (IQWSA) is explained which estimates the weighted sum of a function f: $\{0, 1, ..., 2n\text{-}1\} \rightarrow [0, 1]$ and it is based on the Quantum Mean Algorithm (QMA) of (Brassard *et al.*, 2011). The algorithm which is based on Grover's Quantum Search Algorithm (QSA) (Imre and Balazs, 2002) is employed here for finding the particular multi-user symbol that minimizes a specific Cost Function (CF) involved in the QWSA (Botsinis *et al.*, 2013).

MATERIALS AND METHODS

System overview: As we mentioned earlier an uplink DS-DCDMA system is investigated. The ith symbol of the kth (k = 1, 2, ..., K) user is denoted by $b_k[i] \rightarrow \{+1,-1\}$ (Fig. 1). This assumption corresponds to the simplest scenario where symbols remain real-valued although we use the complex equivalent description. We call this type of modulation Binary Phase Shift Keying (BPSK). Its alphabet consists of only two elements. Of course higher level modulations with a larger symbol alphabet such as 16 quadrature amplitude modulation can be applied.

The information bit stream $\{b_k\}$ each user is encoded into the stream $\{c_k\}$ by a convolution encoder which is passed through pseudo-random bit-based interleavers. The interleaved bits $\{u_k\}$ are spread by the spreading gold codes $C = [c_0, c_1, c_2, ..., c_{k-1}]$ are modulated onto the symbols $\{x_k\}$ which are transmitted over Rayleigh channels. The channel matrix H is assumed to be perfectly estimated at the BS by CSMU estimator. On the other hand, the coloured noise imposed at the receiver, along with the during the propagation is unknown. Let us consider a multiuser system supporting k users with M-ary modulation scheme. The matched filter outputs are described by:

$$y_k[i] = \int_{iTs}^{(i+1)Ts} r(t)\, s_k(t - iTs)\, dt \tag{1}$$

The classic optimal MUD that accepts soft inputs and provides soft outputs is the one that computes the bit LLRs of every bit of every symbol of each user. Since, only continuous electromagnetic waveforms can be transmitted in the radio channel in practice each chip has to be multiplied with the so-called chip elementary waveform denoted by $\{g_k(t)\}$. Thus the analog version of the chip sequence is referred to as the user continuous signature waveform:

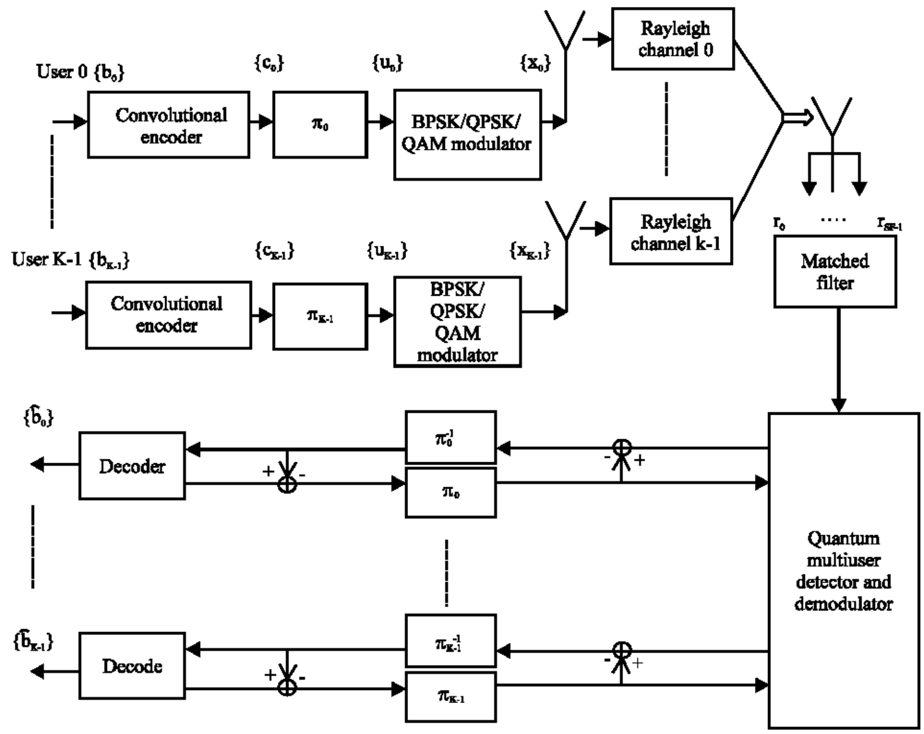

Fig. 1: An uplink DS-CDMA system

$$S_k(t) = \sum_{q=0}^{PG-1} c_k[q] g_k(t - qT_c) \quad (2)$$

where, T_c stands for the time duration of one chip. Obviously members of $\{s_k(t)\}$ are orthogonal concerning the symbol length T_s, i.e:

$$\int_0^{Ts} S_k(t) S_l(t) dt = 0 \quad (3)$$

And normalized:

$$\int_0^{Ts} R^2(S_k(t))dt + \int_0^{Ts} Z^2(S_k(t))\, dt = 1 \quad (4)$$

thus, the output signal of the k_{th} user related to the ith symbol, denoted by $v_k(t)$ is given as:

$$V_k(i,t) = b_k[i]\, S_k(t) \quad (5)$$

Let us assume that each burst consist of R+1 symbols. Therefore, we introduce vector $b_k = [b_k[0], ..., b_k[R]$ denoting the data symbols of the kth user in a certain burst. Thus the kth user's signal during this burst can be expressed as:

$$V_k(t) = \sum_{i=0}^{R} b_k(i)\, s_k(t - iT_s) \quad (6)$$

The channel distortion from the kth user point of view is modelled via an impulse response function as if the channel were a filter:

$$h_k(i,t) = a_k[i]\delta(t - \tau_k) \quad (7)$$

where, $a_k[i] = A_k[i]\ \alpha_k[i]$ with $A_k[i]$ and real $\alpha_k[i]$. The $\alpha_k[i]$ comprises phenomena causing the random nature of the channel and it is called fading. The $A_k[i]$, $\alpha_k[i]$ and τ_k are typically independent random variables while let us suppose as the worst case that they are uniformly distributed about the following regions:

$$A_k[i] \in [-A, A]; \alpha_k[i] \in [0, 2\pi]; \tau_k \in [0, T_s] \quad (8)$$

Similarly, we do not consider Gaussian noise because CDMA systems are strongly interference limited thus Gaussian noise has marginal influence on detection. Finally, we assume that τ_k remains constant during each burst while $a_k[i]$ varies from symbol to symbol. The channel not only delays and distorts the transmitted signal but also adds together all the signals originating from other users, hence we are able to describe the received signal at the base station via convolving the channel input with its impulse response in the following manner:

$$r(t) = \sum_{k=1}^{K}\sum_{i=0}^{R} h_k(i,t)\times v_k(i,t) = \sum_{k=1}^{K}\sum_{k=0}^{R} \alpha_k[i]b_k[i]s_k(t-iTs-\tau_k) \quad (9)$$

