International Journal of System Signal Control and Engineering Application 4 (2): 26-33, 2011
ISSN: 1997-5422

Active Controller Design for Generalized Projective Synchronization of Four-Scroll Chaotic Systems

[1]P. Sarasu and [2]V. Sundarapandian
[1]Faculty of Computer Science and Engineering,
[2]Research and Development Centre, Vel. Tech. Dr. R.R. and Dr. S.R. Technical University,
Avadi, 600-062 Chennai, Tamil Nadu, India

Abstract: This study investigates the active controller design for Generalized Projective Synchronization (GPS) of identical Liu-Chen 4-scroll chaotic systems, identical Lu-Chen-Cheng 4-scroll chaotic systems and non-identical Liu-Chen and Lu-Chen-Cheng 4-scroll chaotic systems. The GPS synchronization results for the 4-scroll chaotic systems have been derived using the active control method and established using Lyapunov stability theory. Since, the Lyapunov exponents are not required for these calculations, the active control method is a very effective and convenient method for achieving Generalized Projective Synchronization (GPS) of the 4-scroll chaotic systems addressed in this study. Numerical simulations are presented to demonstrate the effectiveness of the synchronization results derived in this study for the 4-scroll chaotic systems.

Key words: Active control, chaos, generalized projective synchronization, 4-scroll systems, Liu-Chen system, Haotic system

INTRODUCTION

Chaotic systems are non-linear dynamical systems which are highly sensitive to initial conditions. The sensitive nature of chaotic systems is commonly addressed as the butterfly effect (Alligood *et al*., 1997). Chaos is an interesting non-linear phenomenon and it has been intensively and extensively studied in the last three decades. Chaos theory has wide applications in several fields such as physical systems (Lakshmanan and Murali, 1996), chemical systems (Han *et al*., 1995), ecological systems (Blasius *et al*., 1999), secure communications (Cuomo *et al*., 1993; Kocarev and Parlitz, 1995; Liao and Tsai, 2000), etc.

Chaos synchronization is a phenomenon that may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem.

In most of the chaos synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the master or drive system and another chaotic system is called the slave or response system then the idea of the chaos synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically. Since, the seminal research by Pecora and Carroll (1990) on the synchronization of chaotic systems, a variety of impressive approaches have been proposed for chaos synchronization such as OGY method (Ott *et al*., 1990), sampled-data feedback method (Murali and Lakshmanan, 2003), time-delay feedback method (Park and Kwon, 2003), active control method (Huang *et al*., 2004; Chen, 2005; Sundarapandian, 2011a, i, j), adaptive control method (Park *et al*., 2007; Jia and Tang, 2009; Sundarapandian, 2011b, c, e), Backstepping method (Xiau-Qun and Jun-An, 2003; Park, 2006; Vincent, 2008), Sliding mode control method (Utkin, 1993; Sundarapandian and Sivaperumal, 2011; Sundarapandian, 2011d, f), etc.

In Generalized Projective Synchronization (GPS) of chaotic systems (Zhou *et al*., 2010), the chaotic systems can synchronize up to a constant scaling matrix. Complete synchronization (Huang *et al*., 2004), anti-synchronization (Emadzadeh and Haeri, 2005; Al-Sawalha and Noorani, 2009; Sundarapandian and Karthikeyan, 2011a, b; Sundarapandian, 2011h), hybrid synchronization (Sundarapandian, 2011g), projective synchronization (Mainieri and Rehacek, 1999) and generalized synchronization (Wang and Guan, 2006) are

Corresponding Author: P. Sarasu, Faculty of Computer Science and Engineering,
Vel. Tech. Dr. R.R. and Dr. S.R. Technical University, Avadi, 600-062 Chennai, Tamil Nadu, India

particular cases of generalized projective synchronization. GPS has important applications in areas like secure communications and secure data encryption. In this study, researcher deploy active control method so as to derive new results for the Generalized Projective Synchronization (GPS) for identical and different Liu-Chen 4-scroll systems and Lu-Chen-Cheng 4-scroll chaotic systems. Explicitly using active non-linear control and Lyapunov stability theory, we achieve generalized projective synchronization for identical Liu-Chen 4-scroll chaotic systems (Liu and Chen, 2004), identical Lu-Chen-Cheng 4-scroll chaotic systems (Lu *et al*., 2004) and non-identical Liu-Chen and Lu-Chen-Cheng 4-scroll chaotic systems.

