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Fuzzy Kalman Filtering of the Slam Problem Using Pseudo-Linear Models with Two-Sensor Data Association

Chandima Dedduwa Pathiranage, Keigo Watanabe and Kiyotaka Izumi
Department of Advanced Systems Control Engineering,
Graduate School of Science and Engineering, Saga University 1-Honjomachi,
Saga 840-8502, Japan

Abstract: This study describes a Takagi-Sugeno (T-S) fuzzy model based solution to the SLAM problem. A less error prone vehicle process model is used to improve the accuracy and the faster convergence of state estimation. Vehicle motion is modeled using vehicle frame translation derived from successive dead-reckoned poses as a control input. Nonlinear process model and observation model are formulated as pseudo-linear models and rewritten with a composite model whose local models are linear according to T-S fuzzy model. Linear Kalman filter equations are then used to estimate the state of the local linear models. Combination of these local state estimates results in global state estimate. Stability of the fuzzy observer is addressed through the assessment of local covariance estimates. Data association to correspond features to the observed measurement is proposed with two sensor frames obtained from two sensors. The above system is implemented and simulated with Matlab to claim that the proposed method yet finds a better solution to the SLAM problem. The proposed method shows a way to use nonlinear systems in Kalman filter estimator without using Jacobian matrices. Pseudo-linear model which preserves the original information in nonlinear systems avoids direct linearization as used in EKF. It is found that a fuzzy logic based approach with the pseudo-linear models provides a remarkable solution to state estimation process because fuzzy logic always stands for a better solution.

Key words: Simultaneous localization and mapping, pseudo-linear modeling, fuzzy Kalman filtering, T-S fuzzy model, data association, stability and consistency

INTRODUCTION

Introducing an alternative technique to solve the simultaneous localization and mapping (SLAM) problem is yet another step forward to advancement of the ongoing research activities. In the domain of solutions to the SLAM problem, there can be alternative and advanced solutions but not yet identified. Accuracy of the solution depends on the design of estimator and the data association scheme used to correspond the environmental features to the observed measurements. Data association is the toughest handling part of SLAM problem. Low computationally complex and simpler scheme for obtaining the solution attracts attention in such a problem. Extraction of a solution to the SLAM problem featuring above merits is described in this study for a vehicle and landmarks system.

The SLAM problem (Durrant-Whyte and Bailey, 2006; Bailey and Durrant-Whyte, 2006), also known as concurrent mapping and localization (CML) problem, is often recognized in the robotics literature as one of the key challenges in building autonomous capabilities for mobile vehicles. The goal of an autonomous vehicle performing SLAM is to start from an unknown location in an unknown environment and build a map (consisting of environmental features) of its environment incrementally by using the uncertain information extracted from its sensors, whilst simultaneously using that map to localize itself with respect to a reference coordinate frame and navigate in real time.

A vehicle capable of performing SLAM using naturally occurring environmental features and capable of running for hours or possibly days in completely unknown and unstructured environments will indeed be invaluable in several key areas of robotics. These include autonomous vehicle operation in unstructured terrain, driver-assistance systems, mining, surveying, cargo handling, autonomous underwater explorations, aviation

Corresponding Author: Chandima Dedduwa Pathiranage, Department of Advanced Systems Control Engineering, Graduate School of Science and Engineering, Saga University 1-Honjomachi, Saga 840-8502, Japan

applications, autonomous planetary exploration and military applications. The first solution to the SLAM problem was proposed by Smith *et al.* (1987). They emphasized the importance of map and vehicle correlations in SLAM and introduced the extended Kalman filter (EKF)-based stochastic mapping framework, which estimated the vehicle pose and the map feature (landmark) positions in an augmented state vector using second order statistics. Although the EKF-based SLAM within the stochastic mapping framework gained wide popularity among the SLAM research community, over time, it was shown to have several shortcomings (Leonard and Durrant-White, 1991; Dissanayake *et al.*, 2001). Notable shortcomings are its susceptibility to data-association errors and inconsistent treatment of nonlinearities.

EKF-based stochastic mapping approach is still considered to be the primary framework of most feature-based stochastic SLAM algorithms. But, the EKF suffers mainly from a linearization problem because it adopts direct linearization of nonlinear models. Here, we propose some remedies to overcome the shortcomings of the EKF algorithm. To preserve the nonlinearity of the system, motion and observation models are represented by the pseudo-linear models (Li and Jikov, 2001; Whitcombe, 1972). This avoids the direct linearization of the system. Discrete time motion model is derived from the dead-reckoned measurements of the vehicle pose as to reduce the error associated with the control inputs. This assures a motion model prone to be less error enabling a faster convergence. We propose a fuzzy Kalman Filter (FKF) state estimator with a pseudo-linear model as an alternative to the EKF and thereby obtain an improved solution to the SLAM problem. Fuzzy logic has been a promising reasoning tool for the nonlinear systems. Fuzzy state estimation is a topic that has received very little attention. Fuzzy Kalman filtering (Chen *et al.*, 1998) is a recently proposed method to the EKF to the case where the linear system parameters are fuzzy variables within intervals.

Data association, registration, or the correspondence problem is one of the extremely difficult problems encountered in SLAM even in static environments and much more challenging in dynamic environments consisting of objects moving at varying velocities. Almost every state estimation algorithm has to deal with the correspondence problem in the form of maximum-likelihood assignment or correlation search in establishing the correspondence between the elements of observations and the available features. Uncertainties in vehicle pose, variable feature densities, dynamic objects in the environment and spurious measurements complicate data association in the SLAM problem in many respects. An efficient data-association scheme must aid feature or track initialization, maintenance, termination and map management. The most widely employed data association algorithm in SLAM is the nearest neighbor data association algorithm (Dissanayake *et al.*, 2001; Bailey, 2002). Nearest neighbor algorithm with single sensor frame is susceptible to false data association.

This study uses the nearest neighbor data association algorithm with two sensor frames that ultimately improves data association, instead of using a single frame. We use two sensor frames to associate an observation to a feature in the SLAM state vector. The feature quality algorithm employed in the study is almost similar to the algorithm used in Dissanayake ***et al.*** (2001). This algorithm is considered to be effective in regard to map management and feature initialization. The proposed T-S fuzzy model (Takagi and Sugeno, 1985) based algorithm to the SLAM problem (FKF-SLAM) has shown that a demanding (not conventional) solution to the SLAM problem exists and it overcomes limitations of the EKF-based SLAM, hinting a new path explored is much suitable for finding an advanced solution to localization and mapping problems.