Let us assume that $\tau_k = 0$ and $a_k = 1$ deterministically (equivalently the channel is regarded as a shortcut or an identity transformation). In this case, the received signal becomes:

$$r(i,t) = \sum_{k=1}^{k} b_k[i]\, s_k(t) \quad (10)$$

Considering the interval belonging to the ith symbol, Bob tries to obtain a fairly good estimation $b_k[i]$ using the orthogonality of signature waveforms according to Eq. 8. This requires multiplication with transmitted signal waveform $s_k(t)$ and integration on $[0, T_s]$. This operation is nothing more than the calculation of the inner product for continuous variables. Bearing in mind the often used notion for this operation in the literature, we call it a matched filter. Let us denote the output of the matched filter in case of the ith symbol:

$$y_k[i] = \int_0^{Ts} r(i,t)s_k(t) \quad (11)$$

$$= \int_0^{Ts} b_k[i]s_k(t)s_k(t)dt + \int_0^{Ts}\sum_{l=1,l\neq k}^{K} b_l[i]\, s_l(t)dt = b_k[i] \quad (12)$$

Thus, theoretically the output of the matched filter contains information only about $b_k[i]$ and its sign can be used to decide which symbol has been sent by applying a comparator. Therefore, Bob can use $y_k[i]$ directly to determine $b_k[i]$ = sgn ($y_k[i]$). In a realistic scenario, the above introduced detector may fail with certain probability. Optimal solutions minimize this probability by possessing additional information. If we insist on using only signature waveform to detect symbols originating from the transmitter then this technique is referred to as single user detection. This approach can be appropriate when the detector is located in a mobile terminal whose computational power is moderated. However, sitting at a base station's receiver module we are allowed to be more pragmatic. Since, all the signals arriving from different users must be detected all the signature waveforms are available. Thus those schemes performing combined detection are called Multi-User Detectors (MUD) (Fig. 2).

Although, MLS-based optimal multi-user detectors are more popular than the MBER based ones because of their less computational complexity, both approaches are far from practical implementations. However, quantum assisted computing exploiting quantum parallelism may help us to attack the optimum MUD problem directly. As

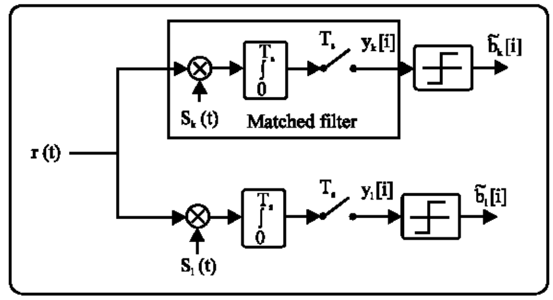

Fig. 2: Block diagram for matched filter

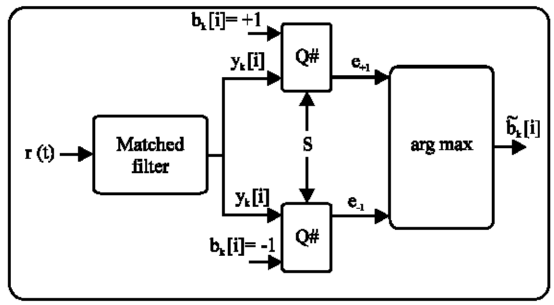

Fig. 3: Block diagram for quantum MUD

we deduced, the computation requires evaluating two conditional probability Density Function (PDF's). Since, we are interested only in the larger PDF the denominators can be omitted. Both numerators require solving a special counting problem. Because all the channel parameters and other symbols are independent and uniformly distributed Bob has to decide whether the number of Z_{+1} or Z_{-1} leading to $y_k[i]$ is bigger which is equivalent to the question whether $b_k[i]$ = +1 or $b_k[i]$ = -1 have the larger probability of being the originator of $y_k[i]$.

In possession of a promising idea and with knowledge about quantum counting next we determine the architecture and initialization parameters of the Quantum based MUD (QMUD) detector. We apply the top-down design principle thus we depict the system concept in Fig. 3. We define two counting circuits according to the two hypotheses, one that assumes $b_k[i]$ = +1 and another for $b_k[i]$ = -1. Their outputs representing the numerators are denoted by:

$$e_{\pm} = \neq (Z_{\pm} : y_k[i] = u'(Z_{\pm})) \quad (13)$$

Each quantum counter is fed with the outcome $y_k[i]$ of the matched filter, the corresponding hypothesis $b_k[i]$ = ±1 and the set S = $\{s_k(t)\}$ of individual signature waveforms of all the active users. Next, the outputs are compared and the result determines estimation. During the design of basic Quantum algorithms, we prepare two quantum registers. The upper one contains n qubits with respect to the size $N = 2^n$ of the database and it is initialized with |γ0 = |0. We feed an n dimensional Hadamard gate with this qregister while the lower qregister is connected to an unknown gate T.

Quantum detection: The Quantum Multi-User Detection (QMUD) technique with quantum detection under quantum multiple-access channel is a promising field in communications. The objective of multi-user detection is to develop a detection technique to minimize the probability of error in noisy and high interference environment. Let's consider, a system with K users transmitting information through the electromagnetic field. Quantum based search algorithm which is based on the quantum parallel computation can find the desired value precisely with O iterations in a large unsorted database which contains N elements. However, to find the desired value in the database, any classical algorithm would need at least O iterations. So, Quantum algorithm, for the system signal detection, can slash algorithm complexity (Bingham, 2000).

