Journal of Engineering and Applied Sciences 7 (2): 159-164, 2012
ISSN: 1816-949X

A Biomechanical Approach for the Study of Two-Phase Blood Flow Through Stenosed Artery

Sapna Ratan Shah
Department of Mathematics, Harcourt Butler Technological Institute, 08002 Kanpur, India

Abstract: This biomechanical approach of the study is to investigate the effects of peripheral layer viscosity on physiological characteristics of blood flow through stenosed artery using two-phase model are investigated. The hemodynamics behavior of the blood flow is influenced by the presence of the arterial stenosis. Newtonian and Herschel-Bulkley Fluid Models are considered in the peripheral layer and central layer region, respectively. The governing equations have been solved with help of boundary conditions and results are displayed graphically for different flow characteristics. It is found that the resistance to flow decreases as stenosis shape parameter increases and increases as stenosis length, stenosis size, peripheral layer viscosity increases.

Key words: Herschel-bulkley Fluid Model, Newtonian fluid, stenosis, peripheral layer viscosity, hemodynamics, resistance to flow, India

INTRODUCTION

There are many evidences that vascular fluid dynamics plays a major role in the development and progression of arterial diseases, one of the most widespread diseases in lungs. Arteries are narrowed by the development of stenosis. Stenosis denotes the narrowing of the artery due to the development of arteriosclerosis plaques. The presence of stenosis can lead to serious circulatory disorders. There is strong evidence that hydrodynamic factors such as resistance to flow, wall shear stress and apparent viscosity may play a vital role in the development and the progression of arterial stenosis. Many researchers (Young, 1968; Caro, 1981; Shukla *et al*., 1980) feel that the hydrodynamic factors may be helpful in the diagnosis, treatment and fundamental understanding of many disorders (Clark, 1976) has made experimental studies with different models of stenosis.

However, the models do not account for the size effects due to the suspension of blood cells in plasma. It should be noted that in the case of an advanced stenosis, the size of the artery reduces considerably. In such a case, a Newtonian fluid cannot represent blood because the size effects influence the flow characteristics significantly. With the advent of the fact that rheologic properties and the flow behaviour of blood are of immense importance in the fundamental study of arterial stenosis.

Shukla *et al.* (1980) have studied the effect of stenosis on the resistance to flow through artery by considering the behaviour of blood as a power-law fluid and a Casson fluid. Murata (1998) has proposed a sedimentation model in which he considered constant values of hematocrit and Newtonian viscosity in the circular core region, containing red cell aggregates. A theoretical model for sedimentation of red cell aggregates in narrow horizontal tubes have proposed by Secomb and El-Kareh (1994) in which they modelled the core region as a solid cylinder moving inside the tube.

A little attention (Chaturani and Ponnalagarsamy, 1986; Lee, 1990; Misra *et al*., 1993; Tandon and Rana, 1995) has been made to study the effect of stenosis through tubes with double constriction on physiological fluid flows. The present study describes two fluids model for blood flow through an artery. In this study, the effects of peripheral layer viscosity on physiological characteristics of blood through the artery with mild stenosis have been studied. To study the influence of stenosis shape parameter (m) through an artery in blood flow a suitable geometry is considered such that the axial shape of the stenosis can be changed just by varying a parameter. In this model, the suspension of erythrocytes in the core region is assumed to be non-Newtonian fluid and peripheral plasma layer is treated as Newtonian fluid.

MATERIALS AND METHODS

Analysis of the problem: Consider the axisymmetric flow of blood in a uniform circular tube with an axially non-symmetric but radially symmetric mild stenosis. The geometry of the stenosis as shown in Fig. 1 is assumed to be manifested as:

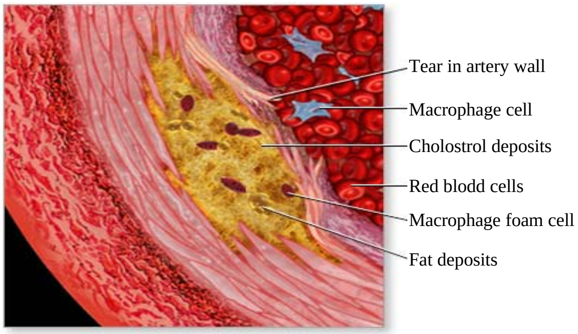

Fig. 1: Antherosclerosis (cut section of artery)

$$\frac{R(z)}{R_0} = 1 - A[L_0^{(m-1)}(z-d) - (z-d)^m], d \le z \le d + L_0 \quad (1)$$
$$= 1, \qquad \text{otherwise}$$

Where:

$R(z)$ and R_0	=	The radius of the capillary with and without stenosis, respectively
L_0	=	The stenosis length
d	=	Indicates its location
$m \ge 2$	=	A parameter determining the stenosis shape and is referred to as shape parameter

Axially, symmetric stenosis occurs when m = 2 and a parameter A is given by:

$$A = \frac{\delta}{R_0 L_0^m} \frac{m^{m/(m-1)}}{(m-1)}$$

Where, δ denotes the maximum height of stenosis at $z = d + L_0/m^{1/(m-1)}$. $\delta/R_0 << 1$. The function $R_1(z)$ representing the shape of the central layer assumed as:

$$\frac{R_1(z)}{R_0} = \alpha, - A_1[L_0^{(m-)}(z-d) - (z-d)^m], d \le z \le d + L_0$$
$$= \alpha, \qquad \text{otherwise} \quad (2)$$

$$A_1 = \frac{\delta_1}{R_0 L_0^m} \frac{m^{m/(m-1)}}{(m-1)}$$

Where, δ_1 denotes the maximum bulging of interface at $z = d + L_0/m^{1/(m-1)}$ due to the presence of stenosis and α is the ratio of the central core radius to the tube radius in the unobstructed region (Fig. 2).

Conservation equation and boundary condition: The equation of motion for laminar and incompressible, steady, fully developed, one-dimensional flow of blood whose viscosity varies along the radial direction in a capillary is:

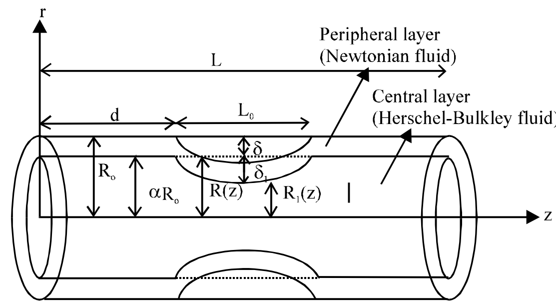

Fig. 2: Geometry of stenosed artery with peripheral layer

$$\left(-\frac{dP}{dz}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left[r\mu\left(\frac{\partial\mu}{\partial r}\right)\right] = 0 \quad (3)$$

where (z, r) are (axial, radial) co-ordinates with z measured along the axis and r measured normal to the axis of the capillary. Following boundary conditions are introduced to solve the Eq. 3:

$$\begin{aligned} &\frac{\partial u}{\partial r} = 0, \text{ at } r = 0, \quad u = 0, \text{ at } r = R(z) \\ &P = P_0, \text{ at } z = 0, \quad P = P_L, \text{ at } z = L \\ &\tau \text{ is finite, at } r = 0 \end{aligned} \quad (4)$$

To see the effect of peripheral layer viscosity on the stenosis shape parameter, resistance to flow, shear stress and apparent viscosity, researchers consider the viscosity function as follows:

$$\begin{aligned} &\mu = \mu_1, \quad 0 \le r \le R_1(z) \\ &\mu = \mu_2, \quad R_1(z) \le r \le R(z) \end{aligned} \quad (5)$$

where μ_1 and μ_2 are the viscosities of the central and the peripheral layers, respectively.