MATERIALS AND METHODS

Consider the chaotic system described by the dynamics:

$$\dot{x} = Ax + f(x) \tag{1}$$

Where:
$x \in R^n$ = The state of the system
A = The n×n matrix of the system parameters
$f: R^n \to R^n$ = The non-linear part of the system

We consider the system (Eq. 1) as the master or drive system. As the slave or response system, we consider the following chaotic system described by the dynamics:

$$\dot{y} = By + g(y) + u \tag{2}$$

Where:
$y \in R^n$ = The state of the system
B = The n×n matrix of the system parameters
$g: R^n \to R^n$ = The non-linear part of the system
$u \in R^n$ = The controller of the slave system

If A = B and f = g then x and y are the states of two identical chaotic systems. If A≠B or f≠g then x and y are the states of two different chaotic systems. In the active control approach, we design a feedback controller u which achieves the Generalized Projective Synchronization (GPS) between the states of the master system (Eq. 1) and the slave system (Eq. 2) for all initial conditions x (0), z $(0) \in R^n$. For the GPS of the systems (Eq. 1 and 2), the synchronization error is defined as:

$$e = y - Mx \tag{3}$$

Where:

$$M = \begin{bmatrix} a_1 & 0 & \cdots & 0 \\ 0 & a_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_n \end{bmatrix} \tag{4}$$

In other words, we have:

$$e_i = y_i - a_i x_i \ (i = 1, 2, ..., n) \tag{5}$$

From Eq. 1-3, the error dynamics is easily obtained as:

$$\dot{e} = By - MAx + g(y) - Mf(x) + u \tag{6}$$

The aim of GPS is to find a feedback controller u so that:

$$\lim_{t \to \infty} \|e(t)\| = 0 \ \text{for all} \ e(0) \in R^n \tag{7}$$

Thus, the problem of Generalized Projective Synchronization (GPS) between the master system (Eq. 1) and slave system (Eq. 2) can be translated into a problem of how to realize the asymptotic stabilization of the system (Eq. 6). So, the objective is to design an active controller u for stabilizing the error dynamical system (Eq. 6) at the origin. We take as a candidate Lyapunov function:

$$V(e) = e^T Pe \tag{8}$$

where, P is a positive definite matrix. Note that $V: R^n \to R^n$ is a positive definite function by construction. We assume that the parameters of the master and slave system are known and that the states of both systems (Eq. 1 and 2) are measurable. If we find a feedback controller u so that:

$$\dot{V}(e) = -e^T Qe \tag{9}$$

Where, Q is a positive definite matrix then $\dot{V}: R^n \to R^n$ is a negative definite function. Thus by Lyapunov stability theory (Sundarapandian, 2011g), the error dynamics (Eq. 6) is globally exponentially stable and hence, the condition (Eq. 7) will be satisfied. Hence, GPS is achieved between the states of the master system (Eq. 1) and the slave system (Eq. 2).

Systems description: In this study, the 4-scroll chaotic systems considered in this research, viz. Liu-Chen 4-scroll chaotic systems (Liu and Chen, 2004) and Lu-Chen-Cheng 4-scroll chaotic systems (Lu *et al*., 2004). The Liu-Chen system is described by the dynamics:

$$\begin{aligned} \dot{x}_1 &= ax_1 - x_2x_3 \\ \dot{x}_2 &= -bx_2 + x_1x_3 \\ \dot{x}_3 &= -cx_3 + x_1x_2 \end{aligned} \tag{10}$$

Where, x_1-x_3 are the states and a-c are positive, constant parameters of the system. The Liu-Chen system (Eq. 10) exhibits a chaotic attractor (4-scroll attractor) when the parameter values are taken as:

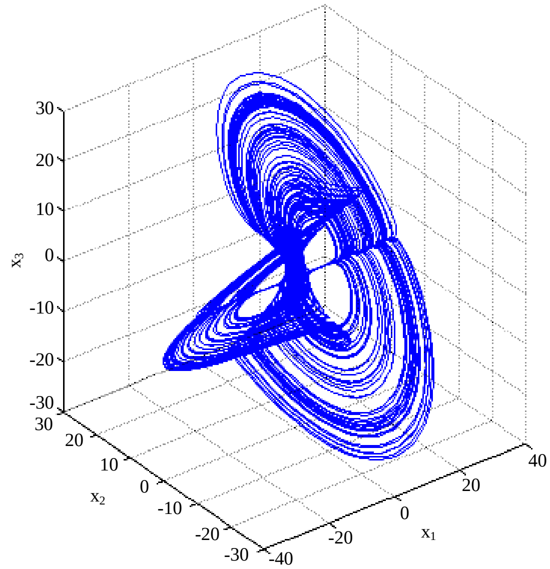

Fig. 1: The Liu-Chen 4-scroll chaotic attractor

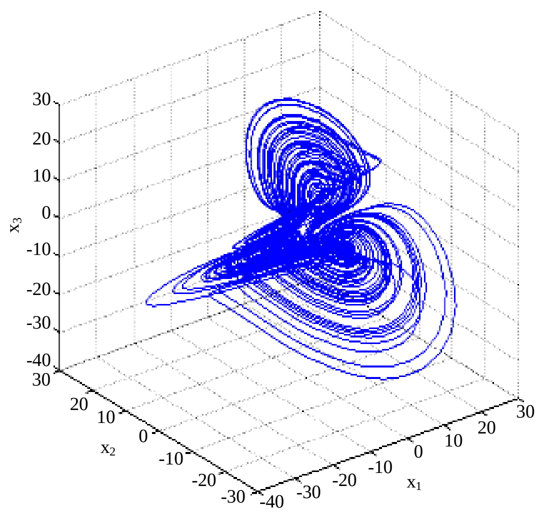

Fig. 2: The Lu-Chen-Cheng 4-scroll chaotic attractor

$$a = 0.4,\ b = 12 \text{ and } c = 5$$

The 4-scroll chaotic attractor of the Liu-Chen system (Eq. 10) is showed in Fig. 1. The Lu-Chen-Cheng system is described by the dynamics:

$$\begin{aligned} \dot{x}_1 &= px_1 - x_2x_3 \\ \dot{x}_2 &= -qx_2 + x_1x_3 + s \\ \dot{x}_3 &= -rx_3 + x_1x_2 \end{aligned} \qquad (11)$$

Where, x_1-x_3 are the states and p-s are positive, constant parameters of the system. The Lu-Chen-Cheng system (Eq. 11) exhibits a chaotic attractor (4-scroll attractor) when the parameter values are taken as:

$$p = 20/7,\ q = 10,\ r = 4 \text{ and } s = 5$$

The 4-scroll chaotic attractor of the Lu-Chen-Cheng system (Eq. 11) is shown in Fig. 2.

RESULTS AND DISCUSSION

Generalized projective synchronization of identical Liu-Chen 4-scroll chaotic systems

Theoretical results: In this study, researchers apply the active non-linear control method for the Generalized Projective Synchronization (GPS) of two identical Liu-Chen 4-scroll chaotic systems (Liu and Chen, 2004). Thus, the master system is described by the Liu-Chen dynamics:

$$\begin{aligned} \dot{x}_1 &= ax_1 - x_2x_3 \\ \dot{x}_2 &= -bx_2 + x_1x_3 \\ \dot{x}_3 &= -cx_3 + x_1x_2 \end{aligned} \qquad (12)$$

Where, x_1-x_3 are the states and a-c are positive, constant parameters of the system. The slave system is described by the controlled Liu-Chen dynamics:

$$\begin{aligned} \dot{y}_1 &= ay_1 - y_2y_3 + u_1 \\ \dot{y}_2 &= -by_2 + y_1y_3 + u_2 \\ \dot{y}_3 &= -cy_3 + y_1y_2 + u_3 \end{aligned} \qquad (13)$$

Where, y_1-y_3 are the states and u_1-u_3 are the active non-linear controls to be designed. For the GPS of Liu-Chen systems (Eq. 12 and 13), the synchronization error e is defined by:

$$\begin{aligned} e_1 &= y_1 - a_1x_1 \\ e_2 &= y_2 - a_2x_2 \\ e_3 &= y_3 - a_3x_3 \end{aligned} \qquad (14)$$

Where, the scales a_1-a_3 are real numbers. The error dynamics is obtained as:

$$\begin{aligned} \dot{e}_1 &= ay_1 - y_2y_3 - a_1(ax_1 - x_2x_3) + u_1 \\ \dot{e}_2 &= -by_2 + y_1y_3 - a_2(-bx_2 + x_1x_3) + u_2 \\ \dot{e}_3 &= -cy_3 + y_1y_2 - a_3(-cx_3 + x_1x_2) + u_3 \end{aligned} \qquad (15)$$

We choose the non-linear controller as:

$$\begin{aligned} u_1 &= -ay_1 + y_2y_3 + a_1(ax_1 - x_2x_3) - k_1e_1 \\ u_2 &= by_2 - y_1y_3 + a_2(-bx_2 + x_1x_3) - k_2e_2 \\ u_3 &= cy_3 - y_1y_2 + a_3(-cx_3 + x_1x_2) - k_3e_3 \end{aligned} \qquad (16)$$

Where, the gains k_1-k_3 are positive constants. Substituting Eq. 16 into Eq. 15, the error dynamics simplifies to:

$$\begin{aligned}\dot{e}_1 &= -k_1e_1\\ \dot{e}_2 &= -k_2e_2\\ \dot{e}_3 &= -k_3e_3\end{aligned} \tag{17}$$

We consider the quadratic Lyapunov function defined by:

$$V(e) = \frac{1}{2}e^Te = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \tag{18}$$

Which is a positive definite function on R^3. Differentiating Eq. 18 along the trajectories of Eq. 17, we get:

$$\dot{V}(e) = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 \tag{19}$$

Which is a negative definite function on R^3. Thus by Lyapunov stability theory (Hahn, 1967), the error dynamics (Eq. 17) is globally exponentially stable and hence, we arrive at the following result.

Theorem 1: The active feedback controller (Eq. 16) achieves global chaos Generalized Projective Synchronization (GPS) between the identical Liu-Chen 4-scroll chaotic systems (Eq. 12 and 13).

