A New Approach for Evaluation of the Energy Losses when Starting-Up Separately Excited DC Motors

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Abstract: In this study, a new approach for evaluating the energy losses at start-up conditions for a separately excited DC motor is developed. It is slightly different from the known methods of evaluation the starting losses described in most references of electric drive. Dynamic equations and analytical results have been derived. The obtained results ensure the justification of accuracy of this approach in evaluation of the losses. In further studies, it is expected to extend the approach for the series excitation DC motors and also for the asynchronous motors.

Keywords: Dynamic equations, energy, electric drive, DC motors, asynchronous motors, Jordan

INTRODUCTION

It is known that the motor is the main convertor of electrical energy into mechanical energy. Therefore, in addressing issues of improving the effectiveness of such transformation is the first necessary to provide arrangements so as to reduce energy losses in the electric drive (Fitzgerald et al., 2003; Hadamard, 1923). They occur both in steady and transient states in the motor. The relation between them depends on the load diagram and they are varying widely. For electric drives with frequent starting and braking, they constitute a significant proportion of the total energy losses for a continuous running duty most of the losses are static. The derived relations for calculation the energy losses can identify opportunities to minimize them which in addition to energy conservation is one of the most important ways to improve the reliability of the electric drive because of the reduction of its heat and sudden loads (Krein, 1998; Hadef and Mekideche, 2009).

To simplify the calculations and identify the main factors affecting on the energy of starting, we will not take into account losses in the excitation circuit, steel and mechanical losses.

The latter as is often done, refer to the shaft of the motor, the field of excitation is the same. This study concentrates on the losses in the power circuit caused by an armature current of the starting process. To determine them by using the conventional approach, the problem is solved in three stages considering the example of the no-load motor starting from the standstill to the no-load speed (Hadef and Mekideche, 2009; Favennec et al., 2002).

MATERIALS AND METHODS

The 1st stage is concerning with the energy consumed by an electric motor from the power supply at the starting period is determined in accordance with the relation (Krein, 1998; Hadef and Mekideche, 2009; Favennec et al., 2002):

\[ A = \int_0^{t_f} UdI = J\omega_0 \]  

Where:
- \( U \) = The voltage applied to the armature
- \( I \) = The armature current
- \( J \) = The total moment of inertia of all rotating parts
- \( \omega_0 \) = The angular velocity at no-load

The 2nd stage is determined by the energy imparted by the rotating mass:

\[ A_m = \int_0^{t_f} P_n dt = \int_0^{t_f} E dt = \frac{J\omega_0^2}{2} \]  

where, \( E \) is the induced emf. Energy losses in the resistance of the armature circuit are defined as the difference between Eq. 1 and 2.

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\[ A_d = \frac{1}{2} \int_0^t R dt = A - A_m = \frac{J_0^2}{2} \quad (3) \]

In contrast to evaluate the energy losses in the armature circuit by using the Eq. 3. We take into account as in Eq. 1, the fact that the value of armature current is:

\[ I = \frac{J}{K_m} \frac{d\omega}{dt} \quad (4) \]

where, \( K_m \) is the motor constant.

**Determination of the motor acceleration:** To determine the derivation of the angular velocity, we use the equation of dynamics (movement) for the separately excited DC motor driven through armature circuit:

\[ (T_m S^2 + T_n S + 1)\alpha(t) = \frac{U}{K_e} - \frac{R}{K_e K_m}(T_m S + 1)M_s \quad (5) \]

Where:
- \( T_m = L/R \) (Electromagnetic time constant)
- \( L \) = The inductance of the armature circuit
- \( T_n = JR/K_e K_m \) (Electro-mechanical time constant)
- \( S = d\omega/dt \) (The symbol of differentiation)
- \( K_e = K_m \) (The constant of electric motor)
- \( M_s = \) Load torque

Since, \( T_s << T_m \) Eq. 5 can be simplified to:

\[ (T_m S + 1)\alpha(t) = \frac{U}{K_e} - \frac{R}{K_e K_m}M_s \quad (6) \]

Starting the motor without load if \((M_s = 0)\) then Eq. 6 transforms to:

\[ (T_m S + 1)\alpha(t) = \frac{U}{K_e} \quad (7) \]

Solution of the Eq. 7 gives the following:

\[ \alpha = \alpha_0 \left[ 1 - e^{-\frac{t}{T_m}} \right] \quad (8) \]

