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The Generalized Projective Synchronization of Hyperchaotic Lorenz and Hyperchaotic Qi Systems via Active Control

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Abstract: This study investigates the active controller design for Generalized Projective Synchronization (GPS) of identical Hyperchaotic Lorenz Systems, identical Hyperchaotic Qi Systems and Non-Identical Hyperchaotic Lorenz and Hyperchaotic Qi Systems. The GPS synchronization results for the Hyperchaotic Systems have been derived using the active control method and established using Lyapunov Stability theory. Since, the Lyapunov exponents are not required for these calculations, the active control method is a very effective and convenient method for achieving Generalized Projective Synchronization (GPS) of the Hyperchaotic Systems addressed in this study. Numerical simulations are shown to demonstrate the effectiveness of the synchronization results derived in this study for the Hyperchaotic Systems.

Key words: Active control, hyperchaos, generalized projective synchronization, Hyperchaotic Lorenz System, Hyperchaotic Qi System, Hyperchaotic Systems

INTRODUCTION

Chaotic Systems are Nonlinear Dynamical Systems which possess some special features such as being extremely sensitive to small variations of initial conditions having bounded trajectories in the phase space and so on. The sensitive nature of Chaotic Systems is commonly called as the butterfly effect (Alligood *et al*., 1997). The chaos phenomenon was first observed in weather models by Lorenz (1963).

Hyperchaotic System is usually defined as a Chaotic System having more than one positive Lyapunov exponent. Since, Hyperchaotic System has the characteristics of high capacity, high security and high efficiency, it has the potential of broad applications in nonlinear circuits, neural networks, lasers, secure communications, biological systems and so on. The hyperchaos phenomenon was first observed by Rossler (1979).

Synchronization of Chaotic Systems is a phenomenon that may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two Chaotic Systems is seemingly a very challenging research problem.

In most of the chaos synchronization approaches, the master-slave or drive-response formalism is used. If a particular Chaotic System is called a Master or Drive System and another Chaotic System is called a Slave or Response System, then the idea of chaos synchronization is to use the output of the Master System to control the Slave System so that the output of the Slave System tracks the output of the Master System asymptotically.

Chaos is an interesting nonlinear phenomenon and it has been intensively and extensively studied in the last three decades. Chaos theory has wide applications in several fields such as physical systems (Lakshmanan and Murali, 1996), chemical systems (Han *et al*., 1995), ecological systems (Blasius *et al*., 1999), secure communications (Cuomo *et al*., 1993; Kocarev and Parlitz, 1995), etc.

The seminal work by Pecora and Carroll (1990) is followed by a variety of impressive approaches for chaos synchronization such as the Sampled-Data Feedback Synchronization method (Yang and Chua, 1999), the OGY method (Ott *et al*., 1990), the time-delay feedback method (Park and Kwon, 2003), the active control method (Ho and Hung, 2002; Sundarapandian, 2011a, b), the adaptive control method (Chen and Lu, 2002; Sundarapandian, 2011c, d), the backstepping method (Mascolo and Grassi, 1999; Tan *et al*., 2003), the sliding mode control method (Utkin, 1977; Sundarapandian, 2011e) and others. In generalized projective synchronization (Zhou *et al*., 2010), the chaotic systems can synchronize up to a constant scaling matrix. Complete synchronization (Sundarapandian, 2011f), anti-synchronization

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(Sundarapandian, 2011g), hybrid synchronization (Sundarapandian, 2011a-g), projective synchronization (Mainieri and Rehacek, 1999) and generalized synchronization (Wang and Guan, 2006) are special cases of the generalized projective synchronization. The Generalized Projective Synchronization (GPS) has applications in secure communications.

This study addresses the design of active controllers for the Generalized Projective Synchronization (GPS) of identical hyperchaotic Lorenz Systems (Jia, 2007), identical Hyperchaotic Qi Systems (Chen *et al*., 2007) and non-identical Hyperchaotic Lorenz and Hyperchaotic Qi Systems. The synchronization results derived in this paper are established using the Lyapunov Stability theory (Hahn, 1967).