Grover Quantum Search algorithm for MUD: Grover algorithm objective is increasing probability amplitude of target quantum states by the unitary transformation of initial equal amplitude superposition state while at the same time reducing probability amplitude of other off-target quantum states. In the end, the more probability amplitude of target quantum states the bigger probability the right target is searched (namely measured). Grover's Quantum Search algorithm is based on the quantum computing. A bit in traditional computing only can be expressed as one state (0 or 1). Different from the traditional computing, quantum computing uses quantum bit (qubit) instead of the bit. In quantum computing, the orthonormal basis $|0\rangle$ and $|1\rangle$ are taken to represent the classical bit values 0 and 1, respectively. A qubit can be presented in a linear superposition of $|0\rangle$ and $|1\rangle$. In quantum system, quantum register (qregister) is the quantum mechanical analogue of a classical processor register. An n-bit qregister can present the 2n quantum states. Because of "quantum parallelism", any linear hermitian operator U (U is unitary matrix) operating on a qregister is equivalent to parallel processing for all 2n quantum states (Fig. 4).

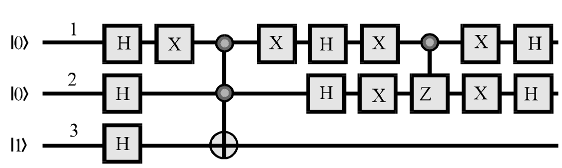

Fig. 4: Circuit diagram for Grover algorithm

Firstly, design two databases, the first database which has 2ns registers save all possible sending sequence. As necessary judgment information, the correlation matrix RS and receiving signal stored in the computer. We can get 2ns decision values by each may be sending sequence and correlation matrix and channel matrix H are calculated according to ky _H×k2: These judgment values put in the second database. Message sending sequences in the first database and judgment values in the second database constitute one-to-one relationships. Message sequences which correspond to minimum decision values are calculated according to ky_H×k2 and are the sending sequences which are detected by the best detection scheme. Therefore, the following work is looking for the smallest judgment value and judging sending sequence.

We quantize every chip of the Kth user's codeword in a quantum register of length N_{ch} where the number representation is not significant at the evaluation of the received symbol. In our model, we prepare for user k two quantum registers φ_k^1 and φ_k^0 each corresponding to transmitted bit "1" and "0" with an overall length Nq = Nch. It is important to notice that the effects of a multipath channel and additive noise are contained in the registers; moreover the density function of the noise does not need to be known a priori. Let, V denote a vector space spanned by $|v_i\rangle$, I = 1, 2, ..., 2Nq orthonormal computational states where $\langle v_i|v_j\rangle = 0$ and $\langle v_i|v_j\rangle = 1$. For Vi = j is hold. The number of stored states in quantum registers $|\varphi_1^k\rangle$ and $|\varphi_0^k\rangle$ is denoted with Ns1 and 0. If the register $|\varphi_1^k\rangle$ contains the desired state.

Grover algorithm can solve this problem. Initialize the system by $H^{\otimes n}\ |\ 0\rangle^{\otimes n}$ (W denote the Walsh Hadamard transformation, $W = H^{\otimes n} = H\otimes H\otimes \ldots \otimes H$. So, we can get an equal superposition state of n-bit register:

$$H^{\otimes n} c^{\otimes n} = \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n} |x\rangle = |s\rangle \tag{14}$$

Apply the oracle, suppose that we wish to search through a search space of 2n elements. Rather than search the search the elements directly, we concentrate on index x to those elements which is just a number in the range 0 to N-1:

$$f(x) = \begin{cases} f(x) = 1, \text{if } x \text{ satisfy the condition} \\ f(x) = 0, \text{ otherwise} \end{cases} \tag{15}$$

The oracle transforms the search problem in an unsorted quantum database into a decision problem. It is an operation that defines the solution. It can be represented by a unitary operator O. The U operation is defined as:

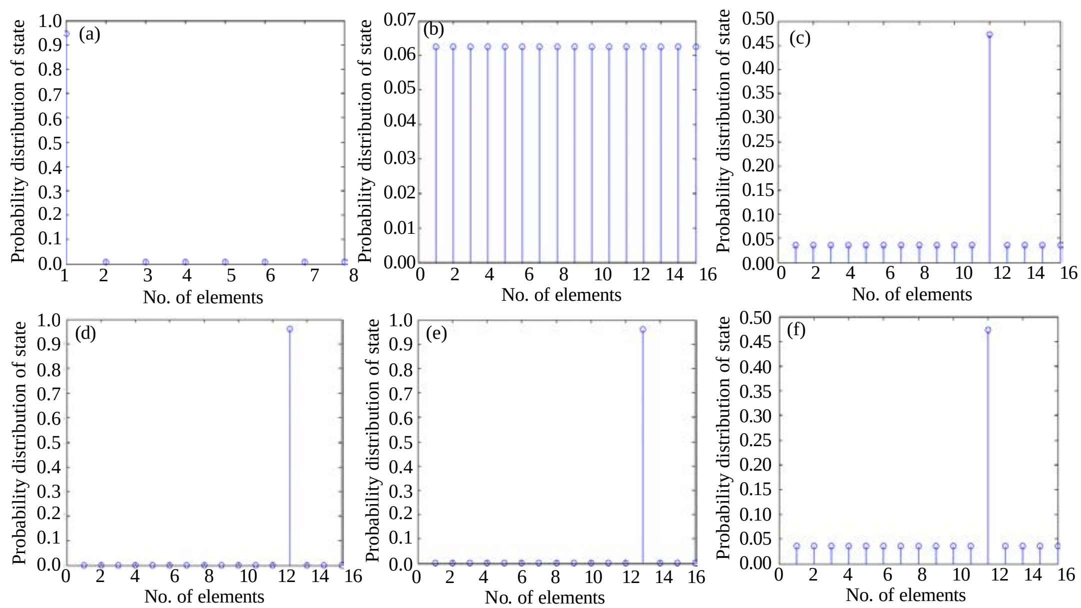

Fig. 5: Probability of observation of quantum states

$$\left|x\right\rangle \xrightarrow{0} (-1)^{f(x)} \int x\rangle \quad (16)$$

The U operation is define as

$$U = WRW = 2|\varphi\rangle\langle\varphi| - 1$$

Where:

$|\varphi\rangle$ = $1/\sqrt{2^n}\sum_{x=0}^{2^n}|x\rangle$
R = $2|0\rangle\langle 0|$-I

In addition to $|0\rangle$, R operation makes every basis state obtain π phase shift. The U operation increase the probability amplitude of solutions while reduce the probability amplitude of non solutions. Performance the Grover iteration G = UO to make sure get a solution with a higher probability when taking measurement. Measurement. After $\pi/4\sqrt{2^n}$ times iteration, observe the q register. And output the result basis state $|0\rangle$ corresponding to f(x) = 1.