Where:

\[ \alpha_0 = \frac{U}{K_e} \]

The time derivative to Eq. 8 has the form:

\[ \frac{d\omega}{dt} = \frac{\alpha_0 \left[ 1 - e^{-\frac{t}{T_m}} \right]}{T_m} \quad (9) \]

**Energy losses of the armature circuit:** Substituting Eq. 9 in Eq. 3 and integrating the latter, this allows determining the energy losses in the resistances of the armature circuit:

\[ A_d = \int_0^t \left( \frac{J_0^2}{K_m T_m} - \frac{1}{2} \right)^2 R dt = \frac{J_0^2 R_0^2}{K_m^2 T_m^2} \]

\[ = \frac{J_0^2 R_0^2}{2K_m T_m} = \frac{J_0^2 R_0^2}{2K_m T_m} = \frac{J_0^2}{2} \quad (10) \]

This result coincides with other methods. The energy losses in Eq. 10 do not depend on the resistance values in the armature circuit, these losses will be the same for the cases of direct starting methods, starting with additional resistors.

To verify the latter, consider the example of starting-up circuit consists of one stage of resistance. For simplicity, the starting-up with the additional resistance is carried out to reach a speed equals the half of no load one \((\omega = 0.5\omega_n)\). Then the resistance is shunted and further acceleration to \(\omega = \omega_n\) would be carried out only at \(R = R_a\). Then the power losses are the sum of losses at two stages of acceleration. The 1st time constant is:

\[ T_{m1} = \frac{JR}{K_e K_m} \]

and the second one is:

\[ T_{m2} = \frac{JR}{K_e K_m} \]

Accordingly, the currents on the first and second stages of acceleration are presented as:

\[ I_1 = \frac{J_1}{K_m T_{m1}}, I_2 = \frac{J_2}{K_m T_{m2}} \quad (11) \]

Then, the energy losses in the motor can be expressed as:

\[ A_d = \int_0^{0.5\omega_n} I_1^2 R_1 dt + \int_0^{0.5\omega_n} I_2^2 R_2 dt \]

\[ = \frac{J_0^2}{2} \frac{1}{0.5 + 1} + \frac{J_0^2}{2} \frac{1}{0.5 + 0.5} \quad (12) \]

\[ = \frac{J_0^2}{4} + \frac{J_0^2}{4} = \frac{J_0^2}{2} \]
A general case of motor starting-up: Consider now a
general case when the start-up of electric drive overcomes
the additional resistance moment, assume for simplicity,
a constant value. Then the solution for the dynamic
equations can be written as:

\[ \alpha(t) = \left[ \frac{U}{K_m} - \frac{R_s}{K_m} M_0 \right] \left( 1 - e^{-\frac{t}{K_m}} \right) \]

or:

\[ K_m \alpha = \frac{d\alpha}{dt} + M_0 \]

Finally:

\[ I = \frac{J}{K_m} \frac{d\alpha}{dt} + \frac{M_0}{K_m} \]

then:

\[ \frac{d\alpha}{dt} = \frac{\alpha}{T_m} + \frac{1}{T_m} \]

RESULTS AND DISCUSSION

Energy losses in the motor during start-up:

\[ A_{st} = R \int_0^{t_w} I^2 dt = R \int_0^{t_w} \left( \frac{J}{K_m} \alpha \frac{1}{e^{-\frac{t}{T_m}}} + \frac{M_0}{K_m} \right)^2 dt \]

\[ = \frac{J \alpha^2}{2} + 2RM_0 \frac{\alpha}{T_m} + \frac{R}{K_m} M_0^2 t_{st} \text{ret} \]

\[ = \frac{J \alpha^2}{4} + 2T_m \alpha^2 M_0 + t_{st}^2 \text{ret} \]

If the 1st term of this sum is self-explanatory on
the 2nd term, it is advisable to stay. It depends on
the duration of the transition process. If for example,
t_{st} = 4T_m, the 2nd term would be:

\[ 2T_m \alpha M_0 = \frac{\alpha M_0}{2} \]

