Reducing PAPR Effect of Coding Schemes with Clipping and Filtering Method on the Performance of OFDM over Noisy Channels

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Abstract: Orthogonal Frequency Division Multiplexing (OFDM) is an attractive technique for wireless communication applications. However, a well-known problem of OFDM is the casual occurrence of high Peak to Average Power Ratio (PAPR) in the time domain signal which reduces the efficiency of transmit high power amplifier. In this study, the investigate through computer simulations, an effective coding schemes of Reed-solomon and concatenated coding with iterative clipping and filtering method on the performance of OFDM including the bit error rate. The bit error rate performance for different modulation schemes is also evaluated for transmissions within an AWGN and fading channels.

Key words: OFDM, clipping, filtering, additive white gaussian noise, fading channel, peak to average power ratio, peak ratio

INTRODUCTION

Orthogonal Frequency-Division Multiplexing (OFDM) is one of the technologies considered for 4G broadband wireless communications due to its robustness against multipath fading and relatively simple implementation compared to single carrier systems. One of the main drawbacks of OFDM is its high Peak to Average Power Ratio (PAPR) (Li and Cimini, 1997). When a high peak signal is transmitted through a nonlinear device such as a High Power Amplifier (HPA) or Digital to Analog Converter (DAC), it generates out of band energy (spectral regrowth) and in-band distortion (constellation tilting and scattering). These degradations may affect the system performance severely. Some other techniques to reduce PAPR on the performance of OFDM are signal processing, coding (random encoding, golay sequence, cyclic code), random scrambling (constellation design), selective mapping, partial transmit sequence. The efficient way of controlling PAPR might be the Clipping and Filtering (CAF) method (Amrani et al., 2008). The clipping process is qualified by the Clipping Ratio (CR), defined as the ratio between the clipping threshold and the rms level of the OFDM signal. Clipping is a nonlinear process that may lead to significant distortion and performance loss. In particular, clipping at the Nyquist sampling rate causes the clipping noise to fall in-band and suffers considerable peak regrowth after digital to analog (D/A) conversion (Jiang and Wu, 2008). Peaks are distorted nonlinearly due to amplifier imperfection. The intermodulation product occurs as effect of nonlinear distortion. They can be interpreted as Inter Carrier Interferences (ICI) and out-of-band radiation. Clipping belongs to simple solutions of this problem. Peaks are cut-off from the signal in simplest case. It is usually followed by filtering to avoid out-of-band radiation (Dinur and Wulich, 2001). So, this clipping technique is very simple to implement. But it requires a low Clipping Ratio (CR) to achieve a significant PAPR reduction. As a consequence, non-linear problems come out again. Out of band problems are eliminated by filtering but the in band noise deteriorates the Bit Error Rate (BER).

The study is focused to reduce PAPR by simplified clipping and filtering method applied for different noisy channels such as AWGN and multipath fading channels. The bit error rate performances for different modulation techniques have been countered. It is shown that clipping and filtering together with FEC coding results in a further improvement in the BER performance and provides further reduction in PAPR.

MATERIALS AND METHODS

OFDM signal and PAPR overview: For an OFDM system with N subcarriers, OFDM signal in baseband notation in discrete form for interval mT_s ≤ t ≤ (m+1) T_s can be expressed as:

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} X_k e^{j2\pi k t}$$

where, f_s = 1/T_s. Replacing t = n · T_s where, T_s = T/N, the discrete time version can be given by:
\[ x_k(t) = \sum_{k=0}^{N-1} X_k e^{j2\pi k t \Delta f} \]  

Where:
\( X_k \) = The symbol carried by the kth subcarrier
\( \Delta f \) = The frequency difference between subcarriers
\( T \) = The OFDM symbol duration

Since the signal which passes through the power amplifier is in the continuous time domain which can be significantly higher than the discrete-time estimate (Shami et al., 2002). Inverse Fast Fourier Transform (IFFT) and Fast Fourier Transform (FFT) are very helpful in OFDM implementation because they can replace complex subchannel modems. IFFT is used at the transmitter and FFT is used at the receiver. If the number of subchannel is denoted by Ns and N-point IFFT is used then the ratio of N/Ns is called oversampling factor. Cyclic Prefix (CP) is required to avoid ISI. CP copies the last part of the OFDM symbol then place it in front of the symbol. CP length is usually about 25% (Pradapet et al., 2008). The PAPR of the signal, \( x(t) \) is then given as the ratio of the peak instantaneous power to the average power written as (Han and Lee, 2005):

\[ \text{PAPR} = \max_{t \in T} \frac{|x(t)|^2}{E[|x(t)|^2]} \]  

where, \(|x(n)|\) returns the magnitude of \( x(n) \), \( E[\cdot] \) is the expectation operator. As more sub-carriers are added, higher peak values may occur hence the PAPR increases proportionally with the number of sub-carriers.

