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EOQ Model for Items with Exponential Distribution Deterioration and Linear Trend Demand under Permissible Delay in Payments

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Abstract: In this study, an order-level inventory model is constructed for deteriorating items with instantaneous replenishment, exponential decay rate and a time varying linear demand without shortages under per-missible delay in payments. It is assumed that a constant fraction of the on-hand inventory deteriorates per unit of time. The exact formulae of the optimal average cost and the lot size are derived without carrying out any approximation over the deterioration rate. Different decision making situations illustrated with the help of numerical examples. Sensitivity analysis of the optimal solution with re-spect to changes in the parameter values is carried out.

Key words: Inventory, economic order quantity, deteriorating items, permissible delay in payments, linear trend demand, sensitivity analysis

INTRODUCTION

In formulating inventory models, two factors of the problem have been of growing interest to the researchers, one being the deterioration of items and the other being the variation in the demand rate.

Demand is the major factor in the inventory management. Therefore, decisions of inventory are to be made because of the present and future demands. As demand plays a key role in modeling of deteriorating inventory, researchers have recognized and studied the variations (or their combinations) of demand from the viewpoint of real life situations. Demand may be constant, time-varying, stock-dependent and price-dependent, etc. The constant demand is valid only when the phase of the product life cycle is matured and also for finite periods of time. Wagner and Whithin (1958) discussed the discrete case of the dynamic version of EOQ. Covert and Philip (1973), Misra (1975), Dave (1979) and Sarma (1987), etc., established inventory models with constant demand rate. The classical no-shortage inventory policy for linear trend in demand was discussed by Donaldson (1977). Ritchie (1980, 1984) discussed the solution of the EOQ formula for a linear increasing time-dependent demand which was obtained by Donaldson (1984). EOQ models for deteriorating items with trended demand were considered by Bahari-Kasani (1989), Goswami and Chaudhuri (1991, 1992), Xu and Wang (1990), Chung and Ting (1993, 1994), Kim (1995), Jalan *et al.* (1996), Jalan and Chaudhuri (1999) and Lin *et al.* (2000), etc. In this extent, Begum *et al.* (2009) developed an inventory model with exponential demand rate, finite production rate and shortages. Silver and Meal (1969) developed an approximate solution procedure for the general case of time-varying demand. Generally, this type of demand exists for some particular goods. Many research articles by Silver (1979), Henery (1979), McDonald (1979), Dave and Patel (1981), Sachan (1984), Deb and Chaudhuri (1986), Murdeshwar (1988) and Hargia (1993), etc., analyzed linear timevarying demand. Later, Ghosh and Chaudhuri (2004, 2006), Khanra and Chaudhuri (2003) and Begum *et al.* (2010), etc., established their models with quadratic time-varying demand. Teng and Chang (2005) established an economic production quantity models for deteriorating items with demand depend on price and stock. The deterioration rate of inventory in stock during the storage period constitutes an important factor which has attracted the attention of researchers. In inventory problems, deterioration is defined as damage, decay, spoilage, evaporation, obsolescence and loss of utility or loss of marginal value of goods that results in decrease the usefulness of the original one. Whitin (1957) is the first reseacher who studied an inventory model for fashion goods deteriorating at the end of a prescribed storage period. An exponentially decaying inventory was developed by Ghare and Schrader (1963). Emmons (1968) established a replenishment model for radioactive nuclide generators.

The assumption of the constant deterioration rate was relaxed by Covert and Philip (1973) who used a two parameter Weibull distribution to represent the distribution of time to deterioration. This model was further generalized by Philip (1974) by taking three parameter Weibull distribution deterioration. Shah and Jaiswal (1977) established an order-level-inventory model for perishable items with a constant rate of deterioration.

Recently, Begum *et al*. (2010) has discussed an EOQ model for the deteriorating items with two parameter Weibull distribution deterioration.

The problem of determining the Economic Order Quantity (EOQ) under the condition of a permissible delay in payment has drawn the attention of researchers in recent times. It is assumed that the supplier (wholesaler) allows a delay of a fixed period for settling the amount owed to him/her. There is no interest charged on the outstanding amount if it is paid within the permissible delay period.