Numerical results: For simulations, the 4th-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the differential Eq. 12 and 13 with the active non-linear controller (Eq. 16). The parameters of the Liu-Chen systems are chosen as:

$$a = 0.4,\ b = 12 \text{ and } c = 5$$

We take the state feedback gains as:

$$k_1 = 3,\ k_2 = 3 \text{ and } k_3 = 3$$

The GPS scales a_i are taken as:

$$a_1 = -2.3,\ a_2 = -3.7 \text{ and } a_3 = 0.8$$

The initial conditions of the master system (Eq. 12) are as:

$$x_1(0) = 12,\ x_2(0) = 4,\ x_3(0) = 23$$

The initial conditions of the slave system (Eq. 13) are taken as:

$$y_1(0) = 6,\ y_2(0) = 30,\ y_3(0) = 15$$

Figure 3 shows the time response of the error states e_1-e_3 of the error dynamical system (Eq. 15) decay to zero exponentially when the active controller (Eq. 16) is deployed. Figure 4 shows the GPS of the systems (Eq. 12 and 13).

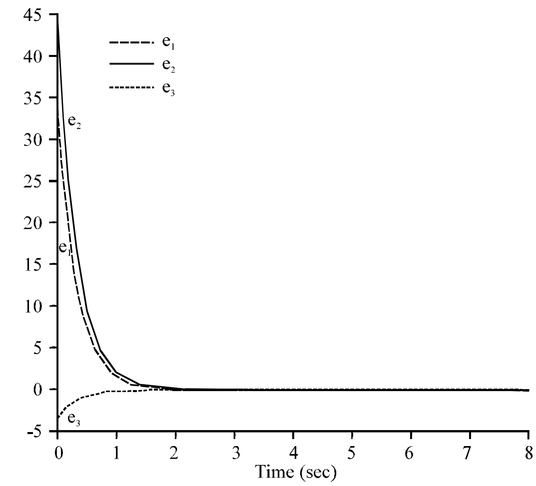

Fig. 3: Time responses of the error states for Liu-Chen systems

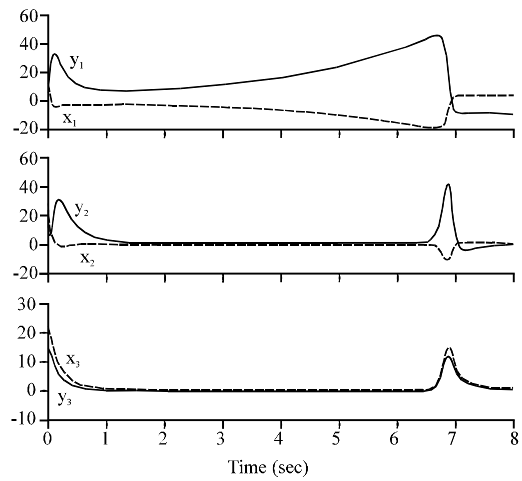

Fig. 4: GPS of the Liu-Chen 4-scroll systems

Generalized projective synchronization of identical Lu-Chen-Cheng 4-scroll chaotic systems

Theoretical results: In this study, we apply the active non-linear control method for the Generalized Projective Synchronization (GPS) of two identical Lu-Chen-Cheng 4-scroll chaotic systems (Lu *et al*., 2004). Thus, the master system is described by the Lu-Chen-Cheng dynamics:

$$\begin{aligned}\dot{x}_1 &= px_1 - x_2x_3\\ \dot{x}_2 &= -qx_2 + x_1x_3 + s\\ \dot{x}_3 &= -rx_3 + x_1x_2\end{aligned} \tag{20}$$

Where, x_1-x_3 are the states and p-s are positive, constant parameters of the system. The slave system is described by the controlled Lu-Chen-Cheng dynamics:

$$\begin{aligned}\dot{y}_1 &= py_1 - y_2y_3 + u_1 \\ \dot{y}_2 &= -qy_2 + y_1y_3 + s + u_2 \\ \dot{y}_3 &= -ry_3 + y_1y_2 + u_3\end{aligned} \quad (21)$$

Where, y_1-y_3 are the states and u_1-u_3 are the active non-linear controls to be designed. For the GPS of Lu-Chen-Cheng systems (Eq. 20 and 21), the synchronization error e is defined by:

$$\begin{aligned}e_1 &= y_1 - a_1x_1 \\ e_2 &= y_2 - a_2x_2 \\ e_3 &= y_3 - a_3x_3\end{aligned} \quad (22)$$

Where, the scales a_1-a_3 are real numbers. The error dynamics is obtained as:

$$\begin{aligned}\dot{e}_1 &= py_1 - y_2y_3 - a_1(px_1 - x_2x_3) + u_1 \\ \dot{e}_2 &= -qy_2 + y_1y_3 + s - a_2(-qx_2 + x_1x_3 + s) + u_2 \\ \dot{e}_3 &= -ry_3 + y_1y_2 - a_3(-rx_3 + x_1x_2) + u_3\end{aligned} \quad (23)$$