VEHICLE MODEL AND ODOMETRY

In the history of SLAM problem, it has been the common practice of generating the motion model with forward velocity and steering angle as control inputs. In this representation, measurement errors in control inputs propagate into the next stage with the same noise strength. But, the present model has control inputs prone to be less error because control inputs to the motion model are derived from the successive dead-reckoned poses, where the current dead-reckoned pose subtracts the immediate previous dead-reckoned pose to generate the control inputs. It is hopeful that this subtracts the common dead-reckoned error so as to generate control inputs with low noise level.

Dead-reckoned odometry measurement: Assume that left and right wheels of radius r mounted on both sides of the rear axle turn amounts $\delta\theta_l$ and $\delta\theta_r$ in one time interval, as shown in Fig. 1. We want to express the change of position of the center of rear axle of the vehicle (δx_o, δy_o) and the change of orientation ($\delta\phi_o$) as a function of $\delta\theta_l$ and $\delta\theta_r$. From the geometrical relationship of Fig. 1, it is easy to see that

$$r\delta\theta_r = (c - L/2)\alpha,\ \ r\delta\theta_l = (c + L/2)\alpha \qquad (1)$$

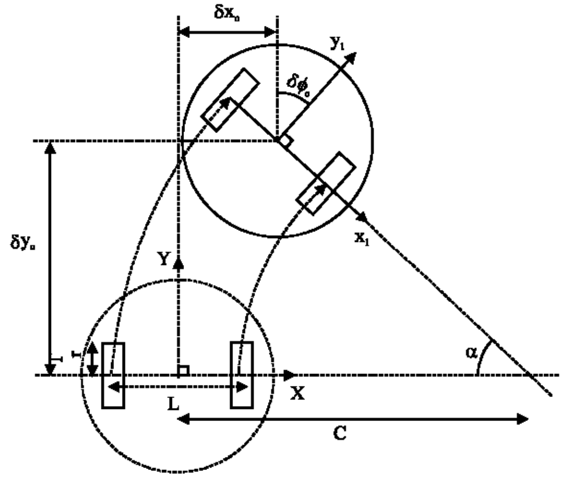

Fig. 1: Geometric construction of rear wheel movements

Solving above 2 equations in Eq. 1 for c and α, we obtain

$$c = \frac{L}{2}\frac{\delta\theta_l + \delta\theta_r}{\delta\theta_l - \delta\theta_r},\ \alpha = \frac{r}{L}(\delta\theta_l - \delta\theta_r) \tag{2}$$

immediately then it yields that

$$\begin{bmatrix} \delta x_o \\ \delta y_o \\ \delta\phi_o \end{bmatrix} = \begin{bmatrix} (1-\cos\alpha)c \\ c\sin\alpha \\ -\alpha \end{bmatrix} \tag{3}$$

The dead-reckoning system in the vehicle simply compounds these small changes in position and orientation to obtain a global position estimate. Starting from an initial nominal frame at each iteration of its sensing loop it deduces a small change in position and orientation and then "adds" this to its last dead-reckoned position. Of course the "addition" is slightly more complex than simple adding. What actually happens is that the vehicle composes successive coordinate transformation. This is an important concept and will be discussed in the following.

We define 2 operators ⊕ (compound) and ⊖ (inverse) to compose multiple transformations (Smith *et al*., 1987). They allow us to express something (perhaps a point or vehicle) described in one frame, in another alternative frame. We can use this notation to explain the compounding of odometry measurements. Figure 2 shows a vehicle with a prior pose x_o (k-1). The processing of wheel rotations between successive readings (via Eq. 3) has indicated a vehicle-relative transformation (i.e., in the frame of the vehicle) $u_o = [\delta x_o, \delta y_o, \delta\phi_o]^T$. The task of

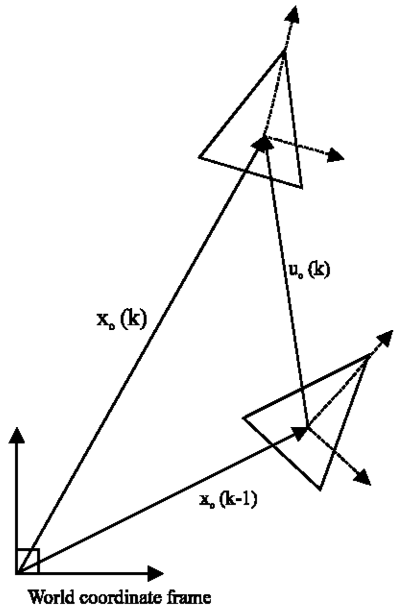

Fig. 2: Deducing a new dead-reckoned state from a prior dead-reckoned state with a local odometry measurement

combining this new motion u_o (k) with the old dead-reckoned estimate x_o (k-1) to arrive at a new dead-reckoned pose x_o (k) is trivial, i.e.,

$$x_o(k) = x_o(k-1) \oplus u_o(k) \tag{4}$$

We have now explained a way that measurements of wheel rotations can be used to estimate dead-reckoned pose. However, we set out to figure out a way that a dead-reckoned pose could be used to form a control input or measurement into a navigation system. In other words, we are given the low-level vehicle software to generate a sequence x_o (1), x_o (2), ..., x_o (k), etc. We want to figure out the low error control inputs to the vehicle motion model ($u_v = [\delta x_o, \delta y_o, \delta\phi_o]^T$) from the successive dead reckoned poses. Compounding x_o (k) to the inverse relationship of x_o (k-1) results in u_v (k) given by

$$u_V(k) = \ominus x_0(k-1) \oplus x_0(k) \tag{5}$$

Looking at Fig. 2, we can see that the transformation u_v (k) is equivalent to going back along x_o (k-1) and forward along x_v (k). This gives us a small control vector u_v (k) derived from two successive dead-reckoned poses that is suitable for use in another navigation algorithm prone to be hopefully less error. Effectively Eq. 5 subtracts out the common dead-reckoned gross error.

We are now in a position to write down the vehicle motion model using successive dead-reckoned poses as a control input:

$$\begin{aligned} x_v(k+1) &= f(x_v(k), u_v(k)) \\ &= x_v(k) \oplus (\ominus x_o(k-1) \oplus x_o(k)) \\ &= x_v(k) \oplus u_v(k) \end{aligned} \quad (6)$$

PSEUDO-LINEAR SYSTEM MODELING

This study describes the formulation of the vehicle motion model and the observation model in pseudo-linear form is described in detail. In our implementation, the vehicle is equipped with two sensors of the same type which can provide measurements (range and bearing) of the location of landmarks with respect to the vehicle. Our main objective of this implementation is to prove that there exists a much better solution to the SLAM problem through the proposed algorithm presented in this study, called FKF-SLAM algorithm, compared to the widely used EKF algorithm. It is further aimed at demonstrating the better performances of the proposed algorithm over the key properties of the SLAM algorithm: convergence, consistency and boundedness of the map error in comparison with the EKF approach. In particular, the implementation shows how generally nonlinear vehicle and observation models are represented by pseudo-linear models and can be readily incorporated into linear Kalman filter equations.