Beyond this period, interest is charged. During this fixed period of permissible delay in payments, the customer (a retailer) can sell the items, invest the revenues in an interest-earning account and earn interest instead of paying off the over-draft which is necessary if the supplier requires settlement of the account immediately after replenishment. The customer finds it economically beneficial to delay the settlement to the least moment of the permissible period of delay. This problem was first studied by Goyal (1985) for a non-deteriorating items having a constant demand rate. Chand and Ward (1987) commented in a brief note on some of the assumptions made by Goyal (1985) in analyzing the cost of funds tied up in inventory. The effects of deterioration of goods in stock on the cost and price components cannot be ignored in practice. The model of Goyal (1985) was extended by Aggarwal and Jaggi (1995) to the case of a deteriorating item. Hwang and Shinn (1997) discussed lot sizing policy for an exponentially deteriorating products under the condition of permissible delay in payments when the demand rate would depend on retail price.

In the present study, the researchers assume that the time dependence of demand is linear. Deterioration rate is assumed to be exponential distribution. The rate of replenishment is infinite and shortages are not allowed. The results presented in this study extend and improve the corresponding results of Aggarwal and Jaggi (1995) by taking into account a time-dependent demand rate. The solution procedure involving different decission making situations is illustrated with the help of numerical examples. Analysis is carried out to study sensitivity of the optimal solution to changes in the values of the different parameters involved in the system.

NOTATION AND ASSUMPTIONS

The mathematical model is developed on the basis of the assumptions and notations.

Notation:

- $R(t)$ = Demand rate which is linearly dependent on time i.e., $R(t) = a+bt$ where $a>0$, $b>0$
- θ = Constant rate of deterioration
- A = Ordering cost per order
- c = Unit purchasing cost per item
- s = Unit selling price per item
- I_e = Interest earned per \$ per year
- I_c = Interest charged per \$ in stocks per year
- M = Permissible period (in years) of delay in settling the accounts with the supplier
- T = Time interval (in years) between two successive orders

Assumptions:

- The demand rate for the item is represented by a linear and continuous function
- Replenishment rate is infinite and replenishment is instantaneous
- The lead time is zero
- Shortages are not allowed
- The distribution of time to deterioration of an item follows the exponential distribution g (t):

$$g(t)=\begin{cases}\theta e^{-\theta t}, \text{ for } t>0\\ 0, \text{ otherwise}\end{cases}$$

Where, θ is called the deterioration rate; a constant fraction θ, assumed to be small of the on-hand inventory gets deteriorated per unit time during the cycle time.

- There is no repair or replenishment of deteriorated units in the given cycle
- $s \geq c$, $I_c \geq I_e$
- When $T \geq M$, the account is settled at time $T = M$ and retailer starts paying for the interest charges on the items in stock with rate I_c. When $T \leq M$, the account is settled at $T = M$ and the retailer does not need to pay interest charge
- The retailer can accumulate revenue and earn interest after his/her customer pays for the amount of purchasing cost to the retailer until the end of the trade credit period offered by the supplier. That is the retailer can accumulate revenue and earn interest during the period N to M with rate I_e under the condition of trade credit

MATHEMATICAL FORMULATION

$$\frac{dI(t)}{dt}+\theta I(t)=-R(t), 0 \leq t \leq T \quad (1)$$

Where:

$$I(0) = Q, I(T) = 0 \quad (2)$$

And:

$$R(t) = a + bt \quad (3)$$

Where, a>0, b>0. The solution of Eq. 1 using Eq. 2 and 3 is:

$$I(t)=\frac{1}{\theta}\left[e^{\theta(T-t)}\left(a-\frac{b}{\theta}+bT\right)-\left(a-\frac{b}{\theta}+bT\right)\right] \quad (4)$$

The initial order quantity at t = 0 is:

$$Q=I(0)=\frac{1}{\theta}\left[e^{\theta T}\left(a-\frac{b}{\theta}+bT\right)-\left(a-\frac{b}{\theta}\right)\right] \quad (5)$$

The total demand during one cycle is:

$$\int_0^T (a+bt)dt=aT+\frac{bT^2}{2}$$

Number of deteriorated units:

$$=Q-\left(aT+\frac{bT^2}{2}\right)$$
$$=\frac{1}{\theta}\left[e^{\theta T}\left(a-\frac{b}{\theta}+bT\right)-\left(a-\frac{b}{\theta}\right)\right]-\frac{T}{2}(2a+bT) \quad (6)$$