We choose the non-linear controller as:

$$\begin{aligned}u_1 &= -py_1 + y_2y_3 + a_1(px_1 - x_2x_3) - k_1e_1 \\ u_2 &= qy_2 + y_1y_3 - s + a_2(-qx_2 + x_1x_3 + s) - k_2e_2 \\ u_3 &= ry_3 - y_1y_2 + a_3(-rx_3 + x_1x_2) - k_3e_3\end{aligned} \quad (24)$$

Where, the gains k_1-k_3 are positive constants. Substituting Eq. 24 into 23, the error dynamics simplifies:

$$\begin{aligned}\dot{e}_1 &= -k_1e_1 \\ \dot{e}_2 &= -k_2e_2 \\ \dot{e}_3 &= -k_3e_3\end{aligned} \quad (25)$$

We consider the quadratic Lyapunov function defined by:

$$V(e) = \frac{1}{2}e^Te = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \quad (26)$$

which is a positive definite function on R^3. Differentiating Eq. 26 along the trajectories of Eq. 25, we get:

$$\dot{V}(e) = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 \quad (27)$$

which is a negative definite function on R^3. Thus by Lyapunov stability theory (Hahn, 1967), the error dynamics (Eq. 25) is globally exponentially stable and hence, we arrive at the following result.

Theorem 2: The active feedback controller (Eq. 24) achieves global chaos Generalized Projective Synchronization (GPS) between the identical Lu-Chen-Cheng 4-scroll chaotic systems (Eq. 20 and 21).

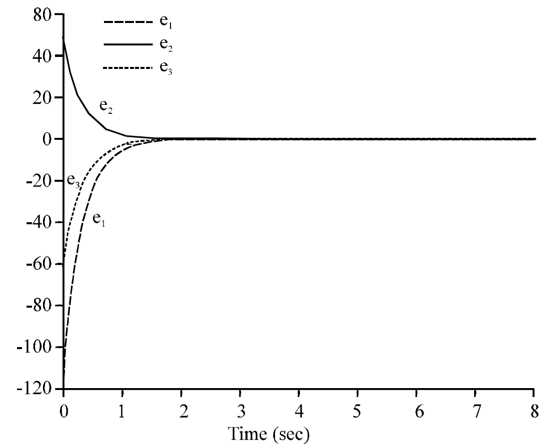

Fig. 5: Time responses of the error states for Lu-Chen- Cheng systems

Numerical results: For simulations, the 4th-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the differential Eq. 20 and 21 with the active non-linear controller (Eq. 24). The parameters of the Lu-Chen-Cheng systems are taken as:

$p = 20/7$, $q = 10$, $r = 4$ and $s = 5$

We take the state feedback gains as:

$k_1 = 3$, $k_2 = 3$ and $k_3 = 3$

The GPS scales a_i are taken as:

$a_1 = 5.2$, $a_2 = -1.9$ and $a_3 = 2.3$

The initial conditions of the master system (Eq. 20) are taken as:

$x_1(0) = 24$, $x_2(0) = 12$, $x_3(0) = 20$

The initial conditions of the slave system (Eq. 21) are taken as:

$y_1(0) = 8$, $y_2(0) = 25$, $y_3(0) = 7$

Figure 5 shows the time responses of the error states e_1-e_3 of the error dynamical system (Eq. 23) decay to zero exponentially when the active controller (Eq. 24) is deployed. Figure 4 shows the GPS of the systems (Eq. 20 and 21).

Generalized projective synchronization of Liu-Chen and Lu-Chen-Cheng 4-scroll chaotic systems

Theoretical results: In this study, we apply the active non-linear control method for the Generalized Projective

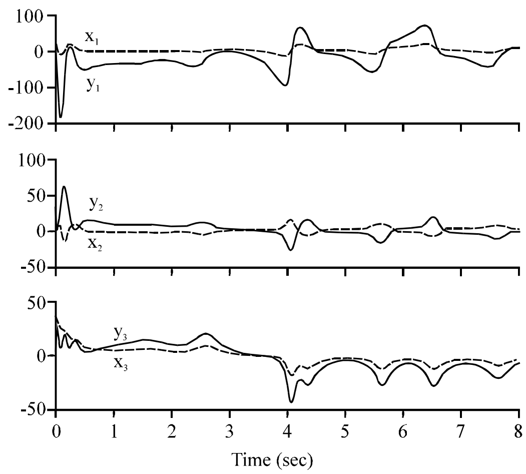

Fig. 6: GPS of the Lu-Chen-Cheng 4-scroll systems

Synchronization (GPS) of two different 4-scroll chaotic systems, amely the Liu-Chen system (Liu and Chen, 2004) as the master system and the Lu-Chen-Cheng system (Lu *et al*., 2004) as the slave system (Fig. 6). Thus, the master system is described by the Liu-Chen dynamics:

$$\begin{aligned} \dot{x}_1 &= ax_1 - x_2x_3 \\ \dot{x}_2 &= -bx_2 + x_1x_3 \\ \dot{x}_3 &= -cx_3 + x_1x_2 \end{aligned} \tag{28}$$