PROBLEM STATEMENT AND METHODOLOGY

Consider the Chaotic System described by the dynamics:

$$\dot{x} = Ax + f(x) \tag{1}$$

where, $x \in R^n$ is the state of the system, A is the n x n matrix of the system parameters and f: $R^n \to R^n$ is the nonlinear part of the system. Researchers consider the system (1) as the Master or Drive System. As the Slave or Response System, we consider the following Chaotic System described by the dynamics:

$$\dot{y} = By + g(y) + u \tag{2}$$

where, $y \in R^n$ is the state of the system, B is the n x n real matrix of the system parameters, g: $R^n \to R^n$ is the nonlinear part of the system and $u \in R^n$ is the controller of the slave system.

If A = B and f = g then x and y are the states of two identical Chaotic Systems. If $A \neq B$ or $f \neq g$ then x and y are the states of two different Chaotic Systems.

In the active control approach, researchers design a feedback controller u which achieves the Generalized Projective Synchronization (GPS) between the states of the master system (1) and the slave system (2) for all initial conditions x (0), z (0)$\in R^n$. For the GPS of the systems (1) and (2), the synchronization error is defined as:

$$e = y - Mx \tag{3}$$

Where:

$$M = \begin{bmatrix} \alpha_1 & 0 & \cdots & 0 \\ 0 & \alpha_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_n \end{bmatrix} \tag{4}$$

In other words, we have:

$$e_i = y_i - \alpha_i x_i, \ (i = 1, 2, ..., n) \tag{5}$$

From Eq. 1-3, the error dynamics is easily obtained as:

$$\dot{e} = By - MAx + g(y) - Mf(x) + u \tag{6}$$

The aim of GPS is to find a feedback controller u so that:

$$\lim_{t \to \infty} \|e(t)\| = 0 \quad \text{for all } e(0) \in R^n \tag{7}$$

Thus, the problem of Generalized Projective Synchronization (GPS) between the Master System (1) and Slave System (2) can be translated into a problem of how to realize the asymptotic stabilization of the system (6). Hence, the objective is to design an active controller u for stabilizing the Error Dynamical System (6) at the origin. Researchers take as a candidate Lyapunov function:

$$V(e) = e^T P e \tag{8}$$

where, P is a positive definite matrix. V: $R^n \to R$ is a positive definite function by construction. We assume that the parameters of the Master and Slave System are known and that the states of both systems (1) and (2) are measurable. If we find a feedback controller u so that:

$$\dot{V}(e) = -e^T Q e \tag{9}$$

where, Q is a positive definite matrix, then $\dot{V}: R^n \to R$ is a negative definite function. Thus, by Lyapunov Stability theory (Hahn, 1967), the error dynamics (6) is globally exponentially stable and hence the condition (7) will be satisfied. Hence, GPS is achieved between the states of the Master System (1) and the Slave System (2).

SYSTEMS DESCRIPTION

The hyperchaotic Lorenz System (Jia, 2007) is described by the dynamics:

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_4 \\ \dot{x}_2 &= -x_1 x_3 + c x_1 - x_2 \\ \dot{x}_3 &= x_1 x_2 - b x_3 \\ \dot{x}_4 &= -x_1 x_3 + d x_4 \end{aligned} \tag{10}$$

where, x_i are the state variables and a-d are positive, constant parameters of the system. The system (10) is hyperchaotic when the parameter values are chosen as:

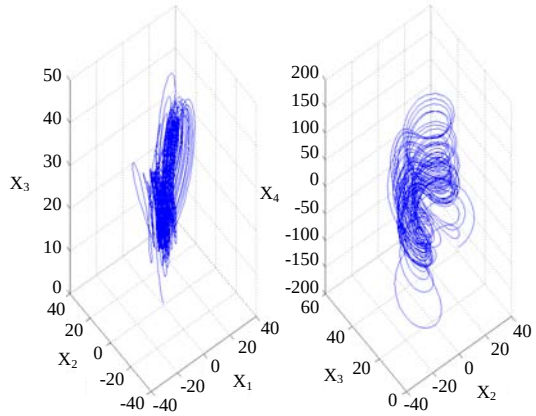

Fig. 1: Phase portrait of the Hyperchaotic Lorenz System

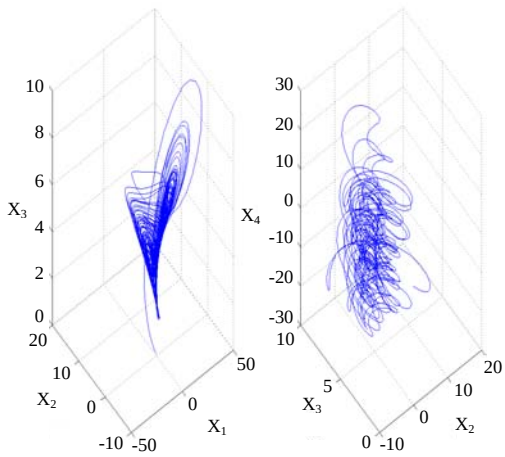

Fig. 2: Phase portrait of the Hyperchaotic Qi System

$$a = 10,\ b = 8/3,\ c = 28,\ d = 1.3$$

Figure 1 shows the phase portrait of the hyperchaotic Lorenz System (10). The Hyperchaotic Qi System (Chen *et al*., 2007) is described by the dynamics:

$$\begin{aligned}\dot{x}_1 &= p(x_2 - x_1) + \varepsilon x_2 x_3\\ \dot{x}_2 &= r x_1 - s x_1 x_3 + x_2 + x_4\\ \dot{x}_3 &= x_1 x_2 - q x_3\\ \dot{x}_4 &= -\lambda x_2\end{aligned} \tag{11}$$

where, x_i (i = 1-4) are the state variables and p-s, ε, λ are positive, constant parameters of the system. The system (11) is hyperchaotic when the parameter values are chosen as:

$$p = 35,\ q = 4.9,\ r = 25,\ s = 5,\ \varepsilon = 35,\ \lambda = 22$$

Figure 2 shows the phase portrait of the hyperchaotic Qi System (11).

GPS OF IDENTICAL HYPERCHAOTIC LORENZ SYSTEMS

Theoretical results: Researchers derive results for the Generalized Projective Synchronization (GPS) of identical hyperchaotic Lorenz systems (Jia, 2007). Thus, the master system is described by the Hyperchaotic Lorenz dynamics:

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + x_4\\ \dot{x}_2 &= -x_1 x_3 + c x_1 - x_2\\ \dot{x}_3 &= x_1 x_2 - b x_3\\ \dot{x}_4 &= -x_1 x_3 + d x_4\end{aligned} \tag{12}$$

Where:

x_1-x_4 = The state variables
a-d = Constant, positive parameters of the system

Also, the Slave System is described by the controlled Hyperchaotic Lorenz dynamics:

$$\begin{aligned}\dot{y}_1 &= a(y_2 - y_1) + y_4 + u_1\\ \dot{y}_2 &= -y_1 y_3 + c y_1 - y_2 + u_2\\ \dot{y}_3 &= y_1 y_2 - b y_3 + u_3\\ \dot{y}_4 &= -y_1 y_3 + d y_4 + u_4\end{aligned} \tag{13}$$

Where:

y_1-y_4 = The state variables
u_1-u_4 = The active controllers of the system

For the GPS of the Hyperchaotic Systems (12) and (13), the synchronization error is defined as:

$$e_i = y_i - \alpha_i x\ ,\quad (i = 1\text{-}4) \tag{14}$$

where, the scales α_1-α_4 are real constants. A simple calculation yields the error dynamics:

$$\begin{aligned}\dot{e}_1 &= a(y_2 - y_1) + y_4 - \alpha_1\left[a(x_2 - x_1) + x_4\right] + u_1\\ \dot{e}_2 &= -y_1 y_3 + c y_1 - y_2 - \alpha_2\left[-x_1 x_3 + c x_1 - x_2\right] + u_2\\ \dot{e}_3 &= y_1 y_2 - b y_3 - \alpha_3\left[x_1 x_2 - b x_3\right] + u_3\\ \dot{e}_4 &= -y_1 y_3 + d y_4 - \alpha_4\left[-x_1 x_3 + d x_4\right] + u_4\end{aligned} \tag{15}$$