The cost of stock holding for one cycle is:

$$=h\int_0^T I(t)dt, \text{ where } h=ph_p$$
$$=\frac{h}{\theta}\left[\frac{\left(a-\frac{b}{\theta}+bT\right)}{\theta}\left(e^{\theta T}-1\right)-T\left\{a-\frac{b}{\theta}+\frac{bT}{2}\right\}\right]$$

Hence, the holding cost per unit time is:

$$=\frac{h}{\theta T}\left[\frac{\left(a-\frac{b}{\theta}+bT\right)}{\theta}\left(e^{\theta T}-1\right)-T\left\{a-\frac{b}{\theta}+\frac{bT}{2}\right\}\right] \quad (7)$$

Case 1

Let T>M: Since, the interest is payable during time (T-M), the interest payable in one cycle is:

$$=pI_p\int_M^T I(t)dt$$
$$=\frac{pI_p}{\theta}\left[\frac{1}{\theta}\left(a-\frac{b}{\theta}+bT\right)\left\{e^{\theta(T-M)}-1\right\}-(T-M)\left\{a-\frac{b}{\theta}+\frac{b}{2}(T+M)\right\}\right]$$

Hence, interest payable per unit time is:

$$=\frac{pI_p}{\theta T}\left[\frac{1}{\theta}\left(a-\frac{b}{\theta}+bT\right)\left\{e^{\theta(T-M)}-1\right\}-(T-M)\left\{a-\frac{b}{\theta}+\frac{b}{2}(T+M)\right\}\right] \quad (8)$$

Interest earned per unit time is:

$$=\frac{pI_e}{T}\int_0^T tR(t)dt=pI_eT\left(\frac{a}{2}+\frac{bT^2}{3}\right) \quad (9)$$

Total variable cost per cycle is = Ordering cost+Cost of deteriorated units+Inventory holding cost+Interest payable beyond permissible period-interest earned during the cycle

Hence, total variable cost per unit time in this case is given by:

$$C_1(T)=\frac{A}{T}+\frac{p}{T}\left[\frac{1}{\theta}\left\{e^{\theta T}\left(a-\frac{b}{\theta}+bT\right)-\left(a-\frac{b}{\theta}\right)\right\}-\frac{T}{2}(2a+bT)\right]+$$
$$\frac{h}{\theta T}\left[\frac{\left(a-\frac{b}{\theta}+bT\right)}{\theta}\left(e^{\theta T}-1\right)-T\left\{a-\frac{b}{\theta}+\frac{bT}{2}\right\}\right]+$$
$$\frac{pI_p}{\theta T}\left[\frac{1}{\theta}\left(a-\frac{b}{\theta}+bT\right)\left\{e^{\theta(T-M)}-1\right\}-(T-M)\left\{a-\frac{b}{\theta}+\frac{b}{2}(T+M)\right\}\right]-$$
$$pI_eT\left(\frac{a}{2}+\frac{bT^2}{3}\right) \quad (10)$$

The researchers have now to minimize $C_1(T)$ for a given value of M. The necessary and sufficient conditions to minimize C_1 (T) for a given value of M are respectively:

$$\frac{dC_1(T)}{dT}=0 \quad (11)$$

And:

$$\frac{d^2C_1(T)}{dT^2}>0 \quad (12)$$

After simplification:

$$\frac{dC_1(T)}{dT}=0$$

yields the following nonlinear equation in T:

$$\begin{aligned}&2b\theta\left[p\theta+h+pI_pe^{-\theta M}\right]T^2e^{\theta T}+\\&2\theta\begin{bmatrix}p\theta\left(a-\frac{b}{\theta}\right)+h\left(a-\frac{b}{\theta}\right)+\\pI_p\left(a-\frac{b}{\theta}\right)e^{-\theta M}\end{bmatrix}Te^{\theta T}\\&-2\left(a-\frac{b}{\theta}\right)\left[p\theta+h+pI_pe^{-\theta M}\right]e^{\theta T}-\\&\frac{4}{3}pI_eb\theta^2T^2-\theta\begin{bmatrix}b(h+p\theta)\\+paI_e\theta+pbI_p\end{bmatrix}T^2\\&2\left(a-\frac{b}{\theta}\right)\left[h+p\theta-pI_p\theta M+pI_p\right]-\\&2s\theta^2-pbI_p\theta M^2=0\end{aligned} \quad (13)$$

By solving Eq. 13 for T, we obtain the optimal cycle length $T = T_1^*$ provided it satisfies Eq. 12. The EOQ q_0^* for this case is given by:

$$q_0^*(T_1^*)=\frac{1}{\theta}\left[e^{\theta T_1^*}\left(a-\frac{b}{\theta}+bT_1^*\right)-\left(a-\frac{b}{\theta}\right)\right]$$

The minimum annual variable cost $C_1(T_1^*)$ is then obtain from Eq. 10 for $T = T_1^*$.