Case 2: Herschel-Bulkley Fluid Model: The stress-strain relation of Herschel-Bulkley fluid is given as:

$$\begin{aligned} &f(\tau) = \left(-\frac{du}{dr}\right) = \frac{1}{\mu}(\tau - \tau_0)^n, \quad \tau \ge \tau_0 \\ &f(\tau) = \left(-\frac{du}{dr}\right) = 0, \quad \tau \le \tau_0 \end{aligned} \quad (6)$$

$$\tau = \left(-\frac{dp}{dz}\frac{r}{2}\right), \quad \tau_0 = \left(-\frac{dp}{dz}\frac{R_c}{2}\right)$$

Where:

μ	=	Herschel-Bulkley viscosity coefficient
τ_0	=	Yield stress
τ	=	Shear stress

R_c = The radius of the plug-flow region
u = The axial velocity along the z direction
n = The flow behavior index

The relation correspond to the vanishing of the velocity gradients in regions in which the shear stress τ is less than the yield stress τ_o, this implies a plug flow wherever $\tau \le \tau_0$ when the shear rates in the fluid are very high, $\tau \ge \tau_o$, the power-law fluid behavior is indicated.

Solution of the problem: The flow flux Q at any cross section is defined as:

$$Q = \int_0^{R(z)} 2\pi r u du = \int_0^{R(z)} \pi r^2 \left(-\frac{du}{dr}\right) dr \quad (7)$$

On using Eq. 3, 6 and boundary condition (Eq. 4), researchers get:

$$\begin{aligned} Q_1 &= -\left(\frac{p}{2\mu}\right)^{1/n} [(r-R_c)^{1/n}] \\ Q_1 &= \int_0^{R_1(z)} \pi r^2 \left(\frac{-du}{dr}\right) dr = \left(\frac{\pi P R_1^4(z)}{8\mu_1}\right) \\ Q_2 &= \int_{R_1(z)}^{R(z)} \pi r^2 \left(\frac{-du}{dr}\right) dr = \frac{\pi P}{8\mu_2}[R^4(z) - R_1^4(z)] \end{aligned} \quad (8)$$

value of $f(\tau)$ from Eq. 1 in Eq. 7,

$$Q = \frac{\pi}{2}\left(\frac{P}{2\mu}\right)^{1/n} \frac{R^{(3+\frac{1}{n})}}{(1+\frac{1}{n})} f(y) \quad (9)$$

Where:

$$f(y) = \begin{bmatrix} 2(1-\frac{R_c}{R})^{((1/n)+1)} - \frac{4}{((1/n)+2)}(1-\frac{R_c}{R})^{((1/n)+2)} + \\ \frac{4}{((1/n)+2)((1/n)+3)}(1-\frac{R_c}{R})^{((1/n)+3)} - \\ ((-1)^{((1/n)+3)}(\frac{R_c}{R}))) \end{bmatrix}$$

$$\bar{y} = \left(\frac{R_c}{R}\right) << 1$$

Using Eq. 8, researchers have:

$$P = \left(-\frac{dp}{dz}\right) = \frac{2\mu}{R^{(1+3n)}}\left(\frac{2Q}{\pi f(\bar{y})}(1+\frac{1}{n})\right)^n \quad (10)$$

to determine λ, Researchers integrate Eq. 11 for the pressure P_L and P_o are the pressure at z = 0 and z = L, respectively where L is the length of the tube:

$$\Delta P = P_L - P_0 = \frac{2\mu}{\pi R_0^{1+3n}}\left(2Q\left(\frac{1}{n}+1\right)\right)^n \int_0^L \frac{dz}{(R(z)/R_0)^{(1+3n)}(f(\bar{y}))^n} \quad (11)$$

The resistance to flow is given by the coefficient λ is defined as follows:

$$\lambda = \frac{(P_L - P_0)}{Q} \quad (12)$$

$$\lambda_0 = \frac{2\mu}{R_0^{1+3n}}\left(\frac{2Q(1+\frac{1}{n})}{\pi}\right)^n (M) \quad (13)$$

$$M = \left(\int_0^d \frac{dZ}{(f_0)^n} + \int_d^{d+L_0} \frac{dZ}{\left(\frac{R(z)}{R_0}\right)^{1+3n}(f(\bar{y}))^n} + \int_{d+L_0}^L \frac{dZ}{(f_0)^n}\right)$$