In the following, the vehicle state is defined by $x_v = [x, y, \phi]^T$, where, x and y are the coordinates of the center of the rear axle of the vehicle with respect to some global coordinate frame and ϕ is the orientation of the vehicle axis. The landmarks are modeled as point landmarks and represented by a Cartesian pair such that $m_i = [x_i, y_i]^T$, i = 1,..., N. Both vehicle and landmark states are registered in the same frame of reference.

The pseudo-linear process model: Figure 3 shows a schematic diagram of the vehicle in the process of observing a landmark. The dead-reckoned measurements obtained from successive vehicle frames can be used to predict the vehicle state from the previous state. The discrete-time vehicle process model can be obtained according to the Eq. 6 and expressed in the following form:

$$\begin{bmatrix} x(k+1) \\ y(k+1) \\ \phi(k+1) \end{bmatrix} = \begin{bmatrix} x(k) \\ y(k) \\ \phi(k) \end{bmatrix} + \begin{bmatrix} \cos(\phi(k)) & -\sin(\phi(k)) & 0 \\ \sin(\phi(k)) & \cos(\phi(k)) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta x(k) \\ \delta y(k) \\ \delta\phi(k) \end{bmatrix} \quad (7)$$

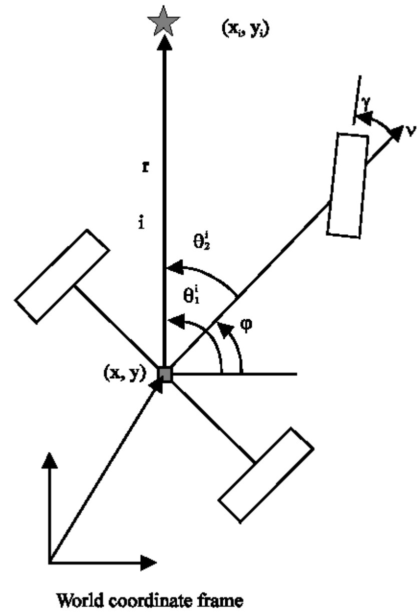

Fig. 3: Vehicle in process of observing a feature

which can be represented by the discrete-time pseudo-linear vehicle motion model expressed by

$$x_v(k+1) = x_v(k) + B_v(k)\, u_v(k) \quad (8)$$

for use in the prediction stage of the vehicle state estimator.

The landmarks in the environment are assumed to be stationary point targets. The landmark process model is thus

$$\begin{bmatrix} x_i(k+1) \\ y_i(k+1) \end{bmatrix} = \begin{bmatrix} x_i(k) \\ y_i(k) \end{bmatrix} \quad (9)$$

for all landmarks i = 1,..., N. Equation 7 together with Eq. 9 defines the process model of the vehicle-landmarks. To represent the process model in the proposed SLAM algorithm, the vehicle-landmarks augmented state vector can then be represented in the following pseudo-linear form:

$$x(k+1) = x(k) + B(k)u(k) \quad (10)$$

Where,

$$x(k) = [x_v^T(k) \quad m^T(k)]^T,$$

$$B(k) = \begin{bmatrix} B_v^T(k) & 0_1^T \end{bmatrix}^T \text{ and } u(k) = u_v(k)$$

in which θ_1 is a null matrix.

Observation model with two sensors: We adopt a rather different technique to model the observation model for a particular landmark. In our problem setting, sensor arrangements can be described as follows. The mobile robot vehicle is equipped with two sensors of the same type. Two sensors are fixed at the center of the rear vehicle axle so that one sensor starts reading measurements from the x axis (horizontal axis) and the other from the center axis of the vehicle. The vertical axes of 2 sensors are assumed to be aligned. Each sensor returns range r_i (k) and bearing θ_i (k) measurements to a particular landmark i at an instant. The range measurements and bearing measurements are taken from the center of rear vehicle axle where the vehicle position (x, y) is taken. Referring to Fig. 3, the observation model for ith landmark with respect to the sensor 1 is written as

$$r_1^i(k) = \sqrt{(x_i - x(k))^2 + (y_i - y(k))^2} + v_{r_1}(k) \quad (11)$$

$$\theta_1^i(k) = \arctan\left(\frac{y_i - y(k)}{x_i - x(k)}\right) + v_{\theta_1}(k) \quad (12)$$

and for the sensor 2

$$r_2^i(k) = \sqrt{(x_i - x(k))^2 + (y_i - y(k))^2} + v_{r_2}(k) \quad (13)$$

$$\theta_2^i(k) = \arctan\left(\frac{y_i - y(k)}{x_i - x(k)}\right) - \phi(k) + v_{\theta_2}(k) \quad (14)$$

We propose an observation model for the ith landmark derived from above 2 measurement models as:

$$z_i(k) = [r_i(k), \theta_i(k), \beta_i(k)]^T \quad (15)$$

Where,

$$r_i(k) = r_2^i(k) = \sqrt{(x_i - x(k))^2 + (y_i - y(k))^2} + v_r(k) \quad (16)$$

$$\theta_i(k) = \theta_1^i(k) = \arctan\left(\frac{y_i - y(k)}{x_i - x(k)}\right) + v_\theta(k) \quad (17)$$

$$\beta_i(k) = \theta_1^i(k) - \theta_2^i(k) = \phi(k) + v_\beta(k) \quad (18)$$

where, v_r and v_θ are the white noise sequences associated with the range and bearing measurements with zero means and standard deviations σ_r, σ_θ respectively. v_β is also assumed to be white with zero mean and standard deviation σ_β. It should be mentioned here that the subtraction in Eq. 18 results in lower noise angle β compared to the noise in bearing angle because Eq. 18 subtracts out common noise in bearing angle. The covariance matrix R_z for the observation model given by Eq. 16-18 is then in the form:

$$R_z = \begin{bmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \sigma_\theta^2 & 0 \\ 0 & 0 & \sigma_\beta^2 \end{bmatrix} \quad (19)$$

We want to express this observation model given by Eq. 16-18 in pseudo-linear.

