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Modeling Water Movement in Furrow Irrigation

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Abstract: A green element formulation was developed for horizontal flow as found in furrow irrigation. The model was tested on clay and sandy soils. An integral equation was developed using a one-dimensional horizontal equation for flow through unsaturated soils. The integral equation was implemented by discretize the soil column into elements. The values of the soil water content and the fluxes were obtained directly from the formulation. The soil water content profiles obtained were compared with that of vertical infiltration under the same condition.

Key words: Furrow irrigation, clay soil, sandy soil, soil water

INTRODUCTION

Processes such as ground water recharge and pollution are mainly through vertical movement of soil water. However, in situations such as furrow irrigation, water get to the plants roots through lateral movement.

Philip[1] termed lateral movement of soil water as absorption. Parlange[2] made a clear distinction between absorption and infiltration. He referred to movement of water without the influence of gravity as absorption, while movement under influence of gravity as infiltration.

Hillel[3] identified the three components of soil water potentials responsible for water movement to be gravitational, pressure (or-matric) and osmotic potentials. Mathematically, it can be represented as

$$\phi_t = \phi_g + \phi_p + \phi_o \quad (1)$$

Where ϕ_t is the total potential
ϕ_p is the pressure potential
ϕ_o is the osmotic potential.

The gradient of the soil-water potential is the driving force for water movement. The gradients of the components are not always equally effective. For example, to have osmotic potential gradient, a semi permeable membrane to induce liquid flow must be present. Osmotic effect is important in the interaction between plant roots and the soil and vapour diffusion processes. The gravitational potential arises from the body's position in the gravitational force field. A force equal to the weight of the body attracts the body towards the earth's center.

Matric potential (negative pressure) of soil water results from the adsorptive and capillary forces due to the soil matrix[3].

Vertical infiltration into unsaturated soils occur under the influence of both matric potential and gravity potential gradients. The potential gradient that predominates depends on the initial and boundary conditions and on the stage of the process considered[3]. For horizontal movement of water, the gravity force is zero and water movement is only by matric potential gradient.

The objective of this study is to develop a green-element numerical simulation model that can be used for lateral movement of water such as in situations like furrow irrigation.

The green-element method is an integral formulation that implements the boundary element method in element by element fashion by dividing the domain into sub domains or elements. Green element method was first presented by Taigbenu[4]. There are now flux-based green element formulations for linear and non linear potential flows, heat transfer and contaminant transport. Some of such formulations include[5-10]. These formulations have comparable accuracy over other formulations. The first derivatives of the primary variable (fluxes) are obtained directly.

Theory: The general flow equation is given as

$$\frac{\partial \theta}{\partial t} = -\nabla.[K\nabla(\Psi - Z)] \quad (2)$$

Where
θ is volumetric water content (cm^3cm^{-3})
t is time (h)
k is hydraulic conductivity (cmh^{-1})
ψ is matric suction head (cm)
z is the gravitational head (cm)

For vertical flow ∇Z is unity and zero for horizontal flow. For horizontal flow, Eq. 2 becomes

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x}\left(K\frac{\partial\Psi}{\partial x}\right) \quad (3)$$

where x is distance along the direction of flow (cm).

$\frac{\partial\theta}{\partial t}$ is the matric suction gradient

Expanding the matric suction gradient we have

$$\frac{\partial\Psi}{\partial x} = \frac{\partial\Psi}{d\theta}\frac{\partial\theta}{\partial x} \quad (4)$$

wher $\frac{\partial\theta}{dx}$ e is the wetness gradient

$\frac{d\Psi}{d\theta}$ is the reciprocal of the specific water capacity

$$\text{i.e.,}\quad C(\theta) = \frac{d\theta}{d\Psi} \quad (5)$$

and

$$D = K\frac{d\Psi}{d\theta} \quad (6)$$

Where D is soil water diffusivity (cm^2s^{-1})
Introducing Eq. 6 into Eq. 3 gives

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial\theta}{\partial x}\right) \quad (7)$$

Aribisala[9] developed a green element formulation for vertical flow. The formulation is given as.

$$R_{ij}\ \theta_j + (L_{ij} - V_{in}j\ \phi_n)\ \varphi_j + V_{inj}\ \psi_j k_n + U_{inj}\ \Psi_n \frac{D\theta_j}{d_t} = 0; \qquad i,n,j = 1, 2 \quad (8)$$

Where

$$R_{ij} = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} = (-1)^{i+j-1} \quad (9)$$

$$L_{ij} = \begin{pmatrix} K_m & -(K_m + L) \\ (K_m + L) & -K_m \end{pmatrix} \quad (10)$$

L is the domain length
K_m is the longest length of the spatial element.

$$V_{inj} = \int_{z_1}^{z_2} (|Z_i - Z| + L_m)\frac{(\Omega j\Omega n)}{Ls\varsigma}ds; \quad i,j,n=1,2 \quad (11)$$