$$f_0 = \begin{bmatrix} 2(1-\bar{y}_1)^{(1+\frac{1}{n})} - \frac{4}{(1/n+2)}(1-\bar{y}_1)^{(2+\frac{1}{n})} + \\ \frac{4}{(2+\frac{1}{n})(3+\frac{1}{n})}\left((1-\bar{y}_1)^{(3+\frac{1}{n})} - \left((-1)^{(3+\frac{1}{n})}\bar{y}_1\right)\right) \end{bmatrix}$$

where;

$$\bar{y}_1 = \left(\frac{R_c}{R_0}\right)$$

When there is no stenosis in artery then $R = R_0$, the resistance to flow:

$$\lambda_N = \frac{2\mu}{R_0^{1+3n}}\left(\frac{2Q(1+\frac{1}{n})}{\pi}\right)^n \frac{L}{(f_0)^n} \quad (14)$$

From Eq. 12 and 13, the ratio of ($\lambda/\lambda N$) is given as:

$$\lambda = \frac{\lambda_0}{\lambda_N} = 1 - \frac{L_0}{L} + \frac{(f_0)^n}{L}\int_d^{d+L_0} \frac{dz}{(R(z)/R_0)^{1+3n} f(\bar{y})^n} \quad (15)$$

The apparent viscosity (μ_0/μ) is defined as follow:

$$\mu_{app} = \left(\frac{1}{(R(z)/R_o}\right)^{1+3n} f(\bar{y}))$$

$$Q_1 = \int_0^{R_1(z)} \pi r^2 \left(\frac{-du}{dr}\right) dr = \left(\frac{\pi P R_1^4(z)}{8\mu_1}\right) \quad (16)$$

$$Q_2 = \int_{R_1(z)}^{R(z)} \pi r^2 \left(-\frac{du}{dr}\right) dr = \frac{\pi P}{8\mu_2}[R^4(z) - R_1^4(z)] \quad (17)$$

The total flux, Q is: $Q = Q_1+Q_2$ and Q is written as:

$$Q = \frac{\pi}{8}\frac{P}{\mu_2}[R^4(z) - (1-\pi)R_1^4(z)] \quad (18)$$

Where:

$$\mu = \frac{\mu_2}{\mu_1}$$

From Eq. 18, the pressure gradient is written as follows:

$$P = (8\mu_2 Q / \pi[R^4(z) - (1-\mu)R_1^4(z)] \quad (19)$$

To determine λ, researcher integrate Eq. 19 for the pressure P_L and P_0 which are the pressures at z = 0 and z = L, respectively where L is the length of the tube. The resistance to flow is defined as follows:

$$\lambda_0 = \left(\frac{P_L - P_0}{Q}\right) \quad (20)$$

Let λ_N is the resistance to flow for Newtonian fluid with no stenosis then:

$$\lambda_N = (8\mu_1 L / \pi R_0^4) \quad (21)$$

From Eq. 20 and 21, researcher have:

$$\lambda = \frac{\lambda_0}{\lambda_N} = 1 - \frac{L_0}{L} + \frac{1-(1-\mu)\alpha^4)}{L}\int_d^{d+L_0}(dz/[(\frac{R(z)}{R_0})^4 - (1-\mu)(\frac{R_1(z)}{R_0})^4]) \quad (22)$$

Equation 20 can be rewritten as:

$$Q = (\pi P R^4 / 8\mu_{app})$$

where, μ_{app} is the apparent total tube flow viscosity given by:

$$\mu_{app} = \frac{\mu}{[1-(1-\mu)\alpha^4]}\frac{1}{(R(z)/R_0)^4} \quad (23)$$

The shearing stress at the maximum height of the stenosis can be written as:

$$\tau_s = \left(4\mu_2 Q\left(1-\frac{\delta}{R_0}\right)\pi R_0^3\left[\left(1-\frac{\delta}{R_0}\right)^4\right] - (1-\mu)\left(\alpha - \frac{\delta}{R_0}\right)^4\right) \quad (24)$$

and the shear stress for Newtonian fluid with no stenosis is as:

$$\tau_N = \left(\frac{4\mu_1 Q}{\pi R_0^3}\right) \quad (25)$$

Now the ratio of shearing stresses at the wall can be written as:

$$\tau = \left(\frac{\tau_S}{\tau_N}\right) = \left(\frac{\mu}{\left[1-(1-\mu)\alpha^4\right]}\left(1-\frac{\delta}{R_0}\right)^3\right) \quad (26)$$