Case 2

T<M: In this case, the customer earns interest on the sales revenue up to the permissible delay period and no interest is payable during this period for the items kept in stock. Interest earned up to T is:

$$pI_e\int_0^T(a+bt)t\,dt=pI_e\left(\frac{a}{2}T^2+\frac{b}{3}T^3\right)$$

and interest earned during (M−T) i.e., up to the permissible delay period is:

$$pI_e(M-T)\int_0^T D(t)dt=pI_e\left(a+\frac{b}{2}T\right)(M-T)T$$

Hence, the total interest earned during the cycle is:

$$\begin{aligned}&pI_e\left(\frac{a}{2}T^2+\frac{b}{3}T^3\right)+pI_e\left(a+\frac{b}{2}T\right)(M-T)T=\\&pI_eT\left\{(bM-a)\frac{T}{2}-\frac{1}{6}bT^2+aM\right\}\end{aligned} \quad (14)$$

Total variable cost per cycle = Ordering cost+Cost of deteriorated units+ Inventory holding cost- Interest earned during the cycle

Hence, the total variable cost per unit time is:

$$\begin{aligned}C_2(T)=&\frac{A}{T}+\frac{p}{T}\left[\frac{1}{\theta}\left\{\begin{matrix}e^{\theta T}\left(a-\frac{b}{\theta}+bT\right)-\\\left(a-\frac{b}{\theta}\right)\end{matrix}\right\}-\frac{T}{2}(2a+bT)\right]+\\&\frac{h}{\theta T}\left[\frac{a-\frac{b}{\theta}+bT}{\theta}\left(e^{\theta T}-1\right)-T\left\{a-\frac{b}{\theta}+\frac{bT}{2}\right\}\right]-\\&pI_e\left[(bM-a)\frac{T}{2}-\frac{1}{6}bT^2+aM\right]\end{aligned} \quad (15)$$

The researchers have now to minimize $C_2(T)$ as before for a given value of M. After simplification:

$$\frac{dC_2(T)}{dt}=0$$

yields the result:

$$\begin{aligned}&-s\theta-p\theta\left[\frac{1}{\theta}\left\{\begin{matrix}e^{\theta T}\left(a-\frac{b}{\theta}+bT\right)-\\\left(a-\frac{b}{\theta}\right)\end{matrix}\right\}-\frac{T}{2}(2a+bT)\right]+\\&p\theta T\begin{bmatrix}\frac{1}{\theta}\left\{be^{\theta T}+\left(a-\frac{b}{\theta}+bT\right)\theta e^{\theta T}\right\}\\-(a+bT)\end{bmatrix}-\\&h\begin{bmatrix}\left(a-\frac{b}{\theta}+bT\right)\left(e^{\theta T}-1\right)\frac{1}{\theta}-\\T\left\{a-\frac{b}{\theta}+\frac{1}{2}bT\right\}\end{bmatrix}+\\&hT\left[\left(a-\frac{b}{\theta}+bT\right)e^{\theta T}+\frac{be^{\theta T}}{\theta}-(a+bT)\right]-\\&pI_e\theta T^2\left\{\frac{1}{2}(bM-a)-\frac{1}{3}bT\right\}=0\end{aligned} \quad (16)$$

The optimal cycle length $T = T_2^*$ which minimizes $C_2(T)$ is obtained by solving Eq. 16 for T by using the Newton-Raphson method, provided:

$$\frac{d^2C_2(T)}{dT^2} > 0$$

The EOQ in this case is given by:

$$q_0^*(T_2^*)=\frac{1}{\theta}\left[e^{\theta T_2^*}\left(a-\frac{b}{\theta}+bT_2^*\right)-\left(a-\frac{b}{\theta}\right)\right]$$

and the minimum annual variable cost $C_2(T_2^*)$ is obtained from Eq. 15 for $T = T_2^*$.