Pseudo-linear observation model: In this study, we present the pseudo-linear measurement model (Li and Jikov, 2001; Watanabe, 1991) that is employed in the feature based estimation for localization and map building. In early works of estimation problems, pseudo-linear measurement models are widely used in bearing only target tracking problems (Aidala, 1979; Lindgren and Gong, 1978). The pseudo-linear measurement method, originated in Whitcombe (1972), attempts to circumvent the bias problems of the EKF by avoiding explicit linearization of the nonlinear measurement model given by Eq. 16-18 in the mixed coordinates. It relies on representing such a measurement model in the following pseudo-linear form:

$$y(z) = H(z)x + v_y(x, v) \quad (20)$$

where the pseudo-measurement vector y (z) matrix H (z) are known functions of the actual measurement z and v_y (x, y) is the corresponding pseudo-measurement error, now state dependent. The underlying idea of the approach is clear. Once a pseudo-linear model (20) is available, a linear Kalman filter can be readily used with y (z), H (z) and R_y (x^*) cov [v_y (x^*, v)] where a common choice of x^* is the predicted state estimate $\hat{x}$. Equations 16-18 can be rearranged by algebraic and trigonometric manipulations to obtain the following model expressed by

$$r_i(k) = (x_i - x(k))\cos(\theta_i(k)) + (y_i - y(k))\sin(\theta_i(k)) + v_r(k) \quad (21)$$

$$0 = (x_i - x(k))\sin(\theta_i(k)) - (y_i - y(k))\cos(\theta_i(k)) + v_\theta^y(k) \quad (22)$$

$$\beta_i(k) = \phi(k) + v_\beta(k) \quad (23)$$

Where,

$$v_\theta^y(k) = r_{i,true}(k) v_\theta(k)$$

The observation model composed of Eq. 21-23 can be expressed in the following pseudo-linear form for the ith landmark:

$$y(z_i) = \begin{bmatrix} r_i(k) \\ 0 \\ \beta_i(k) \end{bmatrix} = H(z_i)x + v_{yi}(x, v) \quad (24)$$

Where,

$$H(z_i) = [H_v \quad 0_1 \quad H_{fi} \quad 0_2] \quad (25)$$

$$H_v = \begin{bmatrix} -\cos(\theta_i(k)) & -\sin(\theta_i(k)) & 0 \\ -\sin(\theta_i(k)) & \cos(\theta_i(k)) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (26)$$

$$H_{fi} = \begin{bmatrix} \cos(\theta_i(k)) & \sin(\theta_i(k)) \\ \sin(\theta_i(k)) & -\cos(\theta_i(k)) \\ 0 & 0 \end{bmatrix} \quad (27)$$

Here, 0_1 and 0_2 are the null matrices. Pseudo-measurement noise vector for the *i*th landmark v_{yi} (x, v) is considered to be white with its covariance expressed in the form:

$$R_{yi}(\hat{x}) = \begin{bmatrix} \sigma_r^2 & 0 & 0 \\ 0 & \hat{r}_i^2 \sigma_\theta^2 & 0 \\ 0 & 0 & \sigma_\beta^2 \end{bmatrix} \quad (28)$$

Note that, predicted value of r_i is used in calculating covariance matrix (R_{yi}) because true value of r_i is not available.

TAKAGI-SUGENO (T-S) FUZZY MODEL

The fuzzy model proposed by Takagi and Sugeno is described by fuzzy IF-THEN rules, which represent local linear input-output relations of a nonlinear system. The *j*th rule of the T-S fuzzy model is of the following form:

Rule J: IF q_1 (k) is F_{j1} and ... and q_g (k) is F_{jg} then

$$\begin{aligned} x(k+1) &= A_j x(k) + B_j u(k) \\ y(k) &= C_j x(k) \quad j = 1, 2, \cdots, r. \end{aligned} \quad (29)$$

F_{jl} is the fuzzy set and r is the number of IF-THEN rules. X (k) $\in \Re^n$ is the state vector, u (k) $\in \Re^m$ is the input vector, y (k) $\in \Re^p$ is the measurement vector. q_1 (k) ~q_g (k) are the premise variables. Given a pair of (x (k), u (k)), the final outputs of the fuzzy systems are inferred as follows:

$$x(k+1) = \frac{\sum_{j=1}^{r} w_j(q(k))\{A_j x(k) + B_j u(k)\}}{\sum_{j=1}^{r} w_j(q(k))}$$

$$= \sum_{j=1}^{r} h_j(q(k))\{A_j x(k) + B_j u(k)\} \quad (30)$$

Where,

$$q(k) = [q_1(k) \cdots q_g(k)] \quad (31)$$

$$w_j(q(k)) = \prod_{l=1}^{g} F_{jl}(q_l(k)) \quad (32)$$

$$\begin{cases} \sum_{j=1}^{r} w_j(q(k)) > 0 \\ w_j(q(k)) \geq 0 \end{cases} \quad j = 1, 2, \cdots, r \quad (33)$$

$$h_j(q(k)) = \frac{w_j(q(k))}{\sum_{j=1}^{r} w_j(q(k))} \quad (34)$$

for all k. F_{jl} (q_l (k)) is the grade of membership of q_l (k) in F_{jl}. From Eq. 30-34 we have

$$\begin{cases} \sum_{j=1}^{r} h_j(q(k)) = 1 \\ h_j(q(k)) \geq 0 \end{cases} \quad j = 1, 2, \cdots, r \quad (35)$$

for all k.

Fuzzy modeling of nonlinear terms: Fuzzy description of nonlinear term cosϕ can be expressed using the procedure described in Tanaka *et al*. (1996). It is assumed that ϕ varies in between $-\pi$ and π. Cosϕ can be rewritten for two cases by using 2 linear models for each case. This is illustrated in Fig. 4. They can be represented as follows:

$$\cos\phi = F_1^1(\phi) \cdot 1 + F_1^2(\phi) \cdot \cos a \quad \text{for} \quad |\phi| \leq \pi/2 \quad (36)$$

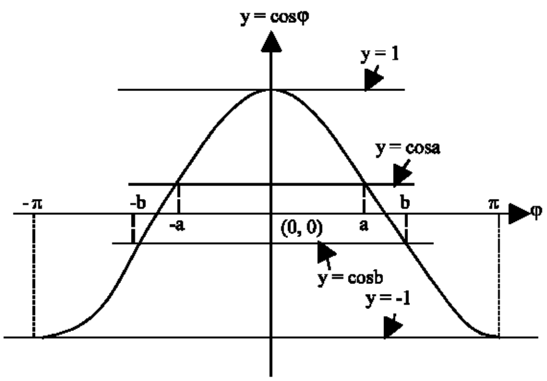

Fig. 4: Approximation of nonlinear term cosϕ

$$\cos\phi = F_2^1(\phi)\cdot(-1) + F_2^2(\phi)\cdot\cos b \ \text{ for } \ \pi/2 < |\phi| < \pi \quad (37)$$