Where, x_1-x_3 are the state variables and a-c are positive parameters of the system. The slave system is described by the controlled Lu-Chen-Cheng dynamics:

$$\begin{aligned} \dot{y}_1 &= py_1 - y_2y_3 + u_1 \\ \dot{y}_2 &= -qy_2 + y_1y_3 + s + u_2 \\ \dot{y}_3 &= -ry_3 + y_1y_2 + u_3 \end{aligned} \tag{29}$$

Where, y_1-y_3 are the states, p-s are positive, constant parameters and u_1-u_3 are the active non-linear controls to be designed. For the GPS of Liu-Chen 4-scroll system (Eq. 28) and Lu-Chen-Cheng systems (Eq. 29), the synchronization error e is defined by:

$$\begin{aligned} e_1 &= y_1 - a_1x_1 \\ e_2 &= y_2 - a_2x_2 \\ e_3 &= y_3 - a_3x_3 \end{aligned} \tag{30}$$

Where, the scales a_1-a_3 are real numbers. The error dynamics is obtained as:

$$\begin{aligned} \dot{e}_1 &= py_1 - y_2y_3 - a_1(ax_1 - x_2x_3) + u_1 \\ \dot{e}_2 &= -qy_2 + y_1y_3 + s - a_2(-bx_2 + x_1x_3) + u_2 \\ \dot{e}_3 &= -ry_3 + y_1y_2 - a_3(-cx_3 + x_1x_2) + u_3 \end{aligned} \tag{31}$$

We choose the non-linear controller as:

$$\begin{aligned} u_1 &= -py_1 + y_2y_3 + a_1(ax_1 - x_2x_3) - k_1e_1 \\ u_2 &= qy_2 - y_1y_3 - s + a_2(-bx_2 + x_1x_3) - k_2e_2 \\ u_3 &= ry_3 - y_1y_2 + a_3(-cx_3 + x_1x_2) - k_3e_3 \end{aligned} \tag{32}$$

Where, the gains k_1-k_3 are positive constants. Substituting Eq. 32 into Eq. 31, the error dynamics simplifies to:

$$\begin{aligned} \dot{e}_1 &= -k_1e_1 \\ \dot{e}_2 &= -k_2e_2 \\ \dot{e}_3 &= -k_3e_3 \end{aligned} \tag{33}$$

We consider the quadratic Lyapunov function defined by:

$$V(e) = \frac{1}{2}e^Te = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \tag{34}$$

which is a positive definite function on R^3. Differentiating Eq. 34 along the trajectories of Eq. 33, we get:

$$\dot{V}(e) = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 \tag{35}$$

which is a negative definite function on R^3. Thus by Lyapunov stability theory, the error dynamics (Eq. 25) is globally exponentially stable. Hence, we arrive at the following result.

Theorem 3: The active feedback controller (Eq. 32) achieves global chaos Generalized Projective Synchronization (GPS) between the non-identical Liu-Chen 4-scroll chaotic system (Eq. 28) and Lu-Chen-Cheng 4-scroll chaotic system (Eq. 29).

Numerical results: For simulations, the 4th-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the differential Eq. 28 and 29 with the active non-linear controller (Eq. 32). The parameters of the Liu-Chen system (Eq. 28) are taken as:

$$a = 0.4, b = 12, c = 5$$

The parameters of the Lu-Chen-Cheng system (Eq. 29) are taken as:

$$p = 20/7, q = 10, r = 4 \text{ and } s = 5$$

We take the state feedback gains as:

$$k_1 = 3, k_2 = 3 \text{ and } k_3 = 3$$

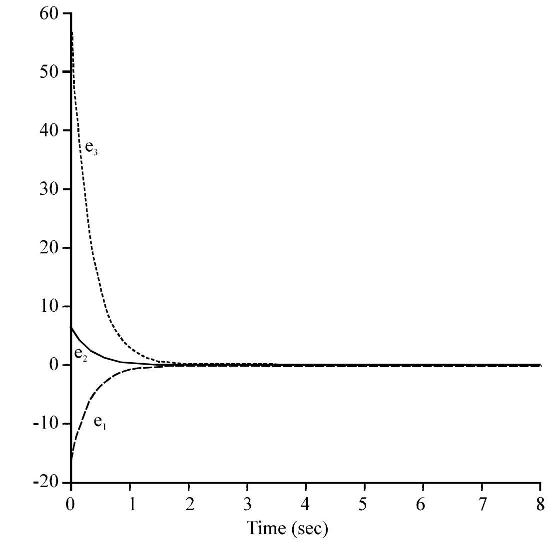

Fig. 7: Time responses of the error states for Liu-Chen and Lu-Chen-Cheng 4-scroll systems