Where
Z is depth (cm)
L_m is highest length of spatial element = km

$$\Omega_1 = 1 - \varsigma,\ \Omega_2 = \varsigma \quad (12)$$

$$\varsigma\ \frac{Z\text{-}Z_1}{L}\text{(Local coordinate system whose origin is at } Z_i\text{)} \quad (13)$$

$$U_{inj} = \int_{z_1}^{z_2} (|Z_i - Z| + L_m)\ (\Omega j\Omega n)ds; \quad i,j,n=1,2 \quad (14)$$

$$\Psi = \frac{d\theta}{dz}$$

$$\phi = \text{InD}$$
$$\Psi = 1/D$$
$$\varphi = \frac{d\theta}{dz}$$

Following the procedure for the development of Eq. 8, the discretized form of the Richard equation for horizontal flow is

$$R_{ij}\ \theta_j + (L_{ij}\text{-}V_{inj}\ \phi_n)\ \varphi_j + U_{inj}\ \Psi_n\ \frac{d\theta}{dt} = 0; \qquad i,n,j = 1,2 \quad (15)$$

The difference between Eq. 8 and 15 is the term carrying the gravity i.e $V_{inj}\ \mathbf{\Psi}_j\ K_n$

MODEL IMPLEMENTATION

A difference approximation Eq. 16 was employed to solve the temporal derivative in Eq. 15 i.e.,

$$\left.\frac{d\theta_j}{d_t}\right|_t t_m + \alpha\Delta t = \frac{\theta j^{(m+1)} - \theta j^{(m)}}{\Delta t},\ 0 \le \alpha \le 1 \quad (16)$$

where t_m is the previous time level
$\Delta t = t_{m+1} - t_m$ is the time step
α = difference weighing factor with value varying between 0 and 1

A fully implicit scheme of $\alpha = 1.0$ was adopted. Combining Eq. 15 and 16 gives

$$\varphi_j^{(m+1,K+1)} = -[(1-\alpha)\,R_{ij} - U_{inj}\left[\frac{\alpha\Psi_n^{m+1,K} + (1-\alpha)\,\Psi_n^m}{\Delta t}\right]\theta_j^m$$

$$-(1-\alpha)[L_{ij} - V_{inj}\,\varphi_n^m]\,\varphi_j^m, \quad i,n.j = 1,2 \qquad (17)$$

$$0 \le 1\ \alpha \le 1$$

To achieve the desired accuracy, convergence tolerance was prescribed as

$$|X^{(k+1)} - X^{(K)}| \le \epsilon \qquad (18)$$

where ϵ is a predetermined small value or convergence tolerance. k and k+1 denote the previous and current iteration levels respectively.

Infiltration into fine textured soils (yolo light clay): The constitutive soil relationships for Yolo light are given by Haverkamp[11] as.

$$K = K_s \frac{A}{A + |\Psi|^{B.}} \qquad (19)$$

$$\theta = \frac{a(\theta_s - \theta_r)}{A + (|n||\Psi|)^b} + \theta_r \qquad (20)$$

$\theta_s = 0.495$, $\theta_r = 0.124$

$a = 739$; $b = 4$ for $\iota \ge -1$cm

Initial and boundary conditions are

$$t < 0,\ z \ge 0,\ \theta_n = 0.2376\ cm^3cm^{-3} \qquad (21)$$

$$t \ge 0,\ z = 0,\ \theta_o = 0.4950\ cm^3cm^{-3} \qquad (22)$$

The flow domain was discretized into 40 equal linear elements of size 1cm. A time step of 0.001388 h and convergence tolerance $\epsilon = 10^{-7}$ was adopted.

Convergence was achieved within 5 iterations. The result is compared with[9] in Fig. 1.

Infiltration into coarse textured soils (sandy soil): The constitutive soil relationships for sandy soil as given by Haverkamp[11] are

$$K = K_s \frac{A}{A + |\Psi|^{B.}} \qquad (23)$$

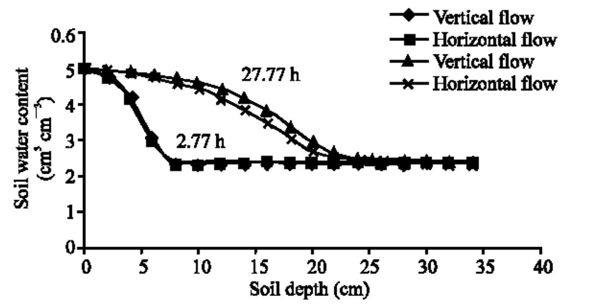

Fig. 1: Comparism of water content profiles in the vertical and horizontal directions in sandy soil