The last term of losses \( t_{st}^2 \text{ret} \) in the traditional formula
is missing. Let’s try to estimate its value in comparison
with the 2nd term. Suppose that for some motor: \( \omega_s = 150 \)
rad sec\(^{-1} \), \( M_0 = 100 \text{ Nm}, K_m = 1.4 \) and \( R_s = 0.05 \text{ ohm} \) then:

\[ t_{st}^2 \text{ret} = \left( \frac{M_0}{K_m} \right)^2 R_s 4T_m = \left( \frac{100}{1.4} \right)^2 \times 0.05 \times 4T_m = 1020T_m \]

For the 2nd term:

\[ 2T_m \times 150 \times 100 = 30,000 T_m \]

Thus, the ratio of the 2-3rd term of energy losses
is about 30 and the latest in the 1st approximation can be
neglected. Then when the talk is about the approximate
parity, the assessment of energy losses in the traditional
and the present method is almost the same in calculation,
although the proposed method is more accurate. From the
analysis of that expressions, a crucial way to reduce
losses is achieved, the decline of moment inertia of the
rotating masses. However, this is not always possible,
since a greater extent is determined by the properties
of the mechanisms, a lesser extent to reduce the losses in the
armature circuit affects by additional resistance. With its
growth acceleration becomes slow and losses are
increasing to overcome the resistance moment during
start-up period.

However, direct starting to full voltage value is also
often not valid because of high currents and sudden
loads on the mechanism. So by trying on the basis of the
before mentioned method for determining the loss, the
impact on the nature of these are the changes in supply
voltage which are considered before, it was assumed
constant. Now, consider the possibility of changing by
the given laws (Hadeif and Mekideche, 2009). First,
suppose that the voltage varies in 2 stages. The 1st stage
will be at the level 0.5U and the 2nd at U. Then:

\[ A_{st} = R \int_0^{t_{w1}} I^2 dt + \int_0^{t_{w2}} I^2 R dt \]

\[ = \frac{J \alpha^2}{4} + 2T_m \alpha^2 M_0 + t_{st}^2 \text{ret} \]

Accordingly:

\[ I_1 = \frac{J}{K_m} \frac{0.5 \alpha}{T_m} \frac{1}{e^{-\frac{t}{T_m}}} \]

\[ I_2 = \frac{J}{K_m} \frac{0.5 \alpha}{T_m} \frac{1}{e^{-\frac{t}{T_m}}} \]

Substituting the values of currents in Eq. 15 and its
integration gives the following results:

\[ A_{st} = \frac{J \alpha^2}{8} + \frac{J \alpha^2}{8} = \frac{J \alpha^2}{4} = 0.5 \frac{J \alpha^2}{2} \]

From this, it follows that the energy losses compared
with starting at full voltage, decreased in 2 times. When
the starting is with 4 equal steps of voltage then the electrical losses is as follows:

\[ A_d = \frac{J_0^2}{2} \left( \frac{1}{K^1} + \frac{1}{K^2} + \cdots + \frac{1}{K^n} \right) = 0.25 \frac{J_0^2}{2} \]

Extending the above results to start when K (is number of steps) at identical levels of voltage, we get the following:

\[ A_d = \frac{J_0^2}{2} \left( \frac{1}{K^1} + \frac{1}{K^2} + \cdots + \frac{1}{K^n} \right) = \frac{J_0^2}{2} \left( \frac{1}{K} \right) = \frac{J_0^2}{2} \frac{1}{K} \quad (16) \]

Thus, by increasing in the number of steps of the voltage up to infinity, the electrical losses decrease to zero. All these results obtained by the analytical method were verified by simulation on a PC. When the study is using a PC, it can provide a complete mathematical description of the form Eq. 5 but we stopped at the simplistic Eq. 6. This study is done to compare the results of analytical and model of research. The results for the selected motor were similar. This study pays special attention to the explicit agreement of the results when starting an electric motor with a constant resistance moment (Eq. 11). When investigated using PC, it is obtained that the total value of losses for the analytical calculation and summation of these losses are same.

In the study on the PC, an experimented result is obtained with the law of linear increasing of the voltage in the form the value of \( U_1 = k_t \) of an armature to value of \( U_1 = U_{ik} \). The results are shown in Table 1. Table 1 shows that with increasing the time of motor starting at no-load conditions, the losses would be reduced.

**CONCLUSION**

The general conclusion is that the proposed method of estimating the starting losses can be used along with known methods. But for more complicated starting-up conditions, simulation on a PC to obtain accurate results is strongly recommended.

**REFERENCES**