RESULTS AND DISCUSSION

The model shown before contributes to the fact that blood possesses an inbuilt mechanics of reducing drag due to the presence of peripheral layer. Therefore, incorporation of a cell free layer of plasma and a central core of thickly concentrated suspension of cells with higher viscosity ($\mu_2>\mu_1$) describes the simplest representation of blood in small diameter vessels. The results obtained in this study consist of the expression for resistance to flow (λ) in Eq. 22, expression for apparent viscosity (μ_{app}) in Eq. 23 and expression for shear stress in Eq. 26 and displayed graphically. Fig. 3 and 4 shows the variation of resistance to flow with stenosis size, stenosis length, stenosis shape parameter and peripheral layer viscosity. It is observed from Fig. 3-6 that the resistance to flow decreases as stenosis shape parameter increases while it increases as stenosis size and peripheral layer viscosity increases. A slight change in the stenosis size (radius of the artery) brings about a noticeable change in the resistance to flow (Lerche, 2009). It is found by Chakravarty and Mandal (2001) that the peripheral layer viscosity of blood in diabetic patients is higher than in

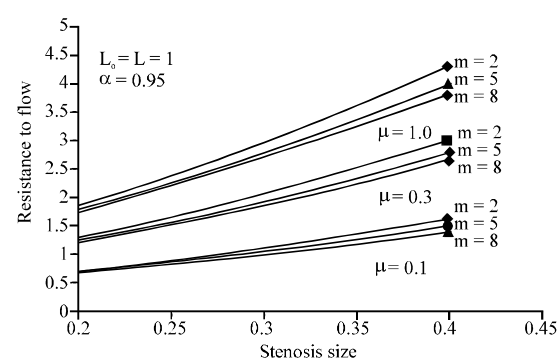

Fig. 3: Variation of resistance to flow with stenosis size for different values of stenosis shape parameter

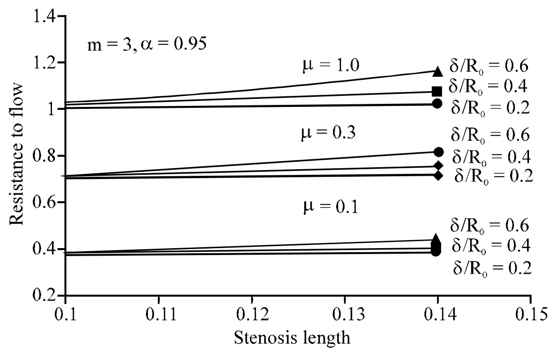

Fig. 4: Variation of resistance to flow with stenosis length for different values of stenosis ize

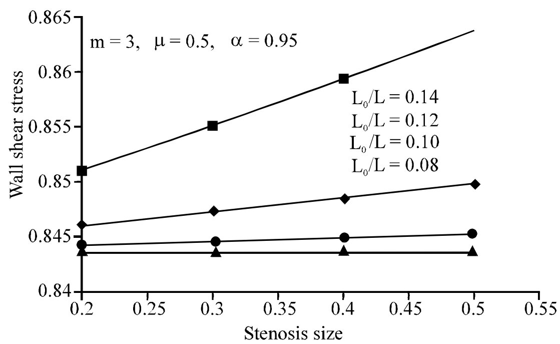

Fig. 5: Variation of wall shear stress with stenosis size for different values of stenosis length