We consider the active nonlinear controller defined by:

$$\begin{aligned}u_1 &= -a(y_2 - y_1) - y_4 + \alpha_1\left[a(x_2 - x_1) + x_4\right] - k_1 e_1\\ u_2 &= y_1 y_3 - c y_1 + y_2 + \alpha_2\left[-x_1 x_3 + c x_1 - x_2\right] - k_2 e_2\\ u_3 &= -y_1 y_2 + b y_3 + \alpha_3\left[x_1 x_2 - b x_3\right] - k_3 e_3\\ u_4 &= y_1 y_3 - d y_4 + \alpha_4\left[-x_1 x_3 + d x_4\right] - k_4 e_4\end{aligned} \tag{16}$$

where, the gains k_1-k_4 are positive constants. Substitution of (16) into (15) yields the closed-loop error dynamics:

$$\dot{e}_1 = -k_1 e_1 \quad , \quad \dot{e}_2 = -k_2 e_2 \qquad (17)$$
$$\dot{e}_3 = -k_3 e_3 \quad , \quad \dot{e}_4 = -k_4 e_4$$

Theorem 1: The active controller (16) achieves Generalized Projective Synchronization (GPS) between the identical Hyperchaotic Lorenz Systems (12) and (13) globally and exponentially.

Proof: We prove this result using the Lyapunov Stability theory. We consider the Quadratic Lyapunov function defined by:

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right) \qquad (18)$$

Which is positive definite on R^4. Differentiating (18) along the trajectories of the system (17), we get:

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 \qquad (19)$$

Which is a negative definite function on R^4. Thus, by the Lyapunov Stability theory (Hahn, 1967), it follows that the error dynamics (17) is globally exponentially stable. This completes the proof.

Numerical results: For the numerical simulations, the fourth-order Runge-Kutta method with time step $h = 10^{-8}$ is used to solve the two systems of differential Eq. 12 and 13 with the active controller (16). The parameters of the identical Hyperchaotic Lorenz Systems are chosen as:

$$a = 10, b = 8/3, c = 28, d = 1.3$$

The initial values of the master system (12) are chosen as:

$$x_1(0) = 5, x_2(0) = 11, x_3(0) = 28, x_4(0) = 20$$

The initial values of the slave system (13) are chosen as:

$$y_1(0) = 18, y_2(0) = 22, y_3(0) = 7, y_4(0) = 30$$

The GPS scales α_i are taken as:

$$\alpha_1 = 6.2, \alpha_2 = 2.3, \alpha_3 = 3.7, \alpha_4 = 5.6$$

We take the state feedback gains as:

$$k_1 = 4, k_2 = 4, k_3 = 4, k_4 = 4$$

Figure 3 shows the GPS between the identical Hyperchaotic Lorenz Systems (12) and (13). Figure 4 shows the time history of the error states e_1-e_4.

