Chaotic Behaviours in Dynamical Systems with Application to Secure Communication

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Abstract: Attention has recently been focused on irregular aspects of the behaviour of dynamical systems. One of such behavioural modes is chaos which has been widely reported as having a great potential in communications. This study aimed at demonstrating the major concepts in non-linear dynamics and chaos and the application of these properties in communication. The behavioural modes of logistic map which is a simple dynamical system were investigated with different values of system parameters and initial conditions by computer simulation using Microsoft Excel. The simulation results indicated the sensitivities of the logistic map to parameter changes as well as variations in the initial conditions. The noise-like chaotic trajectory obtained for certain parameter value was employed in masking a message signal. The resulting signal confirmed the potential application of chaos in secure communication.

Key words: Dynamical systems, chaos, logistic map, simulations, secure communication, Ibadan

INTRODUCTION

A dynamical system is a set of functions that specify how variables change over time (Clayton, 1997). It is a non-linear system which may be described by a differential equation or a map (difference equation) (Salam et al., 1983; Drazin, 1992). Dynamical systems can exhibit a great variety of behavioural modes some of which are of complex nature (Freire et al., 1984). Over the years, there has been lots of analysis of the regular motion of dynamical systems like the solar system. Attention has recently been focused on irregular aspects of the behaviour. Indeed, most dynamical systems such as weather are highly irregular and notoriously difficult to predict (Harrel II, 1992).

Chaos refers to the unpredictable, seemingly random, motion of trajectories of a dynamical system which was first observed in the two-dimensional Van Der Pol equation with a forcing term (Salam et al., 1983). Chaos is mathematically defined as randomness generated by a simple deterministic system (Pecora and Carroll, 1990). This randomness is a result of the sensitivity of chaotic system to initial conditions. If two identical chaotic systems have slightly different initial conditions, they will diverge from each other to give uncorrelated outputs. However because the systems are deterministic, chaos sometimes has some orderliness inherent in it. Two identical chaotic systems driven by the same signal will produce the same output even though, the output is unpredictable. Thus, chaos can essentially be thought of as deterministic noise. Chaotic dynamics have interested chemical engineers with respect to fluids applications and biotechnology engineers as regards periodic beat patterns in a rabbit heart (Liao and Huang, 1997). However, chaos, most of the time has undesirable effect in mechanical and electrical engineering design and must be averted. Certain effects blamed on noise in electrical systems are examples of chaotic behaviour of a completely deterministic nature (Freire et al., 1984). The investigation of the synchronization of chaotic systems began only recently but it already has some fruitful return, especially in the areas of communication and signal processing (Chen, 1996). The synchronization problem consists of making two systems oscillate in a synchronized manner.

Theory: A measure of behaviour of a dynamical system as a function of time is often referred to as time series. An attractor is the status that the system eventually settles down to. It is a set of values in the phase space to which a system migrates over time or iterations. A phase space or state space is an abstract space used to represent the behaviours of a system and its dimensions are the variables of the system. It is essentially a graph in which each axis is associated with one dynamic variable. When a system settles down to a point, the attractor is called a one-point attractor (Clayton, 1997). When a system settles down to alternating between two points, its attractor is referred to as a two-point attractor. The change from a one-point attractor to a two-point attractor is referred to as a bifurcation or period doubling. Bifurcation is a qualitative change in the behaviour (attractor) of a dynamical system associated with a change in a control.

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parameter (Drazin, 1992; Clayton, 1997). A dynamical system having an infinite-point attractor often called a strange attractor is known as Chaotic System. The Logistic Map or Logistic Difference Equation is a model often used to introduce chaos (Clayton, 1997). Although it is simple, it displays the major chaotic concepts. As an example of the logistic map, consider a Limited Growth (Verhulst) Model which was used in 1845 by Verhulst to predict American population from census data (Harrel II, 1992). This model is expressed as (Clayton, 1997):

\[ x_{n+1} = rx_n[1 - x_n] \]  

(1)

Where:

\( x_{n+1} = \) The new population  
\( x_n = \) The old population  
\( r = \) The rate of population growth  
\( n = \) The time interval

In Eq. 1, \( x_{n+1}, x_n \) is the variables while \( r \) is the parameter. Chaotic attractors being noise-like in appearance can be used to hide information from an unwanted receiver. To recover the information, it is necessary to have another copy of the chaotic system with exact parameters which are used in the transmitter. The chaotic system at the receiver end must synchronize with that at the transmitter end if the exact information transmitted is to be recovered at the receiver end. In general, two periodic systems are referred to as being synchronized if either their phases or frequencies are locked. For Chaotic System, however the notion of frequency and phase are in general not well defined and can thus not be used in characterizing synchronization (Parlitz et al., 1999). A slight variation in the initial conditions causes identical chaotic systems to diverge and fall out of synchronism with each other. Also, any slight change in the parameters results in a change in the trajectory and this will affect synchronization between the transmitter and the receiver and it then becomes impossible to recover the information at the receiver end. Various synchronization techniques have been widely reported in the literature (Pecora and Carroll, 1990; Yang, 2004; Dedieu et al., 1993; Parlitz et al., 1999).