Case 3

T = M: For T = M, both the cost functions $C_1(T)$ and $C_2(T)$ become identical and it is denoted by C(M), say and it is obtained on substituting T = M either in Eq. 10 or in Eq. 15. Thus:

$$C(M)=\frac{A}{M}+\frac{p}{M}\left[\frac{1}{\theta}\left\{\begin{matrix} e^{\theta M}\left(a-\frac{b}{\theta}+bM\right) \\ -\left(a-\frac{b}{\theta}\right)\end{matrix}\right\} -\frac{M}{2}(2a+bM)\right]+ \frac{h}{\theta M}\left[\frac{a-\frac{b}{\theta}+bM}{\theta}\left(e^{\theta M}-1\right)-M\left\{a-\frac{b}{\theta}+\frac{bM}{2}\right\}\right]- \frac{1}{6}pI_eM(3a+2bM) \qquad (17)$$

The EOQ is:

$$q_0^*(M)=\frac{1}{\theta}\left[e^{\theta M}\left(a-\frac{b}{\theta}+bM\right)-\left(a-\frac{b}{\theta}\right)\right]$$

Now in order to obtain the economic operating policy, the following steps are to be followed:

- Step 1: Determine T_1^* from Eq. 13. If $T_1^* \geq M$, obtain $C_1(T_1^*)$ from EQ. 10
- Step 2: Determine T_2^* from Eq. 16. If $T_2^* < M$, evaluate $C_2(T_2^*)$ from Eq. 15
- Step 3: If $T_1^* < M$ and $T_2^* \geq M$ then, evaluate C(M) from Eq. 17
- Step 4: Compare $C_1(T_1^*)$, $C_2(T_2^*)$ and C(M) and take the minimum

NUMERICAL EXAMPLES

The numerical examples given, covers all the three cases that arise in this model:

Example 1 (Cases 1 and 2): Let a = 1000 units year^{-1}, b = 150 units year^{-1}, I_p = 0.15 year^{-1}, I_e = 0.13 per year, s = Rs. 200 per order, h_p = Rs. 0.12 year^{-1}, p = Rs. 20 per unit, M = 0.25 year, θ = 0.20.

Solving Eq. 13, we have $T_1^* = 0.284$ and the minimum average cost is $C_1(T_1^*) = 1283.53$. Solving Eq. 16, the researchers have $T_2^* = 0.206$ and the minimum average cost is $C_2(T_2^*) = 1263.53$. Here $T_1^*>M$ and $T_2^*<M$ both hold and this implies that both the cases 1 and 2 hold. Now $C_2(T_2^*)<C_1(T_1^*)$. Hence, the minimum average cost in this case is $C_2(T_2^*)$ = Rs. 1263.53 where the optimal cycle length is $T_2^* = 0.206$ year <M. The economic order quantity is given by $q_0^*(T_2^*) = 213.82$ units.

Example 2 (Case 1): Let a = 1000 units year^{-1}, b = 150 units year^{-1}, I_p = 0.15 year^{-1}, I_e = 0.13 year^{-1}, s = Rs. 200 per order, h_p = Rs. 0.12 year^{-1}, p = Rs. 20 per unit, M = 0.25 year, θ = 0.01.

Solving Eq. 13 for T, the researchers get $T_1^* = 0.432$ and the minimum average cost is $C_1(T_1^*) = 585.31$. Again, solving Eq. 16, the researchers have $T_2^* = 0.274$ and the corresponding minimum average cost is $C_2(T_2^*) = 793.94$.

Here $T_2^*>M$ which contradicts case 2. In this case $T_1>M$ which is case 1. Therefore, the minimum average cost in this case is $C_1(T_1^*)$ = Rs. 585.31, the EOQ is $q_0^*(T_1^*)$ = 447.23 units and the optimal cycle length is T_ 1 = 0.432 year >M.

Example 3 (Case 2): Let a = 1000 units year^{-1}, b = 150 units year^{-1}, I_p = 0.15 year^{-1}, I_e = 0.13 year^{-1}, s = Rs. 200 per order, h_p = Rs. 0.12 year^{-1}, p = Rs. 40 per unit, M = 0.25 year, θ = 0.20.

Solving Eq. 13 for T, the researchers obtain the optimal value $T_1^* = 0.232$ and the optimal cost $C_1(T_1^*) = 1792.29$. Here, $T_1^*<M$ which contradicts case 1. Again, solving Eq. 16, the researchers have $T_2^* = 0.147$ and the minimum average cost is $C_2(T_2^*) = 1395.29$. In this case, $T_2^*<M$ which is case 2. Therefore, the minimum average cost in this case is $C_2(T_2^*)$ = Rs. 1395.29, the EOQ is $q_0^*(T_2^*) = 150.81$ units and the optimal cycle length is $T_2^* = 0.147$ year <M.