In Fig. 5, the probability distribution of states after each step of the Grover search algorithm for n = 4 qbits is represented in the same way as in Fig. 3. After 3 iterations the probability of measuring the register, searched value is 0.9613. An additional iteration decreases this probability to: 0.5817. Applying Grover algorithm with 12 qbits, the optimal number of iterations is m = 50. After 50 iterations, the probability to find state is 0.999945.

The simulation results are represented in Fig. 5. On the x-axis is the value expressed in the decimal system possible to be stored in the register and on the y-axis the probability of finding that value. Each graph corresponds to the algorithm steps. It can be observed that psi_3 is a superposition of |2≥|10> and |3≥| 11> with equal probability, 0.5, then the first qbit is"1" implement a function which is not constant.

By investigating Grover's Quantum Search algorithm of a simple DS-CDMA with K = 2 employing QPSK modulation scheme with M = 2 states. The channel coefficients are perfectly estimated using optimal channel estimator and spreading codes are used with spreading factor of 31 chips.

The goal of Grover QSA is that of finding the index of desired entries. The figure shows the evolution of Quantum states which depicts the initial amplitude of QIR (Fig. 6). It shows the highest probability solution I QIR accomplished y iterations of QSA.

Improved quantum weighted sum algorithm

IQWSA algorithm: The Quantum Function Register (QFR) consists of (n+1) qubits where n is the number of bits in the (M^{K-L}) = 2-ary symbol minus one, since the bit we want to compute the LLR for is fixed to 0. Hence, we need n = KL $\log_2(M)$-1 = 1 qubit to represent the 2^n = 2 CFEs that appear in the numerator of Eq. 3 and the QFR will have n+1 = 2 qubits in total, superimposed in the N = 2^{n+1} = 4-element all-zero state. The tensor product is defined as (Fig. 7 and 8).

The Quantum Control Register (QCR) consists of l = 8 qubits also in the all-zero state $|0\rangle^{\otimes 8}$. Initially, an l-qubit Hadamard gate is $H^{\otimes l}$ applied to the QCR, resulting in an equiprobable superposition of all 28 states as in:

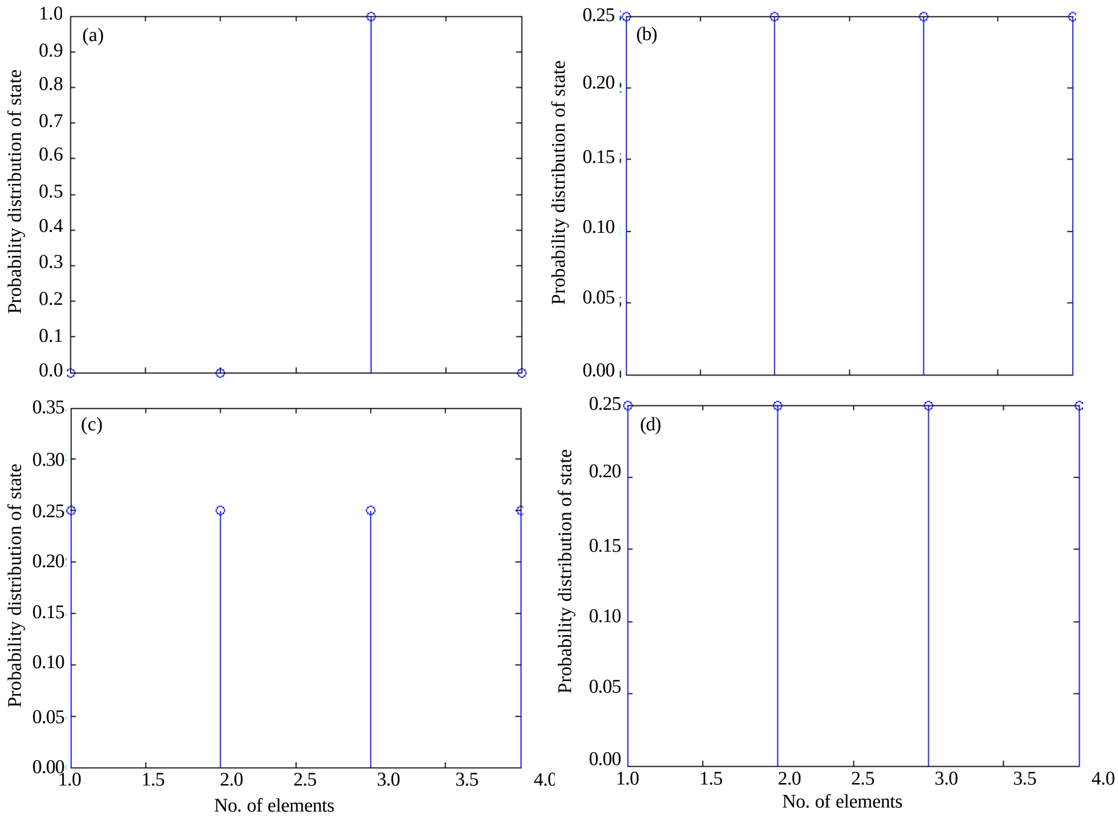

Fig. 6: Probability distribution of quantum state: a) Psi0; b) Psi1; Psi2; Psi3 and d) Psi4