Here, $a = \pi/2-\delta$, $b = \pi/2 + \delta$ and δ is a small positive angle. The membership functions in Eq. 36 and 37 are defined as:

$$F_1^1 = \{\text{about } 0\},\ F_1^2 = \{\text{about } \pm a\},$$
$$F_2^1 = \{\text{about } \pm\pi\} \text{ and } F_2^2 = \{\text{about } \pm b\}$$

Where,

$$F_1^1(\phi), F_1^2(\phi), F_2^1(\phi), F_2^2(\phi) \in [0,1] \quad (38)$$

$$F_1^1(\phi) + F_1^2(\phi) = 1 \quad (39)$$

$$F_2^1(\phi) + F_2^2(\phi) = 1 \quad (40)$$

Solving the above equations gives

$$F_1^1(\phi) = \frac{\cos\phi - \cos a}{1-\cos a} \quad (41)$$

$$F_1^2(\phi) = 1 - F_1^1(\phi) = \frac{1-\cos\phi}{1-\cos a} \quad (42)$$

$$F_2^1(\phi) = \frac{\cos b - \cos\phi}{1+\cos b} \quad (43)$$

$$F_2^2(\phi) = 1 - F_2^1(\phi) = \frac{1+\cos\phi}{1+\cos b} \quad (44)$$

In the same way, sinϕ can also be rewritten by the combination of linear models and can be deduced from the above ϕ by the following formula:

$$\sin\phi = \text{sgn}(\phi)\sqrt{1-\cos^2\phi} \quad (45)$$

Where,

$$\text{sgn}(\phi) = \begin{cases} 1 & \text{if } \phi > 0 \\ -1 & \text{if } \phi < 0 \end{cases} \quad (46)$$

FORMULATION OF FUZZY ALGORITHM IN SLAM PROBLEM

To reduce the computational cost in using the T-S fuzzy model in the SLAM problem, the fuzzification of the process model and the observation model are split into two cases according to the vehicle azimuth angle. A set of fuzzy rules is constructed for each case and is executed based on initial separation of vehicle azimuth angle.

Case 1: If the azimuth angle of the vehicle (ϕ (k)) lies between $-\pi/2$ and $\pi/2$, the jth rule for this case will be of the form:

Local linear system rule j relative to the *i*th landmark: IF ϕ (k) is F^j_ϕ and θ_i (k) is F^j_θ then

$$x_j(k+1) = x(k) + B_j(k)u(k) \quad \text{for} \quad j = 1,2,\cdots,8$$
$$\boldsymbol{y}_{ij}(k+1) = \boldsymbol{H}_{ij}(k+1)\boldsymbol{x}_j(k+1) + \boldsymbol{v}_{\boldsymbol{y}ij}(k+1) \quad (47)$$

F^j_ϕ, F^j_θ ∈ $\{F^1_1, F^2_1, F^1_2, F^2_2\}$ are the membership functions of fuzzy sets of vehicle azimuth angle (ϕ) and bearing angle (θ_i) for the jth rule respectively. B_j is the matrix with its nonlinear elements sectored as discussed in fuzzy description of nonlinear terms and then this results in a linear matrix for fuzzy sets of vehicle azimuth angle (ϕ) for each rule in T-S fuzzy model of the SLAM problem. In the similar way, the nonlinear elements of the matrices H_{ij} are sectored according to the fuzzy sets of bearing angle (θ_i).

Case 2: It is defined for $\pi/2 < |\phi\ (k)| < \pi$ and will be composite of eight similar local linear models as defined above.

Estimation process: In the formulation of T-S fuzzy model based SLAM algorithm, the linear discrete Kalman filter is used to generate local estimates of vehicle and landmark locations for each local linear model defined in T-S fuzzy model. The Kalman filter algorithm proceeds recursively in the 3 stages:

Prediction: The algorithm first generates a prediction for the state estimate, the observation (relative to the ith landmark) and the state estimate covariance at the time k+1 for the jth rule according to

$$\hat{x}_j(k+1|k) = \hat{x}(k|k) + B_j(k)u(k) \quad (48)$$

$$\hat{y}_{ij}(k+1|k) = H_{ij}(k+1)\hat{x}_j(k+1|k) \quad (49)$$

$$P_j(k+1|k) = P(k|k) + B_j(k)Q(k)B_j^T(k) \quad (50)$$

Observation: Following the prediction, the observation $y_i = (k + 1)$ of the *i*th landmark of the true state x (k + 1) is made according to Eq. 24. Assuming correct landmark association, an innovation is calculated for the jth rule as follows:

$$\upsilon_{ij}(k+1) = y_{ij}(k+1) - \hat{y}_{ij}(k+1|k) \quad (51)$$

together with an associated innovation covariance matrix for the jth rule given by

$$S_{ij}(k+1) = H_{ij}(k+1)P_j(k+1|k) H_{ij}^T(k+1) + R_{yij}(k+1) \quad (52)$$

Update: The state update and corresponding state estimate covariance are then updated for the *j*th rule according to

$$\hat{x}_j(k+1|k+1) = \hat{x}_j(k+1|k) + K_j(k+1)\upsilon_{ij}(k+1) \quad (53)$$

$$P_j(k+1|k+1) = P_j(k+1|k) - K_j(k+1)S_{ij}(k+1)K_j^T(k+1) \quad (54)$$

Here, the gain matrix K_j (k + 1) is given by

$$K_j(k+1) = P_j(k+1|k)H_{ij}^T(k+1)\, S_{ij}^{-1}(k+1) \quad (55)$$

Local state estimates are then combined according to the Eq. 30 to obtain the global state estimate for the T-S fuzzy model given by Eq. 47. The global estimate is then obtained by the following equation:

$$\hat{x}(k+1|k+1) = \sum_{j=1}^{8} h_j(q_i(k))\hat{x}_j(k+1|k+1) \quad (56)$$

Where,

$$q_i(k) = [q_{i1}(k)\ q_{i2}(k)] = [\phi(k)\ \theta_i(k)]$$

Propagation of uncertainty for the augmented state error of the T-S fuzzy model is realized by a common covariance which is chosen to be the local covariance that has the ***maximum trace***, i.e., we consider the worst case to assure the stability of the T-S fuzzy model based SLAM algorithm because it is of paramount importance in state estimation using fuzzy algorithm. The common covariance can be formulated as follows:

$$P(k+1|k+1) = \max(\text{trace}\, P_j(k+1|k+1)) \quad \forall j \quad (57)$$

The resulting global state estimate and common covariance are then preceded to the next stage of prediction. Each rule in the T-S fuzzy model takes the global state estimate and the common covariance to generate the next stage prediction. This process is repeated until the required criteria for the state estimation is met.