At the transmitter side, the invertible clipping method reduces the amplitude dynamics and thus the PAPR of the signal that has to be amplified. This result is presented in Complementary Cumulative Distribution Function (CCDF) a term which is defined as follows:

\[ \text{CCDF}(\text{PAPR}(x)) = \text{Prob}(\text{PAPR}(x) > \text{PAPR}_0) \]  

This function represents the probability that the PAPR of the OFDM signal exceeds the threshold \( \text{PAPR}_0 \). This invertible clipping method allows reducing the PAPR of the OFDM signal (Al-Kebbi et al., 2009). Filtering after clipping is therefore, compulsory to limit this spectral regrowth and finally to assure a good system performance. A Cyclic Prefix (CP) is then appended to minimize interblock interference and aid the frequency domain equalizer at the receiver. Digital to Analog (D/A) conversion and analog filtering are performed. The clipping process is characterized by the Clipping Ratio (CR) defined as the ratio between the clipping threshold and the root-mean square (rms) level of the OFDM signal:

\[ \text{CR} = \frac{\text{CL}}{\text{rmslevel}} \]  

Where:
\( \text{CL} \) = The clipping level
\( \text{CR} \) = The clipping ratio

The CR in this simulation has different values depends on modulation schemes (Ochiai and Imai, 2002).

**Concatenated codes**: Reed-Solomon codes are nonbinary cyclic codes with m bit symbols exist for all n and k for which 0<k<n=2^n+2. For the most conventional RS (n, k) code \( (n, k) = (2^m-1, 2^m-1-2t) \). RS codes achieve the largest possible code minimum distance with the same encoder input and output block lengths, the code minimum distance for RS code is given by Gallager (1968):

\[ d_{\text{min}} = n-k+1 \]  

The code is capable of correcting any combination of t or fewer errors where, t can be expressed as (Sklar, 2001):

\[ t = \left[ \frac{d_{\text{min}} - 1}{2} \right] = \left[ \frac{n - k}{2} \right] \]  

The RS decoded symbol-error probability, \( P_e \) in terms of the channel symbol-error probability, \( p \) can be written as follows (Odenwalder, 1976):

\[ P_e = \frac{1}{2^{n-1}} \sum_{j=0}^{2^{n-1}-1} j \left( 1-p \right)^{2^{n-1}-1-j} \]  

where, \( t \) is the symbol error correcting capability of the code and the symbols are made up of m bits each. For convolutional code, the code rate \( r \) where \( k \) is the number of parallel input information bits and \( n \) is the number of parallel output encoded bits is defined as \( r = k/n \). The free distance between a pair of convolutional codewords is the hamming distance between the pair of codewords. The minimum free distance is defined as:

\[ d_{\text{free}} = \min \{ d(y, y') : y \neq y' \} \]

where, \( d(y, y') \) is the hamming distance between convolutional codeword and all the zero-codeword. There
are two error probabilities associated with convolutional codes, event error probability $P_e$, bit error probability $P_b$. For hard-decision decoding, the 1st event error and bit error probabilities are defined as:

$$P_e < T(D,N,J) = 2N^{-1}E_{\gamma = L+1}(1)$$

And:

$$P_b < \frac{dT(D,N,J)}{dN} = 2N^{-1}E_{\gamma = L+1}(1)$$

Where:

$$P = Q \left( \frac{2\pi E_b}{\sqrt{N_0}} \right)$$

And:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{t^2}{2}} dt$$

Instead of using a single block code or convolutional code, it is also possible to combine or concatenate two codes. The main advantage of a concatenated code is its larger coding gain and less hardware complexity compared to a single code. Usually in this system, the inner code is a convolutional code and the outer one is a block code, for example a Reed-Solomon code. The reason is that convolutional coding can better correct random errors while Reed-Solomon code cleans up the relatively few remaining errors in the decoded output of the convolutional decoder. The task of the outer and inner interleaver is to break up bursts of errors as much as possible. Compared with a single-coded system, concatenated coding has more delay because of extra interleaving, encoding and decoding which is a disadvantage for packet communications.

**Model description:** Figure 1 shows the Clipping and Filtering mechanism of OFDM transmitter which has been used in this research.

In this setup, the input binary data stream is ensured against errors with Forward Error Correction codes (FECs) techniques (e.g., RS, CC) that can detect with high probability the error location.

