A New Decomposition to Interpret Curves of Schlumberger Survey

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Abstract: In this study, QR decomposition is utilized for 1D interpretation of Schlumberger sounding data instead of former method to optimize the best fit between the calculated and observed apparent resistivity data. The method is implemented by arbitrary parameters of initial model and uses the damped least-square solution with QR and LU decomposition. The method was tested on three synthetic models comparing with the Singular Value Decomposition (SVD) applicability. Noise also was added to two models. Furthermore, the procedure is employed to interpret real data from Iran and the results are compared with published previous geological information. In all cases, QR and LU Decompositions are quicker than SVD method to determine parameters of models.

Key words: Inversion methods, Schlumberger sounding, SVD, QR, LU decomposition

INTRODUCTION

Electrical detection includes the direction of surface effects produced by electric current flow beneath the earth surface. There are various techniques available than in the other detecting procedures where one use single field of force or anomalous property such as gravitation, magnetism, elasticity, radioactivity. By employing electrical methods, one can measure potentials, currents and electromagnetic fields that take place naturally or introduced artificially into the earth. In addition, the measurements can be made in different ways to reach different results. Principally, it is the enormous variation in electrical resistivity found in different rocks and minerals that makes these techniques possible.

All types of resistivity methods use a synthetic current source which is introduced into the earth through point electrodes or long line contacts; the latter arrangement is infrequently used nowadays. The method is for measuring potentials at other electrodes around the current flow. Because the current is measured as well. It is possible to find an effective or apparent resistivity of the geological structures in the ground. In this regard the resistivity technique is prior theoretically to other electrical procedures because quantitative results are obtained by using a controlled source of particular dimensions. In practice, the maximum potentialities of resistivity are never recognized. The tough difficulty is its high sensitivity to small changes in conductivity near surface; in electronic parlance the noise level is high (Karchani et al., 2015).

The explorations carried out by using DC resistivity sounding method can give good output in the places that have horizontal layers with unsubsstantial lateral variation. The method has been used widely on many geophysical exploration problems. The most advantageous surveys have been carried out in hydrogeological, geothermal, environmental and engineering applications (Batyneh et al., 2001; Mota et al., 2004; Agnesi et al., 2005; Ozuranl and Sahin, 2006; Asfahani and Radwan, 2007; Bauer et al., 2006; Park et al., 2007).

The apparent resistivity data do not yield real resistivity values of the subsurface structures. So, to determine the real resistivity distribution of the subsurface, inversion of the calculated apparent resistivity data should be put into practice. DC resistivity inverse problem was scrutinized for the first time in the early 1930s. Over the last decade, depending on high technological development of computer controlled multi-electrode survey systems and the inversion software, much of the studies performed on DC resistivity method have chiefly focused on 2D and 3D data acquisition and interpretation procedures (Nassir et al., 2000; Bauer et al., 2006; Ekinci et al., 2008). However, 1D techniques can be used to interpret data and may give good approximation of the earth subsurface if the investigation is carried out in areas with slightly sloping or flat lying ground and where the resistivity and thickness variations between the adjacent layers are not tangible.

In this study, we present a MATLAB inversion program that uses damped least-squares solution to interpret Schlumberger sounding curves. The reliability of the present program was tested on both noisy and noise-free synthetic data.
MATERIALS AND METHODS

Schlumberger array is widely used because it is not a complicated electrode configuration. The equation that relates the apparent resistivity data and parameters of the layers is an integral expression. This equation is also used for a forward response for Schlumberger electrode array \( \rho_a \) on a ground with homogeneous, isotropic and horizontal layers. This equation is formulated as follows (Koofoed, 1970):

\[
P_a = S_i \int_0^\infty T(\lambda) J_1(\lambda S) \lambda d\lambda
\]

(1)

Where:
\( s \) = The half current electrode spacing
\( J_1 \) = The first order Bessel function of the first kind
\( \lambda \) = The variable of the integral

Also, \( T(\lambda) \) the resistivity transform function is given by the iterative Eq. 2:

\[
T(\lambda) = \left[ \frac{T_{ic}(\lambda) + \rho_i \tanh(\lambda h_i)}{1 + T_{ic}(\lambda) \tanh(\lambda h_i)} \right]
\]

(2)

\( \lambda \) = The wavelength
\( \rho_i, h_i \) = The resistivity and thickness of the \( i \)th layer, respectively

Recent formula is utilized to calculate the forward response for DC resistivity sounding data.