PERFORMANCE EVALUATION

In this study, we show the simulation results for the FKF-SLAM algorithm with the measurement model derived from two sensor frames for the system composite of Eq. 10 and 24. We investigated the performances of the proposed SLAM algorithm and the conventional EKF-SLAM algorithm when applied to the vehicle-landmarks nonlinear system while keeping all the conditions remain unchanged for the two cases.

Map building: An environment with arbitrarily placed landmarks was simulated with a given vehicle trajectory. Simulation results are depicted in Fig. 5 and 6. Figure 5(a) and 6(a) show the evolution of the map over the time obtained from applying the EKF algorithm and the pseudo-linear model based FKF algorithm respectively. It can be seen that error ellipses in Fig. 6(a) converge to the actual landmark locations faster than that in Fig. 5(a). This observation can be made from Fig. 5(b) and 6(b). A feature that has the same map registration number (where its pose is registered) in the state vector has been indicated in Fig. 5(b) and 6(b) to compare the performances of uncertainty convergence rate between two methods. The selected feature in Fig. 5(b) is detected at the point A_e and it requires d_e time span to reach to a minimum bound in uncertainty since detection A_e. And in Fig. 6(b), for the selected landmark, it takes d_f time span to reach to a minimum bound in uncertainty since detection at A_f. This discloses that the proposed pseudo-linear model based FKF approach has higher convergence rate than the EKF approach ($d_e > d_f$). From Fig. 5(c) and 6(c), it can be observed that the actual landmark state error for all the landmarks obtained from the pseudo-linear model based FKF approach reaches to a minimum bound within a less

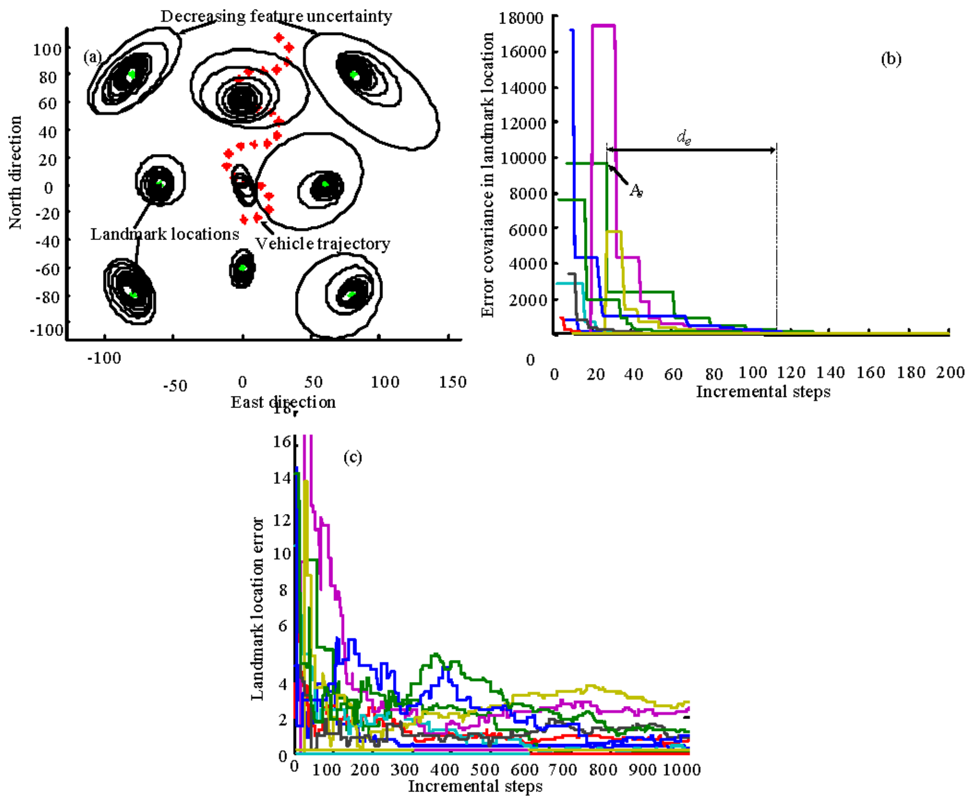

Fig. 5: Feature based map building for a given vehicle trajectory: The EKF approach

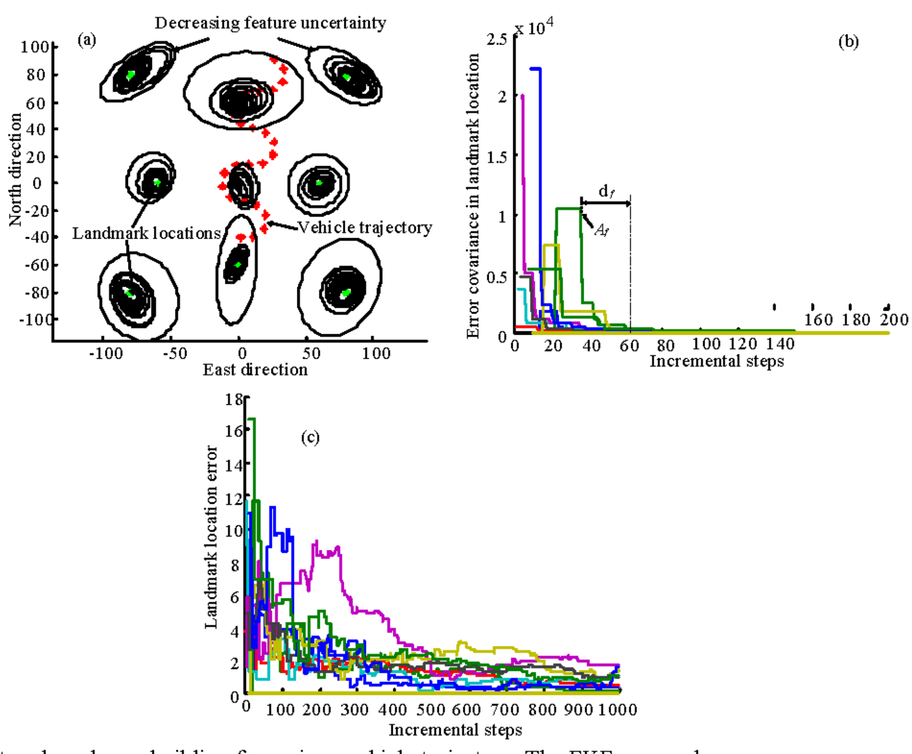

Fig. 6: Feature based map building for a given vehicle trajectory: The FKF approach