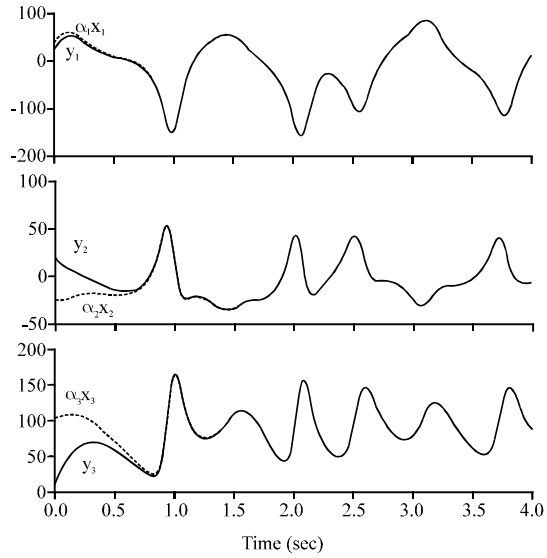

Fig. 3: GPS of the Identical Hyperchaotic Lorenz Systems

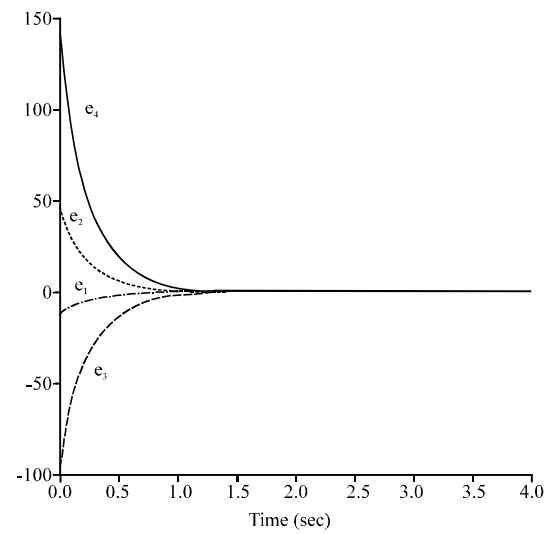

Fig. 4: Time history of the synchronization error

GPS OF IDENTICAL HYPERCHAOTIC QI SYSTEMS

Theoretical results: Researchers derive results for the Generalized Projective Synchronization (GPS) of identical Hyperchaotic Qi Systems (Chen *et al*., 2007). Thus, the Master System is described by the hyperchaotic Qi dynamics:

$$\begin{aligned} \dot{x}_1 &= p(x_2 - x_1) + \varepsilon x_2 x_3 \\ \dot{x}_2 &= r x_1 - s x_1 x_3 + x_2 + x_4 \\ \dot{x}_3 &= x_1 x_2 - q x_3, \quad \dot{x}_4 = -\lambda x_2 \end{aligned} \qquad (20)$$

where, x_1-x_4 are the state variables and p, q, r, s, ε, λ are constant, positive parameters of the system. Also, the

Slave System is described by the controlled hyperchaotic Qi dynamics:

$$\begin{aligned}\dot{y}_1 &= p(y_2 - y_1) + \varepsilon y_2 y_3 + u_1\\ \dot{y}_2 &= ry_1 - sy_1y_3 + y_2 + y_4 + u_2\\ \dot{y}_3 &= y_1y_2 - qy_3 + u_3\\ \dot{y}_4 &= -\lambda y_2 + u_4\end{aligned} \tag{21}$$

Where:

y_1-y_4 = The state variables
u_1-u_4 = The active controllers of the system

For the GPS of the Hyperchaotic Systems (20) and (21), the synchronization error is defined as:

$$e_i = y_i - \alpha_i x_i,\ (i = 1\text{-}4) \tag{22}$$

where the scales α_1-α_4 are real constants. A simple calculation yields the error dynamics:

$$\begin{aligned}\dot{e}_1 &= p(y_2 - y_1) + \varepsilon y_2 y_3 - \alpha_1[p(x_2 - x_1) + \varepsilon x_2 x_3] + u_1\\ \dot{e}_2 &= ry_1 - sy_1y_3 + y_2 + y_4 - \alpha_2[rx_1 - sx_1x_3 + x_2 + x_4] + u_2\\ \dot{e}_3 &= y_1y_2 - qy_3 - \alpha_3[x_1x_2 - qx_3] + u_3\\ \dot{e}_4 &= -\lambda y_2 - \alpha_4[-\lambda x_2] + u_4\end{aligned} \tag{23}$$

We consider the active nonlinear controller defined by:

$$\begin{aligned}u_1 &= -p(y_2 - y_1) - \varepsilon y_2 y_3 + \alpha_1[p(x_2 - x_1) + \varepsilon x_2 x_3] - k_1 e_1\\ u_2 &= -ry_1 + sy_1y_3 - y_2 - y_4 + \alpha_2[rx_1 - sx_1x_3 + x_2 + x_4] - k_2 e_2\\ u_3 &= -y_1y_2 + qy_3 + \alpha_3[x_1x_2 - qx_3] - k_3 e_3\\ u_4 &= \lambda y_2 + \alpha_4[-\lambda x_2] - k_4 e_4\end{aligned} \tag{24}$$

where the gains k_1-k_4 are positive constants. Substitution of (24) into (23) yields the closed-loop error dynamics:

$$\begin{aligned}\dot{e}_1 &= -k_1 e_1 \quad , \quad \dot{e}_2 = -k_2 e_2\\ \dot{e}_3 &= -k_3 e_3 \quad , \quad \dot{e}_4 = -k_4 e_4\end{aligned} \tag{25}$$

Theorem 2: The active controller (24) achieves Generalized Projective Synchronization (GPS) between the identical Hyperchaotic Qi Systems (20) and (21) globally and exponentially.