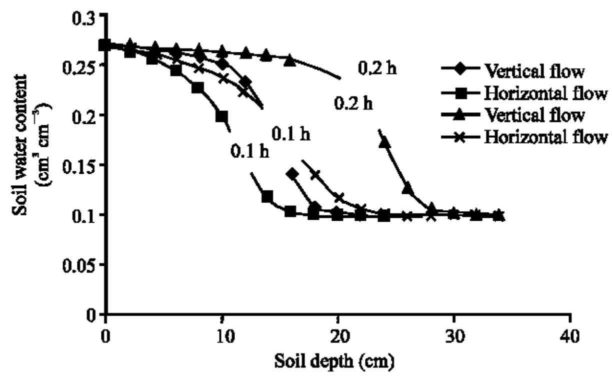

Fig. 2: Comparism of water content profiles in the vertical and horizontal directions in clay soils

$K_s = 34$ cm h^{-1}, $A = 1.175 \times 10^{-6}$, $B = 4.74$

$$\theta = \frac{a(\theta_s - \theta_r)}{a + |\Psi|^{b.}} + \theta_r \qquad (24)$$

$a = 1.61111 \times 10^{-5}$, $\theta_s = 0.287$, $\theta_r = 0.075$, $b = 3.96$

Initial and boundary conditions are

$$t < 0,\ z \ge 0,\ \theta_n = 0.1\ cm^3cm^{-3}$$

$$t \ge 0,\ z = 0,\ \theta_o = 0.267\ cm^3cm^{-3}$$

A soil column discretized into 20 uniform linear elements of size 2cm was simulated using a time step of 0.00277 h and a convergence tolerance of 10^{-6}. Convergence was achieved within 4 iterations. The result is compared with[9] in Fig. 2.

DISCUSSION

The difference between the green element formulation for vertical flow, Eq. 8 and horizontal flow, Eq. 15 is the expression carrying the gravity term i.e $V_{inj}\ \Psi_j K_n$. since the term is an addition in Eq. 8, it implies that for a particular time the amount of infiltrated water in vertical flow will be more than that absorbed in the horizontal direction for the same distance of flow. To confirm this claim, the model for horizontal flow developed in this work was compared with the model for vertical flow that has been validated. The result of the comparisms in Fig. 1 and 2 show more water movement in vertical direction than horizontal direction.

Since clay and sandy soils used as tests cases are extreme soil types, it implies that the model developed in this paper is suitable for furrow irrigation and for all soil types. For clay soils, flow in the vertical and horizontal directions are comparable, whereas there is a marked difference between the horizontal and vertical flows in sandy soils. This implies that gravity force is a great factor when considering flow through sandy soils. This will be of importance when considering materials for earth dams.

CONCLUSION

The non inclusion of the gravity term in the model for horizontal flow affected flow through sandy soils remarkably. Flow through clay soil was affected to a lesser extent by the absence of gravity term. This implies that flow through clay soils either in the horizontal or vertical direction is due mainly to suction gradient. Gravitational gradient is not the driving force in clay soils either in the horizontal or vertical direction. In sandy soils, gravitational gradient predominates in vertical flow while, capillarity predominates in horizontal flow.

The green element method is found to be suitable for modeling field conditions as in initially wet soils in furrow irrigation.

REFERENCES

1. Philip, J.R., 1969a. Theory of Infiltration Advance Hydrosci, 5: 215-290.
2. Parlange, J.Y., 1972. Theory of water movement in soils. 4. Two and Three Dimensional Steady Infiltration. 2: 96-107.
3. Hillel, D., 1982. Introduction to soil physics academic press, Inc. London, pp: 1-364.
4. Taigbenu, A.E., 1990. A More Efficient Implementation of the Boundary Element Theory. Proc. 5th International Conference on Boundary Element Technology (BETECH, 90) New York. Delgwere, pp: 355-366.
5. Taigbenu, A.E., 2004. Green element calculation of non linear heat conduction with a time-dependent fundamental solution, Engineering Analysis with Boundary Element, 28: 52-60.
6. Mangore, E. and A.E. Taigbenu, 2004. Land use Impacts on Quality of Groundwater in Aquifer. Water S.A. 30: 453-464.
7. James, C.S, M.T Kleynhans and A.L. Birkhead, 2004. Predicting Hydro-Ecological Impact in a Seasonal Flood Plain, 5th International Symposium on Ecohydraulics, Madrid, Spain.
8. Taigbenu, A.E., 2001. Unsaturated-Flow Simulation with Green element models, ASCE J. Hydraulic Engineering, 127: 307-312.
9. Aribisala, J.O., 2006. Green Element simulation of water flow in unsaturated soils. J. Enginee. and Appl. Sci., Cairo (In press).
10. Taigbenu, A.E. and O.O. Onyejekwe, 1995. Green element simulations of the transient unsaturated flow equation. Applied Mathematical Modeling 19: 675-684.
11. Haverkamp, R., M. Vauclin, J. Touma, P.J Wierenga and C.A. Vachaud, Comparism of Numerical Simulation models for One-Dimensional Infltration. Soil Sci., Soc. Am. J., 41: 285-294.