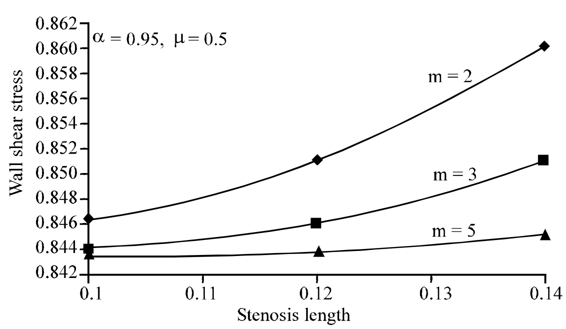

Fig. 6: Variation of wall shear stress with stenosis length for different values of stenosis shape parameter

non-diabetic patients, resulting higher resistance to blood flow. Thus, diabetic patients with higher peripheral layer viscosity are more prone to high blood pressure. Therefore, the resistance to blood flow in case of diabetic patients may be reduced by reducing viscosity of the plasma.

This can be done by injecting saline water to such patients the process is called dilution in medical terms. Figure 5 and 6 consist the results for wall shear stress for different values of stenosis size and stenosis length,

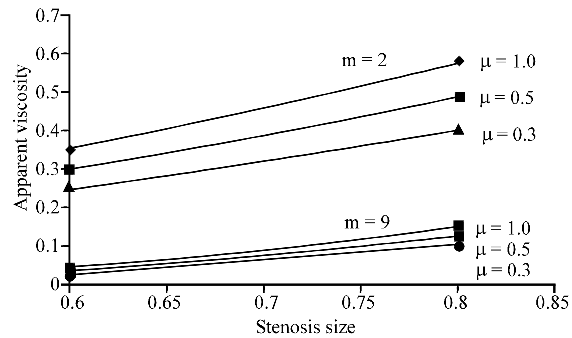

Fig. 7: Variation of apparent viscosity with stenosis size different values of peripheral layer viscosity

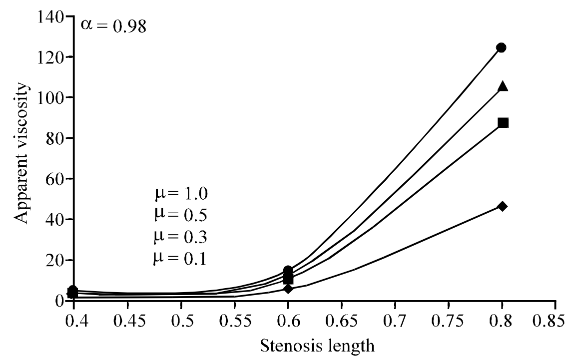

Fig. 8: Variation of apparent viscosity with stenosis length for values of peripheral layer viscosity

stenosis shape parameter and peripheral layer viscosity. It is observed from Fig. 5 and 6 that the wall shear stress decreases as stenosis shape parameter increases but in the case of increasing stenosis size, stenosis length and peripheral layer viscosity wall shear stress is increasing. Figure 7 and 8 highlighted the results for apparent viscosity with the variation of stenosis size, stenosis length, stenosis shape parameter and peripheral layer viscosity. Figure 7 and 8 shows that apparent viscosity increases as stenosis size, stenosis length and peripheral layer viscosity increases. It has also been seen from the graphs that the apparent viscosity decreases as shape parameter increases. These results are qualitative agreement with the observation of Lerche (2009) and Sankar and Hemalatha (2006). In normal human artery, apparent viscosity is found to decrease with the artery radius and is called Fahraeus-Lindquist effect. One may conclude that peripheral layer viscosity plays an important role in lowering the resistance to flow and wall shear stress along the increasing stenosis thickness. In medical practice, several medicines are prescribed to lower the plasma viscosity and by injecting saline water intra-venously.

CONCLUSION

The effect of peripheral layer viscosity on the blood flow in the presence of mild stenosis in the lumen of the artery has been investigated by using Power Law Fluid Model. It has concluded that the resistance to flow, apparent viscosity and wall shear stress have been found to increases with viscosity of peripheral layer but the same are not found to increase as the shape of stenosis increases. The model predicts increase in wall shear stress with peripheral layer viscosity. Predicted trends are found to exist in artery and hence, validate the model. More experimental results are required for further development from clinical point of view.

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