To update iteratively the model parameters, non-linear least-square inversion method is used and in each iteration it uses a correction vector which is the solution of a pack of normal equations. This kind of inversion is an ill-posed problem. So, a small variation in the input data leads to a big variation in the model. Also, by optimizing we can restrict the rudimentary model to be near the true model. The complexity can be reduced by using damping into the set of equations (Roy, 1999). To reduce the DC level of eigenvalues one can add the parameters to the principal diagonal of \( AA^T \) (Raju, 2003). The damped least-squares is as follow (Levenberg, 1944):

\[
\Delta p = (AA^T + \varepsilon^2 I)^{-1} A^T \Delta d
\]

(3)

Where:
\( \Delta p \) = The parameter correction vector
\( A^d \) = The data difference vector
\( A \) = The Jacobian matrix
\( I \) = The identity matrix
\( \varepsilon \) = Damping factor

**SVD decomposition:** Singular Value Decomposition (SVD) is common technique used in many areas of applied sciences including the earth sciences. According to Ekinci and Demirci (2008), it was discussed that SVD can be easily utilized to small scale geophysical problems. It is mathematically robust and numerically stable and also provides other vital information on the state of the model and data thus enabling model resolution and covariance studies (Meju, 1994). However, it must be noted that the use of SVD is not logical for large scale problems. Furthermore, it may has not more precise than others that we will mention it in next section. The large scale problems can be solved by other decompositions or iterative methods. A \( n \times n \) or \( n \times p \) matrix \( A \) can be factored into a product of three other matrices as follows:

\[
A = USV^T
\]

(4)

Where for \( n \) data and \( p \) parameters, \( U \) (\( n \times n \)) and \( V \) (\( p \times p \)) are respectively the data space and parameter space eigenvectors and \( S \) is a \( p \times p \) diagonal matrix containing at most \( r \) non-zero eigenvalues of \( A \) with \( r \leq p \). Suppose that members on main diagonal of \( S \) are presented by \( \lambda_1, \lambda_2, ..., \lambda_r \) which called singular values of \( A \). This factorization is known as SVD of \( A \) (Meju, 1994). If SVD is used in damped least-squares solution, Eq. 4 is obtained:

\[
\Delta p = \left( VS^T V^T + \varepsilon^2 I \right)^{-1} V SU^T \Delta d
\]

(5)

By adding the damping factor to the diagonal elements we obtain:

\[
\left( VS^T V^T + \varepsilon^2 I \right) = V \text{diag} \left[ \lambda_1, \varepsilon + \varepsilon^2 \lambda_2, ..., \varepsilon + \varepsilon^2 \lambda_r \right] V^T
\]

(6)

By inversing the Eq. 6 and substituting it in Eq. 5 is given by as follows:

\[
\Delta p = V \text{diag} \left[ \frac{1}{\lambda_1 + \varepsilon^2}, ..., \frac{1}{\lambda_r + \varepsilon^2} \right] V^T VSU^T \Delta d
\]

(7)

And the parameter correction vector after simplification can be expressed as:

\[
\Delta p = V \text{diag} \left[ \frac{\lambda_1}{\lambda_1 + \varepsilon^2}, ..., \frac{\lambda_r}{\lambda_r + \varepsilon^2} \right] U^T \Delta d
\]

(8)

Equation 8 provides damped least-squares solution with SVD for automated inversion, the common practice is to set the damping factor first to a large positive value. Thus, taking the advantage of the good initial convergence properties of the steepest descent method and thereafter the damping factor is multiplied by a factor.
RESULTS AND DISCUSSION

Synthetic examples: Synthetic models with ideal resistivity distributions of geological relevance were used in the applications. Three different examples consisting of three- or four-layered earth models were preferred. In general as a similar explanation mentioned by (Muirane and Pedersen, 1999), the subsurface models where resistivity increases or decreases with depth are not difficult to resolve. On the other hand, if a resistor layer is embedded between two conductor layers or a conductor layer is embedded between two resistor layers, convergence is more difficult and slow in comparison to first type. We used these types of models for the numerical examples. The synthetic data were generated at a predetermined set of 20 AB/2 values for all models by using of the forward modeling function of the program. The 5% noise level is reasonable for sites with high resistivity contrasts and higher electrode contact resistances (Dahlin, 2001). Thus, models 2 and 3 were contaminated with +5% random noise depending on the magnitude of each data point.

Like (Ekinici and Demirci, 2008) we used homogeneous half spaces for the initial models. The inversion procedure was controlled by two misfit functions in the iteration. When the predetermined error limit was reached the inversion procedure was stopped automatically. The convergences were calculated as an rms error function between the observed and calculated apparent resistivity values at the end of the process as follows:

$$\text{rms} = \left( \sum_{i=1}^{N} \left( d_{\text{obs}} - d_{\text{calc}} \right)^2 \right)^{1/2} / (N)$$ (12)

Where obs stands for observed and cal stands for calculated apparent resistivity values and ND stands for the number of the apparent resistivity data. The inversion results are illustrated as best-fit curves and resistivity-depth distributions for model 1-3 in following.