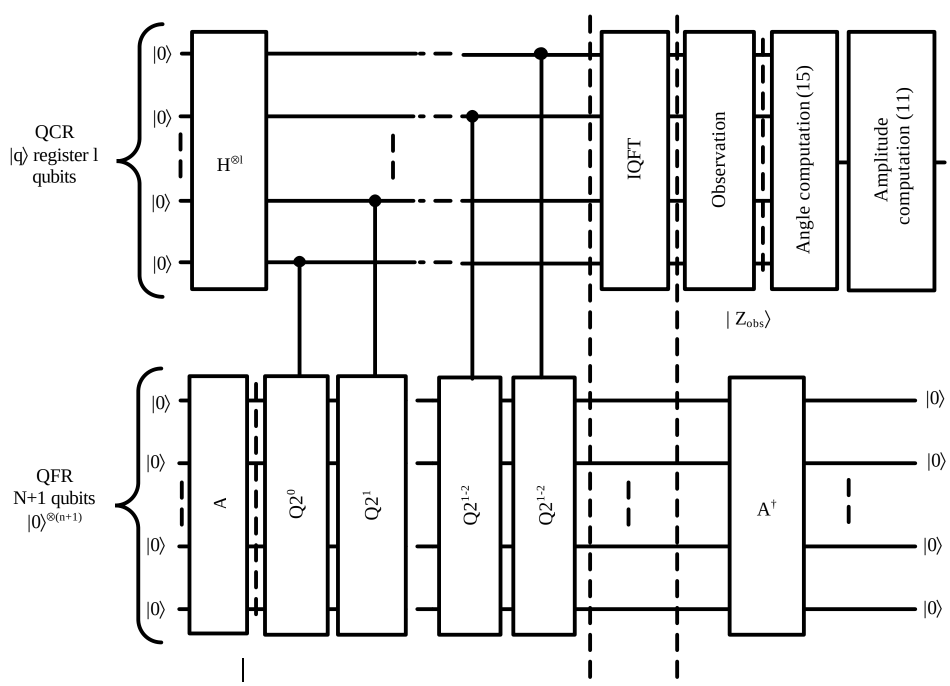

Fig. 7: The IQWSA quantum circuit

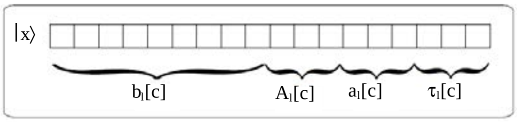

Fig. 8: The quantum states in QCR

$$|q>=H^{\otimes l}|0>^{\otimes 8}=\frac{1}{\sqrt{2^l}}|\psi\rangle=\frac{1}{16}\sum_{\psi=0}^{255}|\psi\rangle \quad (17)$$

Each of the first n = 1 qubits of the QFR is rotated by a qubit-specific rotation gate having an angle which depends on the a priori probability of its corresponding bit as in (Hanzo *et al*., 2003):

$$R_i|0\rangle_i=\sqrt{P(b_{k,l}^m=0)|0\rangle}+\sqrt{P(b_{k,l}^m=1)|1\rangle} \quad (18)$$

where, the i = (kL log 2(M)+l log2(M)+m)th rotation gate is described. Therefore, the initialized QFR is in the state:

$$|\varepsilon\rangle=\sum_{x=0}^{N-1}\sqrt{P(x)|x\rangle}|0\rangle=\sqrt{0.25|0\rangle}|0\rangle+\sqrt{0.75|1\rangle}|0\rangle \quad (19)$$

A unitary operator A_is then applied to $|\varepsilon\rangle$ of (Xu *et al*., 2008) where the operation of A' may be described as:

$$|x\rangle|0\rangle\xrightarrow{A'}|x\rangle(\sqrt{1-f'(x)|0\rangle}+\sqrt{f'(x)|1\rangle} \quad (20)$$

and in our scenario, the resultant state $|\Psi\rangle$ = A'$|\varepsilon\rangle$ is:

$$|\psi\rangle=(\sqrt{0.47|0\rangle}+0|1\rangle)|0\rangle+0.15|0\rangle+0.86|1\rangle)|1\rangle \quad (21)$$

We note that the probability of observing the last qubit of the QFR in the state $|1\rangle$ is equal to a = 0.1592+0.8662 = 0.775 which is equal to the desired weighted sum $P(b_{0,0}^{(0)}=0)|r\rangle/f(x)$. In the general case, applying the operator A = A_(R0, ..., Rn-1, I) to the all-zero state results in (Hanzo *et al*., 2003):

$$|\psi\rangle\sum_{x=0}^{N-1}\sqrt{P(x)(1-f'(x))|x\rangle}|0\rangle+\sum_{x=0}^{N-1}\sqrt{P(x)f(x)|x\rangle}|1\rangle \quad (22)$$

where, $\alpha=\sum_{x=0}^{N-1}P(x)f(x)$ is the desired weighted sum while $|\Psi_1\rangle$ and $|\Psi_2\rangle$ include the wanted and unwanted states, respectively, since the probability of observing ??in a state that belongs $|\Psi_1\rangle$ is equal to the desired sum a. Therefore, after the initialization stage, the QWSA employs the Quantum Amplitude Estimation (QAE) algorithm (Chen *et al*., 2006) for estimating the amplitude of the quantum states that belong to $|\Psi_1\rangle$.

The QCR controls, the Q = AP0A CNOT operators that are applied to the QFR's state $A^\dagger$ where $|\Psi_1\rangle$ is the conjugate transpose of the matrix representation of A (Hanzo *et al*., 2003), P0 flips the sign of the amplitudes of all quantum states except for the all-zero state (Hanzo *et al*., 2003) and CNOT is a Controlled NOT operator that flips the sign of the amplitudes of the states in $|\Psi\rangle$ when the last qubit of those states is equal to $|\Psi\rangle$, therefore marking the wanted states. Every single application of the Q operator to $|\Psi\rangle$ rotates it by 2θ where we have:

$$\theta=\arcsin\sqrt{a}\Rightarrow a=\sin^2\theta \quad (23)$$

It has been shown that $|\Psi\rangle$ Eq. 10 is a scaled version of the eigenvectors of the Q operator. Therefore the eigenvalues $\lambda_\pm$ of Q are connected to the rotation angle θ through $\lambda_\pm = e^{\pm j2\theta}$(Fa *et al*., 2009; Hanzo *et al*., 2003; Xu *et al*., 2008). The eigenvalues of Q that are not equal to ±1 in our scenario are ±λ = -0.55±j0.84. Hence, we may derive that θ = -j/2 ln ($\lambda_\pm$) = 1.08.

It should be noted that ($\lambda_\pm$) and θ is unknown to us in a practical application and we may not exploit it in the simulations but only for comparison with the estimated value. Furthermore, since $|\Psi\rangle$, consists of the eigenvectors of Q, the QFR's states may be considered unaltered and the rotation is performed on the QCR's states instead as shown in Fig. 5.