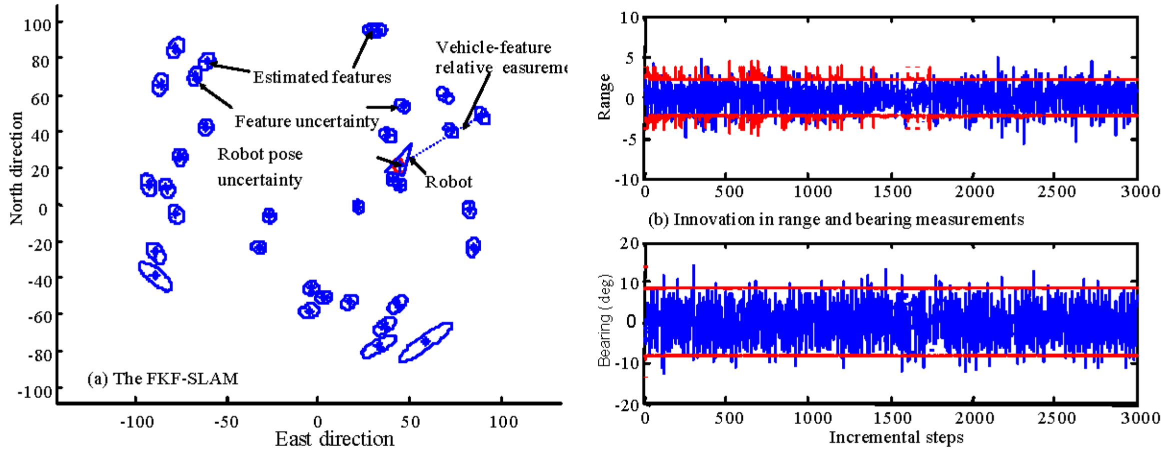

Fig. 7. Feature based simultaneous localization and mapping

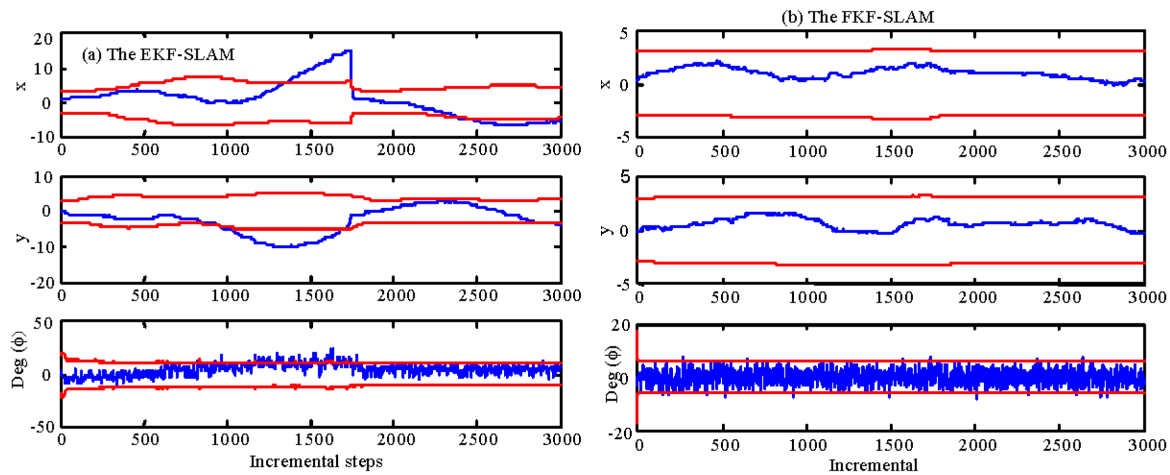

Fig. 8. Estimated error and actual error in vehicle state

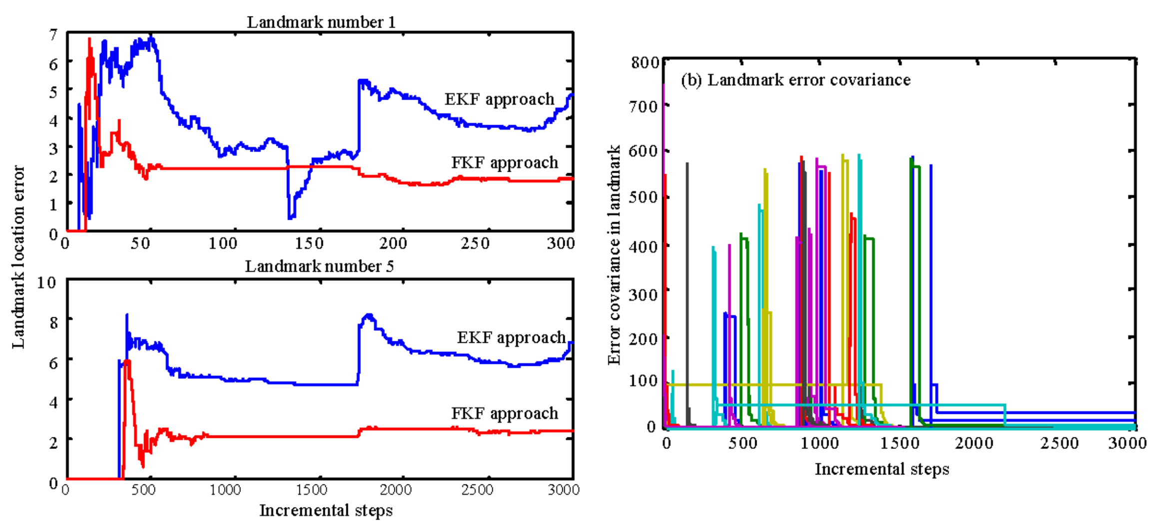

Fig. 9. Landmark location error and covariance for all the detected landmarks

number of time steps compared to that obtained from the EKF algorithm implying map of landmark locations could be accurately constructed faster than that of the EKF estimator.

Simultaneous localization and mapping: The newly described method is applied to the feature based SLAM problem. An environment populated with point landmarks was simulated with the FKF algorithm to generate the state estimates and state errors/covariance. Simulation results are depicted in Fig. 7-9.