Proof: We prove this result using the Lyapunov Stability theory. We consider the quadratic Lyapunov function defined by:

$$V(e) = \frac{1}{2}e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right) \tag{26}$$

Which is positive definite on R^4. Differentiating (26) along the trajectories of the system (25), we get:

$$\dot{V}(e) = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 \tag{27}$$

Which is a negative definite function on R^4. Thus, by the Lyapunov Stability theory (Hahn, 1967), it follows that the error dynamics (25) is globally exponentially stable. This completes the proof.

NUMERICAL RESULTS

For the numerical simulations, the fourth-order Runge-Kutta method with time step $h = 10^{-8}$ is used to solve the two systems of differential Eq. 20 and 21 with the active controller (24). The parameters of the identical Hyperchaotic Qi Systems are chosen as:

$$p = 35,\ q = 4.9,\ r = 25,\ s = 5,\ \varepsilon = 35,\ \lambda = 22$$

The initial values of the Master System (20) are chosen as:

$$x_1(0) = 12,\ x_2(0) = 7,\ x_3(0) = 28,\ x_4(0) = 6$$

The initial values of the Slave System (21) are chosen as:

$$y_1(0) = 9,\ y_2(0) = 17,\ y_3(0) = 22,\ y_4(0) = 18$$

The GPS scales α_i are taken as:

$$\alpha_1 = -6.8,\ \alpha_2 = 5.6,\ \alpha_3 = 4.2,\ \alpha_4 = -3.7$$

We take the state feedback gains as:

$$k_1 = 4,\ k_2 = 4,\ k_3 = 4,\ k_4 = 4$$

Figure 5 shows the GPS between the identical Hyperchaotic Qi Systems (20) and (21). Figure 6 shows the time history of the error states e_1-e_4.

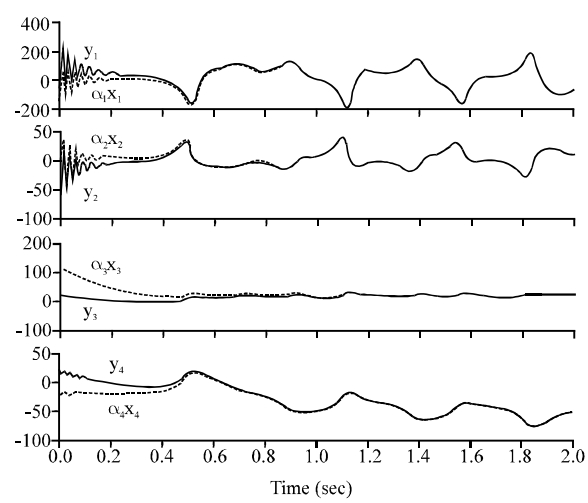

Fig. 5: GPS of the identical Hyperchaotic Qi Systems

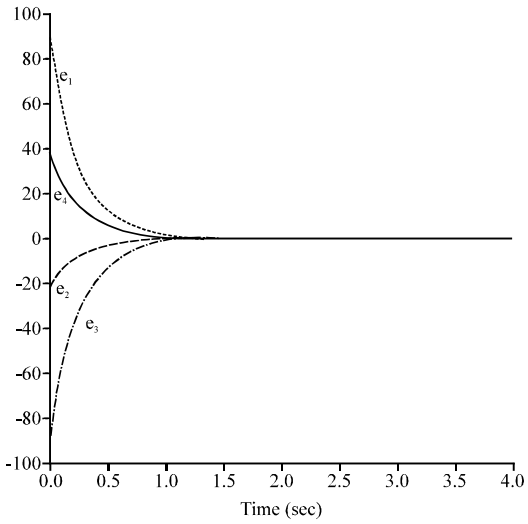

Fig. 6: Time history of the synchronization error

GPS OF NON-IDENTICAL HYPERCHAOTIC LORENZ AND HYPERCHAOTIC QI SYSTEMS

Theoretical results: Researchers derive results for the Generalized Projective Synchronization (GPS) of non-identical hyperchaotic systems, viz., Hyperchaotic Lorenz System (Jia, 2007) and Hyperchaotic Qi System (Chen *et al*., 2007). Thus, the Master System is described by the Hyperchaotic Lorenz dynamics:

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1) + x_4\\ \dot{x}_2 &= -x_1x_3 + cx_1 - x_2\\ \dot{x}_3 &= x_1x_2 - bx_3\\ \dot{x}_4 &= -x_1x_3 + dx_4\end{aligned} \tag{28}$$

Where:

x_1-x_4 = The state variables
a-d = Constant, positive parameters of the system

Also, the Slave System is described by the controlled Hyperchaotic Qi dynamics:

$$\begin{aligned}\dot{y}_1 &= p(y_2 - y_1) + \varepsilon y_2y_3 + u_1\\ \dot{y}_2 &= ry_1 - sy_1y_3 + y_2 + y_4 + u_2\\ \dot{y}_3 &= y_1y_2 - qy_3 + u_3\\ \dot{y}_4 &= -\lambda y_2 + u_4\end{aligned} \tag{29}$$

where, y_1-y_4 are the state variables, p-s, ε, λ are constant, positive parameters of the system and u_1-u_4 are the active controllers of the system. For the GPS of the Hyperchaotic Systems (28) and (29), the synchronization error is defined as:

$$e_i = y_i - \alpha_i x_i,\ (i = 1\text{-}4) \tag{30}$$

where the scales α_1-α_4 are real constants. A simple calculation yields the error dynamics:

$$\begin{aligned}\dot{e}_1 &= p(y_2 - y_1) + \varepsilon y_2y_3 - \\ &\quad \alpha_1\left[a(x_2 - x_1) + x_4\right] + u_1\\ \dot{e}_2 &= ry_1 - sy_1y_3 + y_2 + y_4 - \\ &\quad \alpha_2\left[-x_1x_3 + cx_1 - x_2\right] + u_2\\ \dot{e}_3 &= y_1y_2 - qy_3 - \alpha_3\left[x_1x_2 - bx_3\right] + u_3\\ \dot{e}_4 &= -\lambda y_2 - \alpha_4\left[-x_1x_3 + dx_4\right] + u_4\end{aligned} \tag{31}$$

We consider the active nonlinear controller defined by:

$$\begin{aligned}u_1 &= -p(y_2 - y_1) - \varepsilon y_2y_3 + \\ &\quad \alpha_1\left[a(x_2 - x_1) + x_4\right] - k_1e_1\\ u_2 &= -ry_1 + sy_1y_3 - y_2 - y_4 + \\ &\quad \alpha_2\left[-x_1x_3 + cx_1 - x_2\right] - k_2e_2\\ u_3 &= -y_1y_2 + qy_3 + \alpha_3\left[x_1x_2 - bx_3\right] - k_3e_3\\ u_4 &= \lambda y_2 + \alpha_4\left[-x_1x_3 + dx_4\right] - k_4e_4\end{aligned} \tag{32}$$

where the gains k_1-k_4 are positive constants. Substitution of (32) into (31) yields the closed-loop error dynamics:

$$\begin{aligned}\dot{e}_1 &= -k_1e_1 \quad , \quad \dot{e}_2 = -k_2e_2\\ \dot{e}_3 &= -k_3e_3 \quad , \quad \dot{e}_4 = -k_4e_4\end{aligned} \tag{33}$$

Theorem 3: The active controller (32) achieves Generalized Projective Synchronization (GPS) between the Hyperchaotic Lorenz System (28) and the Hyperchaotic Qi System (29) globally and exponentially.

Proof: We prove this result using the Lyapunov Stability theory. We consider the Quadratic Lyapunov function defined by:

$$V(e) = \frac{1}{2}e^Te = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right) \tag{34}$$

Which is positive definite on R^4. Differentiating (26) along the trajectories of the system (33), we get:

$$\dot{V}(e) = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 \tag{35}$$

Which is a negative definite function on R^4. Thus, by the Lyapunov Stability theory (Hahn, 1967), it follows that the error dynamics (33) is globally exponentially stable. This completes the proof.