For exploiting the periodicity created in the amplitudes of $|\phi_1\rangle$ shown in Fig. 3, the IQFT [2] is applied to $|\phi_1\rangle$. The resultant quantum state $|\phi_2\rangle$ of our system may be described in the computational basis as encapsulated in and the amplitudes of the specific states which $|\phi_1\rangle$ is superimposed in are illustrated in Fig. 5:

$$|\phi_2\rangle\sum_{z=0}^{2^l-1}\sum_{x=0}^{N-1}\beta_{zx0}|z|0\rangle+\sum_{z=0}^{2^l-1}\sum_{x=0}^{N-1}\beta_{zx1}|z|1\rangle \quad (24)$$

$$\beta_{zx1}=\frac{\sqrt{P(x_1)f(x)}}{2^{l+1}\sqrt{1-a}}\mu_{z-} \quad (25)$$

$$\beta_{zx0}=\frac{\sqrt{P(x_0)(1-f(x_0))}}{2^{l+1}\sqrt{1-a}}\mu_{z+} \quad (26)$$

$$\mu_{z\pm}=e^{j\theta}\frac{1-e^{j2\pi(2l\theta/\pi-z)}}{1-e^{j2\pi(\theta/\pi-z/2^l)}}\pm e^{-j\theta}\frac{1-e^{j2\pi(-2^l\theta/\pi-z)}}{1-e^{j2\pi(-\theta/\pi-z/2^l)}} \quad (27)$$

Observing only the QCR translates in observing only the first l = 8 qubits of $|\phi_2\rangle$. The probability of observing the state $|z_{obs}\rangle$ in the QCR of Fig. 2 during a partial measurement is the sum of the probabilities of observing the whole system in a state that has the first l qubits equal to $|z_{obs}\rangle$. Hence, it is equal to the probability of observing each of the 2l = 256 states in our scenario where we have N = 4 is presented in Fig. 5. Due to the IQFT, the probability of observing a state $|z_{obs}\rangle$ is equal to the probability of observing the state 2^l-$|z_{obs}\rangle$ In our scenario, by observing the QCR we obtained $|z_{obs}\rangle = |88\rangle$. The observed state $|z_{obs}\rangle = |88\rangle$ provides an estimate of θ equal to (Hanzo *et al*., 2003; Chen *et al*., 2006).

$$|z_{obs}\rangle \pi \frac{z_{obs}}{2^l} = \pi \frac{88}{256} = 1.08 \quad (28)$$

Finally, according to Eq. 11, the estimated value of the numerator of the LLR in Eq. 4 is $\hat{a} = \sin 2\hat{\theta} = 0.778$. We would have obtained exactly the same estimate if $|z_{obs}\rangle = |256\text{-}88\rangle = |168\rangle$. The estimation error is equal to 0.0026.The error in our scenario is the smallest possible value for l = 8, since we obtained one of the two most likely quantum states when we observed the QCR which corresponds to the best estimate of a. If more qubits were used in the QCR, the estimation error would be smaller as encapsulated in (Xu *et al*., 2008):

$$|\hat{a} - a| \le 2\pi w \frac{\sqrt{a(1-a)}}{2^l} + w^2 \frac{\pi^2}{2^{2l}}, \; w = 1, 2... \quad (29)$$

RESULTS AND DISCUSSION

Computational analysis and simulation: In this study, we evaluate the BER performance of our proposed IQWSA QMUDs in the uplink of DS CDMA systems. The first communication system investigated supports K = 2 users transmitting with different modulation formats like BPSK, QPSK and QAM with M = 4 symbols and the BS station is equipped with P = 4 Aes (Table 1).

For comparison between the IQWSA QMUDs and the ML-MUD, let us firstly investigate a scenario where all users transmit on all the signature codes. Moreover, we will also temporarily assume that all the users have been allocated the same PN sequence code (gold code), therefore they all interfere with each other and hence the resultant system becomes CDMA-OFDM system. The interleaver length is equal to 10240 symbols per user and TCC associated with a rate of R = 1/2.

In Fig. 7, we commence by first presenting the BER performance of BICM-ID systems supporting K = 4, 6 and 8 users employing different modulation formats associated with M = 4. The performance of the IQWSA MUD is compared to that of the ML MUD and to that of the MF detection. We may conclude that the performance of the IQWSA MUD is optimal, since it matches that of the hard ML MUD. The number of CF evaluations performed by the ML MUD of the systems QAM is M^K = 256, 4096 and 65 536, respectively, while the average number of CF evaluations in the same systems when the IQWSA was used is 78, 342 and 1456, respectively. Hence, by using the IQWSA MUD we may achieve optimal performance at a substantially reduced computational complexity.

In a BICM-ID system using R = 1/2 TCCC and supporting K = 2 users employing Gold codes associated with SF = 31 chips each, the performance of the IQWSA

Table 1: BER Vs Eb/No. for BPSK, QPSK and QAM modulation (Rayleigh channel)