Example 4 (Case 3): Let a = 1300 units year^{-1}, b = 100 units year^{-1}, I_p = 0.5 year^{-1}, I_e = 0.01 year^{-1}, s = Rs. 97 per order, h_p = Rs. 0.12 year^{-1}, p = Rs. 40 per unit, M = 0.09 year, θ = 0.3.

In this case $T_1^* = T_2^* = 0.09 = M$ which is case 3. The optimal cost in this case is C (M) = Rs. 2050.56 and the EOQ is $q_0^* = 119.01$ units for M = 0.09 year.

Table 1: Sensitivity of the optimal solution to changes in parameter values

Parameters	Changes (%)	T_1*	C_1 (T_1*)	T_2*	C_2 (T_2*)	Remarks	Solution
a	+50	0.213	2229.626	0.121	1345.365	T_2*<M	C_2 (T_2*)
	+20	0.223	1971.013	0.135	1389.847	T_2*<M	C_2 (T_2*)
	-20	0.246	1605.640	0.164	1374.495	T_2*<M	C_2 (T_2*)
	-50	0.279	1299.320	0.204	1269.928	T_1*>M, T_2*<M	C_2 (T_2*)
b	+50	0.231	1802.880	0.146	1396.504	T_2*<M	C_2 (T_2*)
	+20	0.232	1796.539	0.147	1395.781	T_2*<M	C_2 (T_2*)
	-20	0.233	1788.024	0.147	1394.798	T_2*<M	C_2 (T_2*)
	-50	0.234	1781.591	0.147	1394.047	T_2*<M	C_2 (T_2*)
h_p	+50	0.215	2070.220	0.138	1570.236	T_2*<M	C_2 (T_2*)
	+20	0.225	1906.244	0.143	1466.622	T_2*<M	C_2 (T_2*)
	-20	0.241	1674.154	0.151	1321.984	T_2*<M	C_2 (T_2*)
	-50	0.256	1488.002	0.158	1207.922	T_1*>M, T_2*<M	C_2 (T_2*)
I_e	+50	0.257	1467.189	0.138	924.703	T_1*>M, T_2*<M	C_2 (T_2*)
	+20	0.241	1666.209	0.143	1208.435	T_2*<M	C_2 (T_2*)
	-20	0.224	1913.756	0.151	1580.138	T_2*<M	C_2 (T_2*)
	-50	0.214	2088.299	0.158	1853.247	T_2*<M	C_2 (T_2*)
I_p	+50	0.236	1793.970	0.147	1395.292	T_2*<M	C_2 (T_2*)
	+20	0.234	1793.044	0.147	1395.292	T_2*<M	C_2 (T_2*)
	-20	0.231	1791.388	0.147	1395.292	T_2*<M	C_2 (T_2*)
	-50	0.228	1789.846	0.147	1395.292	T_2*<M	C_2 (T_2*)
p	+50	0.212	2238.779	0.120	1345.124	T_2*<M	C_2 (T_2*)
	+20	0.223	1974.962	0.134	1390.033	T_2*<M	C_2 (T_2*)
	-20	0.246	1600.895	0.164	1373.515	T_2*<M	C_2 (T_2*)
	-50	0.284	1283.529	0.206	1263.527	T_1*>M, T_2*<M	C_2 (T_2*)
s	+50	0.259	2198.801	0.179	2008.266	T_1*>M, T_2*<M	C_2 (T_2*)
	+20	0.244	1960.307	0.161	1655.291	T_2*<M	C_2 (T_2*)
	-20	0.221	1615.720	0.132	1108.257	T_2*<M	C_2 (T_2*)
	-50	0.201	1331.351	0.105	600.396	T_2*<M	C_2 (T_2*)
θ	+50	0.203	2256.024	0.133	1686.162	T_2*<M	C_2 (T_2*)
	+20	0.219	1984.961	0.141	1515.115	T_2*<M	C_2 (T_2*)
	-20	0.248	1588.024	0.154	1270.099	T_2*<M	C_2 (T_2*)
	-50	0.278	1254.317	0.168	1070.399	T_1*>M, T_2*<M	C_2 (T_2*)
M	+50	0.293	1938.772	0.147	738.118	T_1*>M, T_2*<M	C_2 (T_2*)
	+20	0.256	1831.631	0.147	1132.425	T_1*>M, T_2*<M	C_2 (T_2*)
	-20	0.211	1788.009	0.147	1658.157	T_2*<M	C_2 (T_2*)
	-50	0.186	1872.180	0.147	2052.448	T_2*<M	C_2 (T_2*)