Figure 7(a) shows an instant of the FKF-SLAM algorithm running on the vehicle-landmark systems. It can be seen that error ellipses of the features converge to actual landmark locations because the map of the landmark locations is being constructed when the vehicle navigates through the environment. Figure 7(a) exemplifies that a solution to the SLAM problem through the pseudo-linear model based FKF algorithm is well accepted. Figure 7(b) shows the innovation in range measurement and in difference of bearing measurements obtained from two sensor frames together with associated 2σ confidence limit. Innovations are the only available measure to examine online filter behavior when true state values are unavailable. Innovations here indicate that the proposed filter and the models are consistent.

Figure 8 shows standard deviation and error associated with the vehicle state obtained from the two methods. Figure 8(a) shows the vehicle localization results obtained from the EKF-SLAM algorithm. It can be seen that the oscillation in vehicle state errors is higher when the vehicle is cornering (at around 1500 time step) and it needs considerable time to be rebounded by the confidence limits in the estimation error. The true vehicle state error values and the estimated vehicle state error values are higher than those of the FKF-SLAM algorithm. Figure 8(b) shows the true vehicle position and orientation error and the 2σ confidence limit in the estimate error obtained from the FKF-SLAM algorithm. The actual vehicle error is clearly bounded by the confidence limit of estimated vehicle error. Thus, the estimate produced by the FKF-SLAM algorithm is consistent. The estimated vehicle error defined by the confidence limits does not diverge so the estimates produced by the FKF-SLAM algorithm are stable. From Fig. 8(b), it can be seen that the vehicle position error lies well within the confidence limit and the oscillation in the vehicle position error is negligible even if the vehicle is cornering. Vehicle position error is enveloped in a smaller region. That is a highlighting point in the FKF-SLAM. It can be stated that the vehicle orientation can be estimated with higher accuracy. This is implied as the vehicle orientation error is found to be smaller in Fig. 8(b). These results imply that the FKF-SLAM algorithm with two sensor frames based measurement model generates vehicle location estimates which are more consistent, more stable and have well bounded errors compared to those of EKF-SLAM algorithm.

Figure 9 shows the evolution of the actual landmark location error and the covariance of landmark location estimate for all detected landmarks obtained from the FKF-SLAM algorithm. Figure 9(a) shows actual the landmark location error for landmark number 1 and landmark number 5 obtained from the two methods. It can be observed that the actual landmark location error obtained from FKF algorithm reaches to a minimum bound and is smaller than that of EKF algorithm. The landmark location estimates generated by FKF algorithm are thus consistent with actually landmark location errors reach to a minimum bound. Figure 9(b) shows the estimated covariance for all the detected landmarks generated by the FKF-SLAM algorithm. The estimated landmark location errors also decrease monotonically and thus overall error in the map reduces at each observation. The estimated landmark location error for each landmark does not diverge. So, the estimates generated by the proposed filter are stable.

CONCLUSION

A fuzzy logic and pseudo-linear model based solution to the SLAM problem has been proposed in this paper, where the validity of the method was shown with simulation results. The need for direct linearization of nonlinear systems for state estimation is diminished because the newly proposed method performed well and provided a better solution to the SLAM problem. Results obtained from the proposed method were compared with those obtained from widely used EKF algorithm to highlight the merit of the FKF-SLAM algorithm. It was shown that the pseudo-linear model based FKF algorithm provided more satisfactory results over the EKF because the pseudo-linear models did not lose its nonlinearity when employed in the Kalman filter equations. Nearest neighbor data association with two sensor frames further improved the solution. State estimation of vehicle-landmark system was able to be performed with a higher accuracy with two sensor frames based measurement model introduced in this paper. It could be seen that a fuzzy logic based approach with the pseudo-linear models provided a better solution to state estimation process because fuzzy logic has been always standing for a better solution.

REFERENCES

Aidala, V. J., 1979. Kalman filter behavior in bearings-only tracking applications. IEEE. Trans. Aerospace Electronic Syst., 15 (1): 29-39.

Baily, T., 2002. Mobile robot localization and mapping in extensive outdoor environments. Ph.D. dissertation, Australian Center for Field Robotics, Dept. Aerosp., Mech. Mechatron. Eng., Univ. Sydney, Sydney, Australia.

Bailey, T. and H. Durrant-Whyte, 2006. Simultaneous localization and mapping (SLAM): Part II. IEEE. Robotics Automaton Mag., 13 (3): 108-117.

Chen, G., Q. Xie and L.S. Shieh, 1998. Fuzzy Kalman filtering. J. Inform. Sci., 109: 97-209.

Dissanayake, M.W. M.G., P. Newman, S. Clark, H.F. Durrant-Whyte and M. Csorba, 2001. A solution to the simultaneous localization and map building (SLAM) problem. IEEE. Trans. Robotics and Automation, 17 (3): 229-241.

Durrant-Whyte, H. and T. Bailey, 2006. Simultaneous localization and mapping: Part I. IEEE. Robotics Automaton Mag., 13 (2): 99-108.

Leonard, J.J. and H.F. Durrant-White, 1991. Mobile robot localization by tracking geometric beacons. IEEE. Trans. Robotics Automation, 7 (3): 376-382.

Li, X.R. and V.P. Jikov, 2001. A survey of maneuvering target tracking--part III: measurement models. In: Proceeding of SPIE Conference on Signal and Date Processing of Small Targets, San Diego, CA, USA.

Lindgren, A.G. and K.F. Gong, 1978. Position and velocity estimation via bearing observations. IEEE. Tran. Aerospace Electronic Syst., 14 (4): 564-577.

Smith, R., M. Self and P. Cheeseman, 1987. A stochastic map for uncertain spatial relationships. In: Proc. 4th Int. Symp. Robot. Res., pp: 467-474.

Takagi, T. and M. Sugeno, 1985. Fuzzy identification of systems and its applications to modeling and control. IEEE. Trans. Syst. Man, Cybern., 15: 16-132.

Tanaka, K., T. Ikeda and H. Wang, 1996. Robust stabilization of a class of uncertain nonlinear system via fuzzy control. IEEE. Trans. Fuzzy Syst., 4: 1-13.

Watanabe, K., 1991. Adaptive Estimation and Control: Partitioning Approach. London, UK: Prentice-Hall.

Whitcombe, D.W., 1972. Pseudo state measurements applied to recursive nonlinear filtering. In: Proc. 3rd Symp. Nonlinear Estimation Theory and its Application, San Diego, CA, pp: 278-281.