The system model combines the use of concatenated as source coding and CF (Clipping and Filtering) method as shown in Fig. 2. These channel codes improve the bit error rate performance by adding redundant bits in the transmitted bit stream that are employed by the receiver to correct errors introduced by the channel. Major simulation parameters have been shown in Table 1 to constitute the system on the performance of OFDM by FFT point 256 for higher data transmission.

![Fig. 1: Clipping and filtering mechanism of OFDM transmitter](image1)

![Fig. 2: Block diagram of OFDM transmitter with concatenated coding scheme](image2)

**Table 1: Simulation parameters**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulations</td>
<td>QPSK, 8-PSK, 16-QAM</td>
</tr>
<tr>
<td>FFT (size)</td>
<td>256</td>
</tr>
<tr>
<td>Oversampling factor</td>
<td>2</td>
</tr>
<tr>
<td>Error correcting code</td>
<td>Convolutional R = 1/2, Reed-Solomon (255, 239, 8)</td>
</tr>
<tr>
<td>Number of iteration</td>
<td>1x-4x</td>
</tr>
<tr>
<td>Clipping ratio</td>
<td>CR = 2</td>
</tr>
<tr>
<td>Noisy channels</td>
<td>AWGN, Rayleigh, Rician</td>
</tr>
</tbody>
</table>

Control (FEC) consists of a Reed-Solomon (RS) outer code (255, 239, 8) and a rate-compatible Convolutional Code (CC) inner code of 1/2 rated.

Now the convolutionally encoded bits are interleaved further prior to convert into each of the either three complex modulation symbols in QPSK, 8-PSK, 16-QAM modulation and cyclic prefix is added to the data once the data is converted into time domain and the new peak signal is eliminated by using clipping method. To reduce the Out-of-Band (OOB), the signal should then be filtered however, filtering causes peak regrowth.

**RESULTS AND DISCUSSION**

The BER performance was studied over noisy channels with the definition of Eb/No. Some results of clipped filtered OFDM have been demonstrated through computer simulations for N = 256 subcarriers. Figure 3-5
Fig. 3: BER performance for OFDM for situations of concatenated, Reed-Solomon coding schemes with clipping and filtering over AWGN channel.

Fig. 4: BER performance for OFDM for situations of concatenated, Reed-Solomon coding schemes with clipping and filtering over Rayleigh channel.

shows the BER performance vs. Eb/No over AWGN and rayleigh rician fading channel models respectively. As a first step BER analysis of clipping were implemented. The number of 61184 bits is mapped in QPSK, 8-PSK, 16-QAM.

For AWGN the BER performance of OFDM system described, the concatenated coding schemes gives the best results while Reed-Solomon coding with clipping gives worse results as shown in Fig. 3. The results are shown in Table 2.

Figure 4 shows the performance of the same experienced system under Rayleigh channel model.

Compared to the performance under concatenated coding schemes with clipping and filtering, OFDM system reveals significantly degraded error rate as shown in Table 3. On the other hand, Reed-Solomon coding with clipping and filtering produces the worse performance.

Another fading channel model rician is considered to investigate the effect of concatenated and Reed-Solomon coding on the performance of discussed system and the results are shown in Fig. 5.

The results says that concatenated coding system recovers coding advantage in a rich scattering environment and are shown in Table 4.
Table 4: BER performance in rich scattering environment with respect to $E_b/N_0$.

<table>
<thead>
<tr>
<th>Coding with clipping and filtering</th>
<th>$E_b/N_0$ in dB</th>
<th>Bit Error Rate (BER)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>For 16 QAM modulation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concatenated coding</td>
<td>13.5</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>Reed-Solomon coding</td>
<td>14.0</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td><strong>For 8 PSK modulation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concatenated coding</td>
<td>12.0</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>Reed-Solomon coding</td>
<td>19.5</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td><strong>For QPSK modulation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concatenated coding</td>
<td>5.5</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>Reed-Solomon coding</td>
<td>14.0</td>
<td>$10^{-4}$</td>
</tr>
</tbody>
</table>

CONCLUSION

This study has shown that employing coding with clipping and filtering method as a technique to reduce PAPR of OFDM signal can result in further improvement in BER performance, if a forward error correction code, like Concatenated coding was also shown to provide an improved BER performance over convolutional coding. The simulation results are in agreement with coding theory and show that implementation of a better or proper clipping and filtering profile together with FEC can reduce the PAPR of OFDM signals and provide significant improvement in the BER performance.

REFERENCES