Table 2: Results for p = Rs. 20 per unit

M	θ	T_1* or T_2* (years)	C_1 (T_1*) or C_2 (T_2*)	q_0*	Optimal cycle length
0.00	0.01	0.325	1115.97	362.11	T_1*
	0.10	0.277	1415.06	286.81	T_1*
	0.20	0.232	1686.68	241.77	T_1*
0.05	0.01	0.357	972.35	366.84	T_1*
	0.10	0.281	1273.61	290.54	T_1*
	0.20	0.235	1547.23	244.92	T_1*
0.10	0.01	0.367	849.36	378.21	T_1*
	0.10	0.289	1158.93	299.53	T_1*
	0.20	0.242	1440.23	252.49	T_1*

SENSITIVITY ANALYSIS

The researchers now study sensitivity of the solution of the problem to changes in the values of the parameters of the systems. Example 3 is used for this purpose and the results are shown in Table 1. It is found that the solution is not sensitive to changes in the values of the parameters b and Ip. However, it is sensitive to changes in the values of the parameters a, h_p, I_e, p, s, θ and M.

In Table 2, we present the optimal solutions, as M and θ vary, for a less expensive item (p = Rs. 20) when the parameter values are a = 1000, b = 150, h_p = 0.12, I_p = 0.15, I_e = 0.13 and s = 200 in appropriate units. We observe the the following characteristics of the solution:

Table 3: Results for p = Rs. 40 per unit

M	θ	T_1* or T_2* (years)	C_1 (T_1*) or C_2 (T_2*)	q_0*	Optimal cycle length
0.00	0.01	0.252	1569.92	256.62	T_1*
	0.10	0.198	1989.25	203.15	T_1*
	0.20	0.166	2370.09	171.08	T_1*
0.05	0.01	0.257	1293.44	260.36	T_1*
	0.10	0.207	1719.89	207.68	T_1*
	0.20	0.170	2107.29	174.90	T_1*
0.10	0.01	0.271	1072.94	277.27	T_1*
	0.10	0.214	1522.66	219.44	T_1*
	0.20	0.180	1931.33	184.79	T_1*

- The cycle length increases marginally, the order quantity increases slightly and the cost decreases slightly as the credit period M increases, keeping θ fixed
- As the value of θ increases, keeping the credit period M fixed, there is significant reduction in both the cycle length and the order quantity while the cost increases considerably

A similar analysis is made in Table 3 for a more expensive (p = Rs. 40) item. The same type of results is observed in this case also. In Table 4, it is observed that

Table 4: Results for p = Rs. 200 per unit

M	θ	T_1* or T_2* (years)	C_1 (T_1*) or C_2 (T_2*)	q_0*	Optimal cycle length
0.00	0.01	0.114	3485.11	115.14	T_1*
	0.10	0.090	4411.78	91.08	T_1*
	0.20	0.076	5253.46	76.60	T_1*
0.05	0.01	0.125	2285.97	125.95	T_1*
	0.10	0.098	3296.12	99.62	T_1*
	0.20	0.083	4213.92	83.78	T_1*
0.10	0.01	0.152	1589.08	153.52	T_1*
	0.10	0.075	2697.22	75.93	T_2*
	0.20	0.066	3413.56	67.00	T_2*

the cycle length, the order quantity and the average system cost all undergo considerable changes for a very expensive (p = Rs. 200) item.

CONCLUSION

The present model seeks an extension of Aggarwal and Jaggi (1995) work by taking a time-dependent demand rate into consideration. This consideration makes the model more realistic. A linear, time-dependent demand rate implies a steady increase in the demand of the product. This type of demand pattern is observed in the market in the case of many products. Several numerical illustrations are given to explain the solution procedure of the model. Sensitivity analysis is also carried out. Comparison of this model with that of Aggarwal and Jaggi (1995) shows that there are slight changes in the optimal solutions when a time-dependent demand rate is taken into consideration. As a result of the demand rate varying linearly with time, both the cycle length and the order quantity decreases whereas the average system cost increases in comparison to the results of Aggarwal and Jaggi (1995).

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