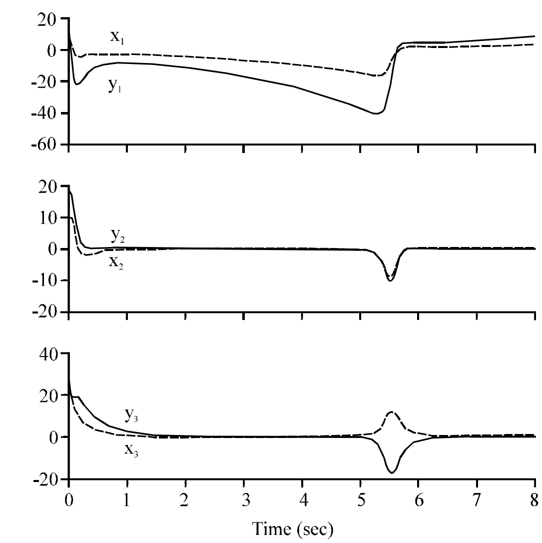

Fig. 8: GPS of the Liu-Chen and Lu-Chen-Cheng 4-scroll

The GPS scales a_i are taken as:

$$a_1 = 2.5,\ a_2 = 1.2 \text{ and } a_3 = -1.5$$

The initial conditions of the master system (Eq. 20) are:

$$x_1(0) = 14,\ x_2(0) = 6,\ x_3(0) = 19$$

The initial conditions of the slave system (Eq. 21) are taken as:

$$y_1(0) = 18,\ y_2(0) = 14,\ y_3(0) = 27$$

Figure 7 shows the time responses of the error states e_1-e_3 of the error dynamical system (Eq. 31) decay to zero exponentially when the active controller (Eq. 32) is deployed. Figure 8 shows the GPS of the non-identical Liu-Chen 4-scroll chaotic system (Eq. 28) and Lu-Chen-Cheng 4-scroll chaotic system (Eq. 29).

CONCLUSION

In this study, we have derived active control laws for achieving Generalized Projective Synchronization (GPS) of the following 4-scroll chaotic systems:

- Identical Liu-Chen 4-scroll systems (2004)
- Identical Lu-Chen-Cheng 4-scroll systems (2004)
- Non-identical Liu-Chen and Lu-Chen-Cheng 4-scroll systems

The synchronization results (GPS) derived in this study for the 4-scroll systems mentioned in 1-3 have been proved, using Lyapunov stability theory. Since, the Lyapunov exponents are not required for these calculations, the proposed active control method is very effective and convenient for achieving GPS of the 4-scroll chaotic systems addressed in this study. Numerical simulations are shown to demonstrate the effectiveness of the synchronization results (GPS) derived in this study.

REFERENCES

Al-sawalha, M.M. and M.S.M. Noorani, 2009. Anti-synchronization between two different hyperchaotic systems. J. Uncertain Syst., 3: 192-200.

Alligood, K.T., T. Sauer and J.A. Yorke, 1997. Chaos: An Introduction to Dynamical Systems. Springer-Verlag, New York.

Blasius, B., A. Huppert and L. Stone, 1999. Complex dynamics and phase synchronization in spatially extended ecological system. Nature, 399: 354-359.

Chen, H.K., 2005. Global chaos synchronization of new chaotic systems via nonlinear control. Chaos Solitons Fractals, 23: 1245-1251.

Cuomo, K.M., A.V. Oppenheim and S.H. Strogatz, 1993. Synchronization of lorenz-based chaotic circuits with applications to communications. Inst. Electr. Electron. Eng. Trans. Circuits Syst. II, 40: 626-633.

Emadzadeh, A.A. and M. Haeri, 2005. Anti-synchronization of two different chaotic systems via active control. World Acad. Sci. Eng. Technol., 6: 62-65.

Hahn, W., 1967. The Stability of Motion. Springer-Verlag, Berlin.

Han, S.K., C. Kerrer and Y. Kuramoto, 1995. Dephasing and bursting in coupled neural oscillators. Phys. Rev. Lett., 75: 3190-3193.

Huang, L., R. Feng and M. Wang, 2004. Synchronization of chaotic systems via nonlinear control. Phys. Lett. A, 320: 271-275.

Jia, L. and H. Tang, 2009. Adaptive control and synchronization of a four-dimensional energy resources system of Jiangsu province. Int. J. Nonlinear Sci., 7: 307-311.

Kocarev, L. and U. Parlitz, 1995. General approach for chaotic synchronization with application to communication. Phys. Rev. Lett., 74: 5028-5031.

Lakshmanan, M. and K. Murali, 1996. Chaos in Nonlinear Oscillators: Controlling and Synchronization. World Scientific, Singapore.