**MATERIALS AND METHODS**

To observe the various behavioural modes of the logistic map, the function is simulated Microsoft Excel using different values of parameter, \( r \) and initial conditions. The time series and phase portraits were obtained by plotting \( x_{n+1} \) against \( n \) and \( x_{n+1} \) against \( x_n \), respectively. To demonstrate the potential application of chaos in communication, a chaotic signal was added to a message signal (a pure sinusoid) to hide the information from unwanted receiver.

**RESULTS AND DISCUSSION**

Plotting \( x_{n+1} \) against \( n \) for \( 0 < r < 1 \) and \( x_0 = 0.5 \) gave the time series graph shown in Fig. 1a, b is a phase portrait for \( r = 0.5 \). Figure 2-5 shows the time series and phase portraits for \( x_0 = 0.5, 1 < r < 3, x_0 = 0.5, r = 3.2, x_0 = 0.5, r = 3.54 \) and \( x_0 = 0.5, r = 3.99 \), respectively.
Fig. 3: For \( r = 3.2 \), a) triangular wave oscillator and b) bifurcation or period doubling.

Fig. 4: Time series and phase portrait for \( r = 3.54, x_i = 0.5 \); a) Time series and b) Phase portrait.

Fig. 5: At \( r = 3.99 \); a) Strange attractor and b) Chaos.

Fig. 6: Time series for \( r = 3.99, x_i = 0.3 \) and \( r = 3.99, x_i = 0.301 \).

The time series in Fig. 6 showed entirely different trajectories for initial conditions \( x_i = 0.3 \) and \( x_i = 0.301 \). The trajectory of the signal was obtained by adding the message signal in Fig. 7 and the chaotic signal in Fig. 8. The graph in Fig. 1 in which \( x_{out} \) goes towards zero as \( n \) increases is usually referred to as a one-point attractor. The system shown in Fig. 1 behaves like an RC circuit where the curve represents an exponentially decaying current or an RL circuit where the curve represents an exponentially decaying voltage. Figure 2 shows that regardless of the starting value, the system displays non-zero one-point attractors. The system illustrated in Fig. 2 behaves like an amplifier with almost constant gain. For \( r = 3.2 \) as shown in Fig. 3, the system settles down to alternating between two points. This is a two-point attractor which is also a periodic attractor.
the system has an unpredictable trajectory indicating that
ds system has an unpredictable trajectory indicating that
chaos has set in. While the behaviours of the systems shown in Fig. 1-4 are predictable because they all have a
finite number of attractors, the system shown in Fig. 5 has an infinite number of attractors and so it is chaotic and its
behaviour is unpredictable.

This system would produce an unpredictable output
no matter the input signal. Another important feature of a
Chaotic System is its sensitivity to initial conditions (Clayton, 1997). Any two identical chaotic systems
having slightly different initial states will diverge from
each other. Consider the time series for \( r = 3.99; x_i = 0.3 \)
and \( r = 3.99; x_i = 0.301 \) and using the first 40 iterations
as shown in Fig. 6. It is observed from Fig. 6 that even a
small change in the initial condition as low as 0.001 causes
the time series to diverge from one another. The chaotic
signal effectively concealed the message signal as shown in
Fig. 9 from unwanted receiver, demonstrating the
application of chaos in secure communication. Although,
the dynamical system modelled in this research is abstract
in nature, it is not impossible to build a practical system
that has the properties of the system. Thus, a Chaotic
System may be used to mask an information signal such
that an unwanted interceptor would not be able to make
any meaning of it. The information is therefore made
secure by employing chaos (Kennedy et al., 2000).
Practical chaos-based secure communication systems
make use of higher dimension systems such as the Lorenz
System and the Chua oscillator which are more complex
and hence, provide higher security of information being transmitted.

**CONCLUSION**

The major behavioural modes of a dynamical system
including chaos have been discussed in this research and
illustrated using computer simulations. The simulation
results show the sensitivity of dynamical systems to both
changes in system parameters as well as the initial
conditions. The applications of the different behavioural
modes of the modelled dynamical system were discussed
with a special attention paid to the application of the
chaotic behaviour of the dynamical system in secure
communication.

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