Model 1: First example demonstrates a noise-free three-layered earth model with decreasing resistivity with depth. In Table 1, although the amounts of errors are equal, the numbers of iterations shows that which method

| Table 1: Observed and calculated apparent resistivity data of the model 1 |
|------------------------|------------------------|------------------------|------------------------|
| Methods | SVD | LU | QR |
| Real values | Calculated | Calculated | Calculated |
| $\phi_1 = 100$ | $\text{Calc}_p = 99.97$ | $\phi_1 = 99.97$ | $\phi_1 = 99.97$ |
| $\phi_2 = 50$ | $\text{Calc}_p = 50.09$ | $\phi_2 = 50.09$ | $\phi_2 = 50.11$ |
| $\phi_3 = 20$ | $\text{Calc}_p = 19.97$ | $\phi_3 = 19.97$ | $\phi_3 = 19.97$ |
| $d_1 = 5$ | $d_{\text{cal}} = 8.33$ | $d_1 = 8.33$ | $d_1 = 8.33$ |
is the best. Left graph has two figures: observed and calculated apparent resistivity curves (Fig. 1) and resistivity-depth distribution of the initial and final models (Fig. 2). It is obvious that observed data are coincided with calculated in (Fig. 1). In (Fig. 2) we can see although the initial guesses were far removed from the true ones, the model parameters were resolved and the program yielded satisfactory results.

**Model 2:** Second example deals with a three-layered earth model consisting of a resistor layer between two conductors. Like Table 1 and 2 presents the best method in speed not in exactness because errors in all three methods are same.

**Model 3:** A four-layered decreasing resistivity with depth model was used in this example (Fig. 3). Like Table 1-3 presents the best method in speed not in exactness because errors in all three methods are same.

**Real data example:** Harsin sheet is located in the Zagros Orogen zone and consists of various lithologic units of Sanandaj-Sirjan folds and over thrust. The majority of these units is allochthonous and is facially different from adjacent structures. Figure 1 and 2 show a part of the 1/100000 Harsin sheet. The highest point in this area is the Shiraz mount with 2700 height meters from the sea level. The lowest point is in the west of the Mal-Amiri village with 1287 height meters from the sea level. The average of rainfall is 600-1000 mm during the year. From the morphologic view, apart from lime altitudes, other parts of the area are covered by small hills. Rivers such as Gamasab and Grezo and also tectonic activity are significantly responsible for forming the altitudes and plains in the area of exploration. Water drainages in the area show the lineaments of faults. The alluvium consists of fine grained fluvial sediments, pebbles with 10 cm of diameter, limestone and Ophiolite. The average slope in the area is 5%. The locations of electric sounds are exactly on the alluvium. The Northern part of the area consists of crashed limestone. Faults in this formation likely have affected the alluvium. Also, Radiolarite accumulation is located in the North-East and South-East of the area.

The location of exploration area is at 3794412 Northern latitude and 741605 Eastern longitude in WGS84, UTM system. Thirty five sounds were implemented.
These sounds are on four profiles with North-South directions. The distance between two adjacent sounds is 50 m on profiles 1-3 and is 100 m on profile 4. Also, the distance between adjacent profiles is 100 m (Fig. 4). The data is collected by Schlumberger array. Figure 5 shows the profiles and sounds.

The geographical locations of sounds are given in Table 4. According to apparent resistivity data and Fig. 6 on this profile the value of apparent resistivity increases respect to the depth and reaches to 90 Ωm. Smooth changes in the amount of the apparent resistivity on this profile reveal that tectonic activity has not had significant impression on the characteristic of layers.

We chose H31 sounding curve on profile 3 to run the inversion programs and find the real amounts of resistivity at various depths and thicknesses of layers. Then, the results are compared with one another. We can see the results for recent data below (Fig. 7 and Table 5).

**Real model:** It is worth to note that, in addition to the fact the number of iteration considerably has been decreased in our method, in this sounding curve interpretation; the error has been diminished as well though its amount may be tiny.
CPU Time Comparison

<table>
<thead>
<tr>
<th>Methods</th>
<th>SVD</th>
<th>QR</th>
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<tr>
<td>Model 1</td>
<td>11.81</td>
<td>4.29</td>
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Fig. 8: A comparison between the time needed to calculate the parameters by SVD and QR methods

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REFERENCES