Liao, T.L. and S.H. Tsai, 2000. Adaptive synchronization of chaotic systems and its application to secure communications. Chaos Solitons Fractals, 11: 1387-1396.

Liu, W. and G. Chen, 2004. Can a three-dimensional smooth autonomous quadratic chaotic system generate a single four-scroll attractor? Int. J. Bifurcation Chaos, 14: 1395-1403.

Lu, J., G. Chen and D. Cheng, 2004. A new chaotic system and beyond: The generalized lorenz-like system. Int. J. Bifurcation Chaos, 14: 1507-1537.

Mainieri, R. and J. Rehacek, 1999. Projective synchronization in three-dimensional chaotic systems. Phys. Rev. Lett., 82: 3042-3045.

Murali, K. and M. Lakshmanan, 2003. Secure communication using a compound signal using sampled-data feedback. Applied Math. Mech., 11: 1309-1315.

Ott, E., C. Grebogi and J.A. Yorke, 1990. Controlling chaos. Phys. Rev. Lett., 64: 1196-1199.

Park, J.H. and O.M. Kwon, 2003. A novel criterion for delayed feedback control of time-delay chaotic systems. Chaos Solitons Fractals, 17: 709-716.

Park, J.H., 2006. Synchronization of Genesio chaotic system via backstepping approach. Chaos Solitons Fractals, 27: 1369-1375.

Park, J.H., S.M. Lee and O.M. Kwon, 2007. Adaptive synchronization of Genesio-Tesi chaotic system via a novel feedback control. Phys. Lett. A, 371: 263-270.

Pecora, L.M. and T.L. Carroll, 1990. Synchronization in chaotic systems. Phys. Rev. Lett., 64: 821-824.

Sundarapandian, V. and R. Karthikeyan, 2011a. Anti-synchronization of the hyperchaotic Liu and hyperchaotic Qi systems by active control. Int. J. Comput. Sci. Eng., 3: 2438-2449.

Sundarapandian, V. and R. Karthikeyan, 2011b. Anti-synchronization of pan and liu chaotic systems by active nonlinear control. Int. J. Eng. Sci. Technol., 3: 3596-3604.

Sundarapandian, V. and S. Sivaperumal, 2011. Sliding mode control based global chaos synchronization of four-scroll attractors. CIIT Int. J. Programmable Device Circ. Syst., 3: 297-302.

Sundarapandian, V., 2011a. Global chaos synchronization of liu and harb chaotic systems by active nonlinear control. Int. J. Comput. Inf. Syst., 1: 8-12.

Sundarapandian, V., 2011b. Adaptive synchronization of uncertain sprott H and I chaotic systems. Int. J. Comput. Inform. Syst., 1: 8-12.

Sundarapandian, V., 2011c. Adaptive control and synchronization of uncertain sprott H system. Int. J. Math. Sci. Comput., 1: 14-18.

Sundarapandian, V., 2011d. Sliding mode controller design for synchronization of Shimizu-Morioka chaotic systems. Int. J. Inform. Sci. Tech., 1: 20-29.

Sundarapandian, V., 2011e. Adaptive control and synchronization of hyperchaotic Newton-Leipnik system. Int. J. Adv. Inform. Technol., 1: 22-33.

Sundarapandian, V., 2011f. Global chaos synchronization of hyperchaotic Newton-Leipnik systems by sliding mode control. Int. J. Inform. Technol. Convergence Serv., 1: 34-43.

Sundarapandian, V., 2011g. Hybrid synchronization of hyperchaotic rossler and hyperchaotic lorenz systems by active control. Int. J. Adv. Sci. Technol., 2: 1-10.

Sundarapandian, V., 2011h. Anti-synchronization of Lorenz and T chaotic systems by active nonlinear control. Int. J. Comput. Inform. Syst., 2: 6-10.

Sundarapandian, V., 2011i. Global chaos synchronization of Liu-Su-Liu and Li systems by active nonlinear control. CIIT Int. J. Digital Signal Process., 3: 171-175.

Sundarapandian, V., 2011j. Global chaos synchronization of harb and pan systems by active nonlinear control. CIIT Int. J. Programmable Device Circuits Syst., 3: 303-307.

Utkin, V.I., 1993. Sliding mode control design principles and applications to electric drives. IEEE Trans. Ind. Electron., 40: 23-36.

Vincent, U.E., 2008. Chaos synchronization using active control and backstepping control: A comparative analysis. Nonlinear Anal.: Modell. Control, 13: 253-261.

Wang, Y.W. and Z.H. Guan, 2006. Generalized synchronization of continuous chaotic systems. Chaos Solitons Fractals, 27: 97-101.

Xiau-Qun, W. and L. Jun-An, 2003. Parameter identification and backstepping control of uncertain system. Chaos, Solitons Fractals, 18: 721-729.

Zhou, P., F. Kuang and Y.M. Cheng, 2010. Generalized projective synchronization for fractional order chaotic systems. Chin. J. Phys., 48: 49-56.