	BER for BPSK modulation				BER for QPSK modulation				BER for QAM modulation			
Average Eb/No.db	MF	MMSE-MUD	ML-MUD	IQWSA-MUD	MF	MMSE-MUD	ML-MUD	IQWSA-MUD	MF	MMSE-MUD	ML-MUD	IQWSA-MUD
1	0.1464	0.072230	0.058060	0.036610	0.146447	0.092248	0.027728	0.006932	0.14645	0.015461	0.058060	0.004188
2	0.1267	0.056310	0.044110	0.028050	0.126733	0.078088	0.020132	0.005033	0.12673	0.013317	0.044110	0.002714
3	0.1085	0.042290	0.032750	0.020980	0.108485	0.064968	0.014322	0.003581	0.10848	0.011367	0.032750	0.001702
4	0.0919	0.031220	0.023790	0.015340	0.091913	0.054223	0.010001	0.002500	0.09191	0.009650	0.023790	0.001035
5	0.0771	0.022020	0.016930	0.010980	0.077137	0.044345	0.006868	0.001717	0.07714	0.008148	0.016930	0.000612
6	0.0642	0.015710	0.011830	0.007710	0.064183	0.036242	0.004648	0.001162	0.06418	0.006754	0.011830	0.000353
7	0.0530	0.010860	0.008130	0.005320	0.052999	0.030054	0.003107	0.000777	0.05300	0.005545	0.008130	0.000198
8	0.0435	0.007450	0.005510	0.003620	0.043474	0.024221	0.002054	0.000514	0.04347	0.004565	0.005510	0.000110
9	0.0355	0.005000	0.003680	0.002430	0.035459	0.019576	0.001347	0.000337	0.03546	3.75E-03	0.003680	5.94E-05
10	0.0288	0.003220	0.002440	0.001610	0.028782	0.015557	0.000876	0.000219	0.02878	3.05E-03	0.002440	3.18E-05
11	0.0233	0.002210	0.001600	0.001060	0.023269	0.012587	0.000567	0.000142	0.02327	2.45E-03	0.001600	1.68E-05
12	0.0187	0.001490	0.001040	0.000690	0.018748	0.010044	0.000365	9.12E-05	0.01875	1.99E-03	0.001040	8.79E-06
13	0.0151	0.000840	0.000670	0.000450	0.015065	0.008042	0.000234	5.84E-05	0.01506	1.60E-03	0.000670	4.56E-06
14	0.0121	0.000590	0.000430	0.000290	0.012078	0.006332	0.000149	3.73E-05	0.01208	1.29E-03	0.000430	2.35E-06
15	0.0097	0.000400	0.000280	0.000190	0.009665	5.07E-03	9.52E-05	2.38E-05	0.00967	1.02E-03	0.000280	1.20E-06
16	0.0077	0.000200	0.000180	0.000120	0.007723	4.03E-03	6.06E-05	1.51E-05	0.00772	7.90E-04	0.000180	6.14E-07
17	0.0062	0.000140	0.000110	7.55E-05	0.006164	3.21E-03	3.85E-05	9.62E-06	0.00616	6.37E-04	0.000110	3.12E-07
18	0.0049	9.40E-05	7.22E-05	4.81E-05	0.004915	2.58E-03	2.44E-05	6.10E-06	0.00491	5.20E-04	7.22E-05	1.58E-07
19	0.0039	5.10E-05	4.59E-05	3.05E-05	0.003916	2.04E-03	1.55E-05	3.86E-06	0.00392	4.22E-04	4.59E-05	8.01E-08
20	0.0031	4.30E-05	2.91E-05	1.94E-05	0.003118	1.68E-03	9.78E-06	2.45E-06	0.00312	3.23E-04	2.91E-05	4.04E-08
21	0.0025	2.20E-05	1.84E-05	1.23E-05	0.002481	1.24E-03	6.19E-06	1.55E-06	0.00248	2.59E-04	1.84E-05	2.04E-08
22	0.0020	2.80E-05	1.17E-05	7.78E-06	0.001974	9.68E-04	3.91E-06	9.78E-07	0.00197	2.10E-04	1.17E-05	1.03E-08
23	0.0016	7.00E-06	7.39E-06	4.92E-06	0.001570	8.27E-04	2.47E-06	6.18E-07	0.00157	1.65E-04	7.39E-06	5.16E-09
24	0.0012	7.00E-06	4.67E-06	3.11E-06	0.001248	6.49E-04	1.56E-06	3.91E-07	0.00125	1.33E-04	4.67E-06	2.59E-09
25	0.0010	0.000000	2.95E-06	1.97E-06	0.000992	5.25E-04	9.87E-07	2.47E-07	0.00099	1.05E-04	2.95E-06	1.30E-09
26	0.0008	5.00E-06	1.87E-06	1.24E-06	0.000789	3.78E-04	6.23E-07	1.56E-07	0.00079	8.41E-05	1.87E-06	6.54E-10

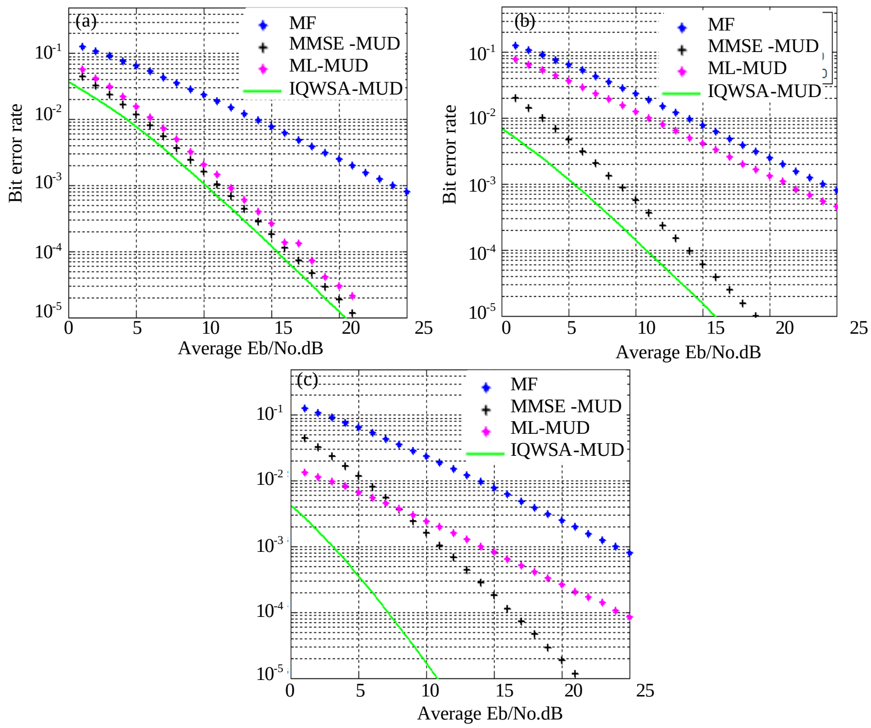

Fig. 9: The BER performance for different modulation formats: a) BER for BPSK modulation; b) BER for QPSK modulation; BER for QAM modulation (Releigh channel)

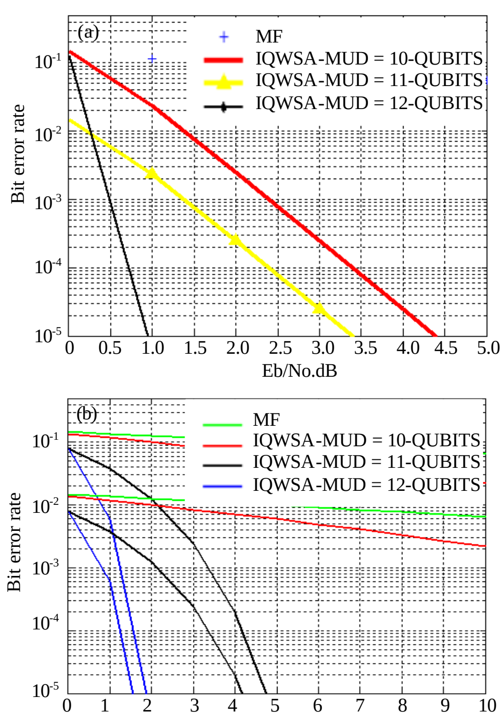

Fig. 10: The BER performance for different Quantum algorithm for k = 2 user and k = 4 user: a,b) BER vsEb/No (Releigh channel)

MUD again matches the ML MUD's as shown in the Fig. 7. The similarity in the BER performance between the quantum and classic ML-MUD is seen in Fig. 7. If more qubits are used in the QCR, the system's performance approaches that of the ML MUD more closely shown in Fig. 8.