Numerical results: For the numerical simulations, the fourth-order Runge-Kutta method with time step is used

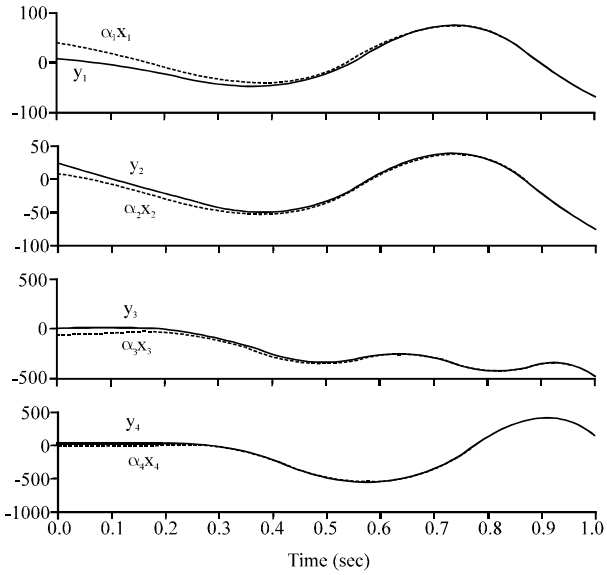

Fig. 7: GPS of the Hyperchaotic lorenz and Hyperchaotic Qi Systems

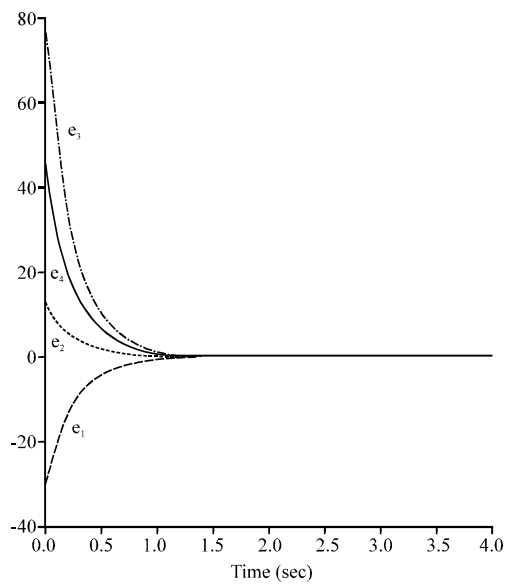

Fig. 8: Time history of the synchronization error

to solve the two systems of differential Eq. 28 and 29 with the active controller (32). The parameters of the Hyperchaotic Lorenz and Hyperchaotic Qi Systems are chosen as:

$$a = 10, b = 8/3, c = 28, d = 1.3$$

$$p = 35, q = 4.9, r = 25, s = 5, \varepsilon = 35, \lambda = 22$$

The initial values of the master system (28) are chosen as:

$$x_1(0) = 15, x_2(0) = 4, x_3(0) = 18, x_4(0) = 20$$

The initial values of the Slave System (29) are chosen as:

$$y_1(0) = 4, y_2(0) = 20, y_3(0) = 6, y_4(0) = 12$$

The GPS scales α_i are taken as:

$$\alpha_1 = 2.3, \alpha_2 = 1.8, \alpha_3 = -3.9, \alpha_4 = -1.7$$

We take the state feedback gains as:

$$k_1 = 4, k_2 = 4, k_3 = 4, k_4 = 4$$

Figure 7 shows the GPS between the non-identical Hyperchaotic Lorenz System (28) and Hyperchaotic Qi System (29). Figure 8 shows the time history of the error states e_1-e_4.

CONCLUSION

In this study, researchers have deployed active control method for achieving Generalized Projective Synchronization (GPS) of the following Hyperchaotic Systems:

- Identical Hyperchaotic Lorenz Systems
- Identical Hyperchaotic Qi Systems
- Non-identical Hyperchaotic Lorenz and Hyperchaotic Qi Systems

The synchronization results (GPS) derived in this study for the Hyperchaotic Systems have been proved using the Lyapunov Stability theory.

Since, the Lyapunov exponents are not required for these calculations, the proposed Active Control Method is very effective and convenient for achieving GPS of the Hyperchaotic systems addressed in this study. Numerical simulations are shown to demonstrate the effectiveness of the synchronization results (GPS) derived in this study.

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