The system characterized in Fig. 9 has a higher complexity than that used in Fig. 10. As the number of qubits increases, the precision in the QWSA becomes better. Hence, the performance is improved and it approaches that of the ML MUD, as it is clearly seen in Fig. 9. In all cases, the QWSA MUD performance has more loosely approached that of the ML MUD for J = 4, because the classic decoder helps mitigate both the probabilistic nature of quantum computation and the errors due to the limited precision of the QWSA.

It should be noted that the number of qubits employed in the QCR unambiguously determines the complexity of the algorithm which is independent of the number of the additive terms (Botsinis *et al*., 2013) or in other words, independent of both the number of users and of the modulation scheme employed. For example, IQWSA MUD employing l = 12 qubits results in a complexity of 182 CF evaluations for each user's each bit. Hence, a QWSA MUD relying on l qubits, M-ary modulation, K users and J MUD-decoder iterations will have an overall complexity order of 0 $C = 2^{(l+3)}$ J $\log_2(M^k)$ (Table 2).

Table 2: Performance comparision BER Vs Eb/No on Rayleigh channel

Eb/No. dB	IQWSA-MUD = 10 QUBITS	MF	IQWSA-MUD = 11 QUBITS	IQWSA-MUD = 12 QUBITS
1	0.146447	0.1346	0.211325	0.07865
2	0.023269	0.1142	0.043565	3.87E-06
3	0.002481	0.0972	0.004926	1.04E-45
4	0.00025	8.23E-02	4.99E-04	0.00E+00
5	2.50E-05	6.69E-02	5.00E-05	0.00E+00
6	2.50E-06	5.69E-02	5.00E-06	0.00E+00
7	2.50E-07	4.85E-02	5.00E-07	0.00E+00
8	2.50E-08	4.03E-02	5.00E-08	0.00E+00
9	2.50E-09	3.39E-02	5.00E-09	0.00E+00
10	2.50E-10	2.74E-02	5.00E-10	0.00E+00
11	2.50E-11	2.26E-02	5.00E-11	0.00E+00
12	2.50E-12	1.94E-02	5.00E-12	0.00E+00
13	2.50E-13	1.61E-02	5.00E-13	0.00E+00
14	2.50E-14	1.23E-02	5.00E-14	0.00E+00
15	2.50E-15	1.07E-02	5.00E-15	0.00E+00
16	2.78E-16	0.0089	0.00E+00	0.00E+00
17	0	0.0071	0.00E+00	0.00E+00
18	0	0.0056	0.00E+00	0
19	0	0.0041	0.00E+00	0
20	0	0.0036	0.00E+00	0
21	0	0.0033	0.00E+00	0

Table 3: Comparision of cost function evaluation

No. of Users-K	CF evaluations MF	ML-MUD	IQWSA-MUD
1	496093.3	3.49E-05	5.82E-45
2	4467088	0.152428	1.35E-30
3	14692327	13.55489	6.18E-23
4	31123852	223.2466	3.60E-18
5	52267182	1520.279	6.44E-15
6	76466855	6150.938	1.49E-12
7	1.02E+08	17839.18	9.29E-11
8	1.29E+08	41276.96	2.40E-09
9	1.56E+08	81356.83	3.31E-08
10	1.82E+08	142506.2	2.87E-07
11	2.08E+08	228274.6	1.77E-06
12	2.32E+08	341168	8.28E-06
13	2.56E+08	482661.5	3.14E-05
14	2.79E+08	653314.3	0.0001
15	3.01E+08	852931.7	0.000278

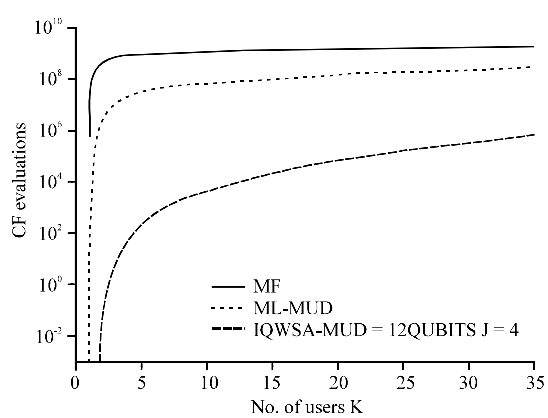

Fig. 11: Computation complexity based on cost function evaluation for classical and quantum methods

If l = 11 is chosen and there are J = 4 iterations between the MUD and the decoder, the IQWSAMUD's complexity becomes less complex than the ML MUD for K>6 users transmitting 8-QAM symbols or for K>10 users employing QPSK modulation (Table 3).

Comparing the proposed IQWSAMUD to the hard-output QMUD that has a complexity of O $[2^{l+1} \log_2 (M^k)]$ CF evaluations operating in a noiseless scenario, we may conclude that its complexity is lower (Fig. 11).

CONCLUSION

In this study, we presented quantum constraints based multiuser detection algorithm which is employed for designing a quantum assisted multiuser detection. Quantum assisted computing exploiting quantum parallelism may help us to show aggression for the optimum MUD problem directly. The process within IQWSA takes place in the quantum domain while the input and outputs are in classical domain enabling to figure out an iterative receiver for the communication system. We showed the computation method to find the error rate using proposed IQWSA algorithm and compared with classical detection methods. The BER curves and performance complexity curves are presented and verified that proposed QMUD is achieved at a reduced computational complexity when compare to classical MUD supporting more number of users and higher modulation formats. Further research plans cover application of other typical channel models and smart handling of entrusted actions in terms of capacity provision.

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